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ABSTRACT This paper describes and analyses the history of the fundamental equation of modern financial economics: the Black-Scholes (or Black-Scholes-Merton) option pricing equation. In that history, several themes of potentially general importance are revealed. First, the key mathematical work was not rule-following but bricolage, creative tinkering. Second, it was, however, bricolage guided by the goal of finding a solution to the problem of option pricing analogous to existing exemplary solutions, notably the Capital Asset Pricing Model, which had successfully been applied to stock prices. Third, the central strands of work on option pricing, although all recognizably 'orthodox' economics, were not unitary. There was significant theoretical disagreement amongst the pioneers of option pricing theory; this disagreement, paradoxically, turns out to be a strength of the theory. Fourth, option pricing theory has been performative. Rather than simply describing a pre-existing empirical state of affairs, it altered the world, in general in a way that made itself more true.

Keywords Black-Scholes, bricolage, option pricing, performativity, social studies of finance

An Equation and its Worlds:

Bricolage, Exemplars, Disunity and Performativity in Financial Economics

Donald MacKenzie

Economics and economies are becoming a major focus for social studies of science. Historians of economics such as Philip Mirowski and the small number of sociologists of economics such as Yuval Yonay have been applying ideas from science studies with increasing frequency in the last decade or so.[1] Established science-studies scholars such as Knorr Cetina and newcomers to the field such as Izquierdo, Lépinay, Millo and Muniesa have begun detailed, often ethnographic, work on economic processes, with a particular focus on financial markets.[2] Actor-network theorist Michel Callon has conjoined the two concerns by arguing that an intrinsic link exists between studies of economics and of economies. The economy is not an independent object that economics observes, argues Callon (1998). Rather, the economy is performed by economic practices. Accountancy and marketing are among the more obvious such practices, but, claims Callon, economics in the academic sense plays a vital role in constituting and shaping modern economies.

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This paper contributes to the emergent science-studies literature on economics and economies by way of a historical case study of option† pricing theory (terms marked † are defined in the glossary in Table 1). The theory is a 'crown jewel' of modern economics: 'when judged by its ability to explain the empirical data, option pricing theory is the most successful theory not only in finance, but in all of economics' (Ross, 1987: 332). Over the last three decades, option theory has become a vitally important part of financial practice. As recently as 1970, the market in derivatives† such as options was tiny; indeed, many modern derivatives were illegal. By December 2002, derivatives contracts totaling US$165.6 trillion were outstanding worldwide, a sum equivalent to around US$27,000 for every human being on earth.[3] Because of its centrality to this huge market, the equation that is my focus here, the Black-Scholes option pricing equation, may be 'the most widely used formula, with embedded probabilities, in human history' (Rubinstein, 1994: 772).

The development of option pricing theory is part of a larger transformation of academic finance. Until the 1960s, the study of finance was a marginal, low status activity: largely descriptive in nature, taught in business schools not in economics departments, and with only weak intellectual linkages to economic theory. Since the 1960s, finance has become analytical, theoretical and highly quantitative. Although most academic finance theorists' posts are still in business schools, much of what they teach is now unequivocally part of economics. Five finance theorists – including two of the central figures discussed here, Robert C. Merton and Myron Scholes – have won Nobel prizes in economics.

This intellectual transformation was interwoven with the rapid expansion of business schools in the US. In the mid-1950s, US business schools were producing around 3000 MBAs annually. By the late 1990s, that figure had risen to over 100,000 (Skapiner, 2002). As business schools grew, they also became more professional and 'academic', especially after the influential Ford Foundation report, *Higher Education for Business* (Gordon and Howell, 1959). At the same time, the importance of the finance sector in the US economy grew dramatically, and increasing proportions of financial assets were held not directly by individuals but by organizations such as mutual funds and pension funds. These organizations formed a ready job market for the growing cohorts of students trained in finance.

The transformation of the academic study of finance is the subject of a fine history by Bernstein (1992), and the interactions between this transformation, the evolution of US business schools, and changing capital markets have been analysed ably by Whitley (1986a, 1986b). However, what the existing literature has not done fully is to 'open the black box' of mathematical finance theory. That – at least for the theory of option pricing – is this paper's goal.[4]

Limitation of space means that the focus of this paper is on the mathematics of option pricing theory and on its intellectual context. The interaction between theory and practice – the processes of the adoption by practitioners of option pricing theory, and the consequences of its adoption

– is the subject of a 'sister' paper (MacKenzie and Millo, forthcoming), although the issue of performativity means that the subject-matter of that paper will be revisited briefly below.

