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THE GEOMETRIC MEAN CRITERION CONTINUED

Henry A. LATANÉ

University of North Carolina, Chapel Hill, NC 27514, USA

Both the portfolio manager and the entrepreneur are faced with repeated non-diversifiable choices with cumulative effects. This note deals with the hierarchy of objectives, goals and criteria which must be used as guides in making such choices.

An *objective* can be reached only in the future. It cannot be used as a basis for choosing among strategies with uncertain outcomes since what strategy will lead to achievement of the objective depends on future events. For example, consider the gambler who has the option to bet \$1 on the toss of a fair coin. In the event of heads he will have \$2 if he bets and \$1 if he does not bet. In the event of tails he will have 0 if he bets and \$1 if he does not bet. The mere fact that this gambler wishes to maximize his money on hand at the end of the toss (his objective) does not give him a rational basis for deciding whether or not to bet.

Since the objective by itself cannot be used as the basis for choosing among strategies with uncertain outcomes *a goal* which can be reached at the time of making the choice is necessary. A goal is necessary whenever the outcome of the choice is uncertain, whether the maximand is expressed in terms of subjective utility or of an objective measure of value. The decision maker who adopts a goal does not forego his objective. He merely chooses the goal as the best available landmark on the road to his objective. They are landmarks which can surely be reached by the decision maker who is confronted with a filled-in matrix which shows the probability of each relevant future occurrence and all combined effects of strategies and future occurrences.

It seems reasonable to assume that the wealth-holder wishes to maximize realized terminal wealth. This is equivalent to realized growth rate and also to realized terminal utility if we accept the premise that more wealth is preferred to less. These three equivalent objectives lead to three sets of *goals and criteria* to be maximized which are not equivalent: (1) expected value, (2) expected utility, and (3) expected growth rate.

The wealth-holder who consistently maximizes expected value will maxi-

mize expected terminal wealth. Expected value is the criterion of classical writers on probability. It also is the goal which led to the first development of utility theory by Daniel Bernoulli because it seemed to him self-evident that a bet which was reasonably priced for a very poor man would be a bargain for a rich and hence that expected value was not a good criterion for either [Latané (1957)].

Expected utility has been the dominant goal in the literature since the publication of *Theory of Games and Economic Behavior* by von Neumann and Morgenstern (1947). They say: 'We have practically defined numerical utility as being that thing for which the calculus of mathematical expectations is legitimate' (p. 28). There is no doubt that if the numerical utilities of the pay-offs are properly weighted and the choice, itself, is utility neutral, the strategy with the maximum expected utility is the proper strategy for a rational man to use. However, it is difficult to identify the underlying utilities and to tell exactly whose utilities are being maximized when institutional decisions are being made. Further, the fact that subjective utilities are intimately related to subjective probabilities makes revealed preferences difficult to identify. As Roy (1952) said many years ago: 'A man who seeks advice about his actions will not be grateful for the suggestion that he maximize expected utility.'

My main interest has been in the third goal – the maximization of the expected growth rate or, what is the same thing, maximization of G, the geometric mean of the probability distribution of returns.

I have never considered G a utility measure except for those who have log utilities. My interest (1957, 1959, 1977, 1978) is in the asymptotic qualities of G and the measurement of the probability of adverse dominance which these qualities make possible when acts are repetitive with cumulative effects. This probability of adverse dominance can be calculated exactly, given the relevant data and the number of repetitions.

It seems to me that this probability of adverse dominance is especially relevant to corporate and other investment decisions and portfolio management where individual subjective utilities of those involved are difficult if not impossible to determine. The cost measured in these probabilities can be calculated for decisions which for utility reasons or otherwise do not maximize the geometric mean. That is, the probabilities permit a direct comparison of the effects of possible courses of action on at least one level. Samuelson (1979) and Ophir (1978, 1979) apparently believe that this information is of no value. So be it. Samuelson (1979) speaks of 'the dead rule'. I believe the announcement of the death is premature.[1]

[1]My position as to the usefulness of G in no sense depends on utility whether bounded or not bounded. Hence I am completely neutral as to the bounds on underlying utility functions. But, unfortunately, these bounds are involved in a note quoting Savage that I included in my 1959

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paper. I understand now that Savage subsequently changed his position on this matter so the note ceases to be relevant.

The origin of this quotation may be of some interest. I met Savage whom I admired greatly while I was a graduate student and he a major expositor of utility theory (1954). He became interested when I pointed out the asymptotic qualities of G and helped me to get my paper published in the Journal of Political Economy. Savage had an opportunity to review the paper before it was published but did not suggest any changes.