In this article, four themes will emerge. I would not describe them as 'findings', because of the limitations on what can be inferred from a single historical case-study, but they may be of general significance. The first theme is bricolage. Creative scientific practice is typically not the following of set rules of method: it is 'particular courses of action with materials at

TABLE 1
Terminology

Arbitrage; arbitrageur	Trading that seeks to profit from price discrepancies; a trader who seeks to do so.
Call	See **option**.
Derivative	An asset, such as a **future** or **option**, the value of which depends on the price of another, 'underlying', asset.
Discount	To calculate the amount by which future payments must be reduced to give their present value.
Expiration	See **option**.
Future	A contract traded as an organized exchange in which one party undertakes to buy, and the other to sell, a set quantity of an asset at a set price on a given future date.
Implied volatility	The **volatility** of a stock or index consistent with the price of **options** on the stock or index.
Log-normal	A variable is log-normally distributed if its natural logarithm follows a normal distribution.
Market maker	In the options market, a market participant who trades on his/her own account, is obliged continuously to quote prices at which he/she will buy and sell options, and is not permitted to execute customer orders.
Option	A contract that gives the right, but not obligation, to buy ('call') or sell ('put') an asset at a given price (the 'strike price') on, or up to, a given future date (the 'expiration').
Put	See **option**.
Riskless rate	The rate of interest paid by a lender who creditors are certain will not default.
Short selling	The sale of a security one does not own, e.g. by borrowing it, selling it, and later repurchasing and returning it.
Strike price	See **option**.
Swap	A contract to exchange two income streams, e.g. fixed-rate and floating-rate interest on the same notional principal sum.
Volatility	The extent of the fluctuations of the price of an asset, conventionally measured by the annualized standard deviation of continuously-compounded returns on the asset.
Warrant	A call **option** issued by a corporation on its own stock. Its exercise typically leads to the creation of new stocks rather than the transfer of ownership of existing stock.

hand' (Lynch, 1985: 5). While this has been documented in overwhelming detail by ethnographic studies of laboratory science, this case-study suggests it may also be the case in a deductive, mathematical science. Economists – at least the particular economists focused on here – are also bricoleurs.[5]

They are not, however, random bricoleurs, and the role of existing exemplary solutions is the second issue to emerge. Ultimately, of course, this is a Kuhnian theme. As is well known, at least two quite distinct meanings of the key term 'paradigm' can be found in Kuhn's work. One – by far the dominant one in how Kuhn's work was taken up by others – is the 'entire constellation of beliefs, values, techniques, and so on shared by the members of a given [scientific] community' (Kuhn, 1970: 175). The second – rightly described by Kuhn as 'philosophically . . . deeper' – is the exemplar, the problem-solution that is accepted as successful and that is creatively drawn upon to solve further problems (Kuhn, 1970: 175; see also Barnes, 1982).

The role of the exemplar will become apparent here in the contrast between the work of Black and Scholes and that of mathematician and arbitrageur[†] Edward O. Thorp. Amongst those who worked on option pricing prior to Black and Scholes, Thorp's work is closest to theirs. However, while Thorp was seeking market inefficiencies to exploit, Black and Scholes were seeking a solution to the problem of option pricing analogous to an existing exemplary solution, the Capital Asset Pricing Model. This was not just a general inspiration: in his detailed mathematical work, Black drew directly on a previous mathematical analysis on which he had worked with the Capital Asset Pricing Model's co-developer, Jack Treynor.

As Peter Galison and others have pointed out, the key shortcoming in the view of the 'paradigm' as 'constellation of beliefs, values, techniques, and so on' is that it overstates the unity and coherence of scientific fields (Galison, 1997; Galison and Stump, 1996). Nowhere is this more true than when outsiders discuss 'orthodox' neoclassical economics, and the nature of economic orthodoxy is the third theme explored here. Black, Scholes, Merton, several of their predecessors, and most of those who in the 1970s subsequently worked on option pricing were all (with some provisos in the case of Black, to be discussed below) recognizably 'orthodox' economists. As others studying different areas of economics have found, however, orthodoxy seems not to be a single unitary doctrine, substantive or methodological (see Mirowski and Hands, 1998; Yonay and Breslau, 2001). For example, Robert C. Merton, the economist whose name is most closely yoked to those of Black and Scholes, did not accept the original version of the Capital Asset Pricing Model, the apparent pivot of their derivation, and Merton reached the Black-Scholes equation by drawing on different intellectual resources. Black, in turn, never found Merton's derivation entirely compelling, and continued to champion the derivation based on the Capital Asset Pricing Model. So *no* entirely unitary 'constellation of beliefs, values, techniques, and so on' can be found.

Economic 'orthodoxy' *is* a reality – attend conferences of economists who feel excluded by it, and one is left in no doubt on that – but it is a reality that should perhaps be construed as a cluster of family resemblances, a cluster that arises from imaginative bricolage drawing on an only partially overlapping set of existing exemplary solutions. 'Orthodox' economics is an 'epistemic culture' (Knorr Cetina, 1999), not a catechism.

A major aspect of Galison's critique of the Kuhnian paradigm (conceived as all-embracing 'constellation') is his argument that diversity is a source of robustness, not a weakness. Though Galison's topic is physics, his conclusion also appears to hold true of economics. Philip Mirowski and Wade Hands, describing the emergence of modern economic orthodoxy in the postwar US, put the point as follows:

> Rather than saying it [neoclassicism] simply chased out the competition – which it did, if by 'competition' one means the institutionalists, Marxists, and Austrians – and replaced diversity with a single monolithic homogeneous neoclassical strain, we say it transformed itself into a more robust ensemble. Neoclassical demand theory gained hegemony by going from patches of monoculture in the interwar period to an interlocking competitive ecosystem after World War II. Rather than presenting itself as a single, brittle, theoretical strand, neoclassicism offered a more flexible, and thus resilient skein. (Mirowski and Hands, 1998: 289; see also Sent, forthcoming)

As we shall see, that general characterization appears to hold for the particular case of option pricing theory.

The final theme explored here, and in the sister paper referred to above (MacKenzie and Millo, forthcoming), is performativity. As we shall see, there is at least qualified support here for Callon's conjecture, albeit in a case that is favourable to the conjecture, since option pricing theory was chosen for examination in part because it seemed a plausible case of performativity. Option pricing theory seems to have been performative in a strong sense: it did not simply describe a pre-existing world, but helped create a world of which the theory was a truer reflection.

It is of course not surprising that a social science like finance theory has the potential to alter its objects of study: the more difficult issue, which fortunately does not need to be breached here, is to specify accurately the non-trivial ways in which natural sciences are performative (see Hacking, 1992a, and from a different viewpoint, Bloor, 2003). That a social science like psychology, for example, has a 'necessarily reflexive character' and that psychologists influence as well as describe 'the psychological lives of their host societies' has been argued by Richards (1997: xii), and Ian Hacking's work (such as Hacking, 1992b and 1995a) also demonstrates the point. As I have argued elsewhere (MacKenzie, 2001), finance is a domain of what Barnes (1983) calls 'social-kind' terms or what Hacking (1995b) calls 'human kinds', with their two-way 'looping effects' between knowledge and its objects.

It is clearly possible in principle, in other words, for finance theory to be performative rather than simply descriptive. However, that does not remove the need for empirical examination. That the theory *can* be performative does not imply that it *has* been performative. Indeed, as we shall see, the performativity of classic option pricing theory is incomplete and historically specific – it did not make itself wholly or permanently true – and exploring the limits and the contingency of its performativity is of some interest.

'Too Much on Finance!'

Options are old instruments, but until the 1970s age had not brought them respectability. Puts† and calls† on the stock of the Dutch East India Company were being bought and sold in Amsterdam when de la Vega discussed its stock market in 1688 (de la Vega, 1957), and subsequently options were widely traded in Paris, London, New York and other financial centres. They frequently came under suspicion, however, as vehicles for speculation. Because the cost of an option was typically much less than that of the underlying stock, a speculator who correctly anticipated price rises could profit considerably by buying calls, or benefit from falls by buying puts, and such speculation was often regarded as manipulative and/or destabilizing. Buying options was often seen simply as gambling, as betting on stock price movements. In Britain, options were banned from 1734 and again from 1834, and in France from 1806, although these bans were widely flouted (Michie, 1999: 22, 49; Preda, 2001: 214). Several American states, beginning with Illinois in 1874, also outlawed options (Kruizenga, 1956). Although the main target in the USA was options on agricultural commodities, options on securities were often banned as well.

Options' dubious reputation did not prevent serious interest in them. In 1877, for example, the London broker Charles Castelli, who had been 'repeatedly called upon to explain the various processes' involved in buying and selling options, published a booklet explaining them, directed apparently at his fellow market professionals rather than popular investors. He concentrated primarily on the profits that could be made by the purchaser, and discussed only in passing how options were priced, noting that prices tended to rise in periods of what we would now call high volatility.† His booklet ended – in a nice corrective for those who believe the late 20th century's financial globalization to be a novelty – with an example of how options had been used in bond arbitrage† between the London Stock Exchange and the Constantinople Bourse to capture the high contango[6] rate prevailing in Constantinople in 1874 (Castelli, 1877: 2, 7–8, 74–77).

Castelli's 'how to' guide employed only simple arithmetic. Far more sophisticated mathematically was the thesis submitted to the Sorbonne in March 1900 by Louis Bachelier, a student of the leading French mathematician and mathematical physicist, Henri Poincaré. Bachelier sought 'to establish the law of probability of price changes consistent with the market' in French bonds. He assumed that the price of a bond, x, followed what we would now call a stochastic process in continuous time: in any time interval, however short, the value of x changed probabilistically. Bachelier constructed an integral equation that a continuous-time stochastic process had to satisfy. Denoting by $p_{x,t}\,dx$ the probability that the price of the bond at time t would be between x and $x + dx$, Bachelier showed that the integral equation was satisfied by:

$$\mathrm{p_{x,t}} = \frac{H}{\sqrt{t}} \exp - (\pi \mathrm{H}^2 \mathrm{x}^2/\mathrm{t})$$

where H was a constant. (For the reader's convenience, notation used throughout this article is gathered together in Table 2.) For a given value of t, the expression reduces to the normal or Gaussian distribution, the familiar 'bell-shaped' curve of statistical theory. Although Bachelier had not demonstrated that the expression was the only solution of the integral equation (and we now know it is not), he claimed that '[e]vidently the probability is governed by the Gaussian law, already famous in the calculus of probabilities'. He went on to apply this stochastic process model – which we would now call a 'Brownian motion' because the same process was later used by physicists as a model of the path followed by a minute particle subject to random collisions – to various problems in the determination of the strike† price of options, the probability of their exercise and the probability of their profitability, showing a reasonable fit between predicted and observed values.[7]

When Bachelier's work was 'rediscovered' by Anglo-Saxon authors in the 1950s, it was regarded as a stunning anticipation both of the modern theory of continuous-time stochastic processes and of late 20th-century finance theory. For example, the translator of his thesis, option theorist A. James Boness, noted that Bachelier's model anticipated Einstein's stochastic analysis of Brownian motion (Bachelier, 1964: 77). Bachelier's contemporaries, however, were less impressed. While modern accounts of the neglect of his work are overstated (Jovanovic, 2003), the modesty of Bachelier's career in mathematics – he was 57 before he achieved a full

TABLE 2
Main Notation

β	the covariance of the price of an asset with the general level of the market, divided by the variance of the market
c	strike† price of option
ln	natural logarithm
N	the (cumulative) normal or Gaussian distribution function
r	riskless† rate of interest
σ	the volatility† of the stock price
t	time
w	warrant or option price
x	stock price
x^*	stock price at expiration† of option

For items marked † see the glossary in Table 1.

professorship, at Besançon rather than in Paris – seems due in part to his peers' doubts about his rigour and their lack of interest in his subject matter, the financial markets. 'Too much on finance!' was the private comment on Bachelier's thesis by the leading French probability theorist, Paul Lévy (quoted in Courtault et al., 2000: 346).

Option and Warrant Pricing in the 1950s and 1960s

The continuous-time random walk, or Brownian motion, model of stock market prices became prominent in economics only from the late 1950s onwards, and did so, furthermore, with an important technical modification, introduced to finance by Paul Samuelson, MIT's renowned mathematical economist, and independently by statistical astronomer M.F.M. Osborne (1959). On Bachelier's model, there was a non-zero probability of prices becoming negative. When Samuelson, for example, learned of Bachelier's model, 'I knew immediately that couldn't be right for finance because it didn't respect limited liability' [Samuelson interview]:[8] a stock price could not become negative. So Samuelson and Osborne assumed not Bachelier's 'arithmetic' Brownian motion, but a 'geometric' Brownian motion, or log-normal[†] random walk, in which prices could not become negative.

In the late 1950s' and 1960s' US the random-walk model became a key aspect of what became known as the 'efficient market hypothesis' (Fama, 1970). Though it initially struck many non-academic practitioners as bizarre to posit that stock price movements were random, the growing number of financial economists argued that all today's information is already incorporated in today's prices: if it is knowable that the price of a stock will rise tomorrow, it would already have risen today. Stock price changes are influenced only by *new* information, which, by virtue of being new, is unpredictable or 'random'.[9] Like Bachelier, a number of these financial economists saw the possibility of drawing on the random walk model to study option pricing. Typically, they studied not the prices of options in general but those of warrants.[†] Options had nearly been banned in the US after the Great Crash of 1929 (Filer, 1959), and were traded only in a small, illiquid, ad hoc market based in New York. Researchers could in general obtain only brokers' price quotations from that market, not the actual prices at which options were bought and sold, and the absence of robust price data made options unattractive as an object of study. Warrants, on the other hand, were traded in more liquid, organized markets, particularly the American Exchange, and their market prices were available.

To Case Sprenkle, a graduate student in economics at Yale University in the late 1950s, warrant prices were interesting because of what they might reveal about investors' attitudes to and expectations about risk levels (Sprenkle, 1961). Let x^* be the price of a stock on the expiration[†] date of a warrant. A warrant is a form of call option: it gives the right to purchase the underlying stock at strike price, c. At expiration, the warrant will therefore be worthless if $x^\star$ is below c, since exercising the warrant would be more expensive than simply buying the stock on the market. If $x^\star$ is higher than c, the warrant will be worth the difference. So its value will be:

$$0 \text{ if } x^\star < c$$

$$x^\star - \mathrm{c} \text{ if } x^\star \geq \mathrm{c}$$

Of course, the stock price $x^\star$ is not known in advance, so to calculate the expected value of the warrant at expiration Sprenkle had to 'weight' these final values by $f(x^\star)$, the probability distribution of $x^\star$. He used the standard integral formula for the expected value of a continuous random variable, obtaining the following expression for the warrant's expected value at expiration:

$$\int_c^\infty (x^\star - c)\, f(x^\star)\, dx^\star$$

To evaluate this integral, Sprenkle assumed that $f(x^\star)$ was log-normal (by the late 1950s, that assumption was 'in the air', he recalls [Sprenkle interview]), and that the value of $x^\star$ expected by an investor was the current stock price x multiplied by a constant, k. The above integral expression for the warrant's expected value then became:

$$kxN\left[\frac{\ln(kx/c) + s^2/2}{s}\right] - cN\left[\frac{\ln(kx/c) - s^2/2}{s}\right] \tag{1}$$

where ln is the abbreviation for natural logarithm, s^2 is the variance of the distribution of $\ln x^\star$, and N is the (cumulative) Gaussian or normal distribution function, the values of which could be found in tables used by any statistics undergraduate.[10]

Sprenkle then argued that the expected value of a warrant would be the price an investor would be prepared to pay for it only if the investor was indifferent to risk or 'risk neutral'. (To get a sense of what this means, imagine being offered a fair bet with a 50 percent chance of winning \$1,000 and a 50 percent chance of losing \$1,000, and thus an expected value of zero. If you would require to be paid to take on such a bet you are 'risk averse'; if you would pay to take it on you are 'risk seeking'; if you would take it on without inducement, but without being prepared to pay to do so, you are 'risk neutral'.) Warrants are riskier than the underlying stock because of their leverage – 'a given percentage change in the price of the stock will result in a larger percentage change in the price of the option' – so an investor's attitude to risk could be conceptualized, Sprenkle suggested, as the price P_e he or she was prepared to pay for leverage. A risk-seeking investor would pay a positive price, and a risk-averse investor a negative one: that is, a levered asset would have to offer an expected rate of return sufficiently higher than an unlevered one before a risk-averse investor would buy it. *V*, the value of a warrant to an investor was then given, Sprenkle showed, by:

$$V = kxN\,[\frac{\ln(kx/c) + s^2/2}{s}] - (1 - P_e)cN\,[\frac{\ln(kx/c) - s^2/2}{s}] \qquad (2)$$

(The right hand side of this equation reduces to expression 1 in the case of a risk neutral investor for whom $P_e = 0$.) The values of *k*, *s*, and P_e were posited by Sprenkle as specific to each investor, representing his or her subjective expectations and attitude to risk. Values of *V* would thus vary between investors, and 'Actual prices of the warrant then reflect the consensus of marginal investors' opinions – the marginal investors' expectations and preferences are the same as the market's expectations and preferences' (Sprenkle, 1961: 199–201).

Sprenkle examined warrant and stock prices for the 'classic boom and bust period' of 1923–32 and for the relative stability of 1953–59, hoping to estimate from those prices 'the market's expectations and preferences', in other words the values of *k*, *s*, and P_e implied by warrant prices. His econometric work, however, hit considerable difficulties: 'it was found impossible to obtain these estimates'. Only by arbitrarily assuming $k = 1$ and testing out a range of arbitrary values of P_e could Sprenkle make partial progress. His theoretically-derived formula for the value of a warrant depended on parameters whose empirical values were extremely problematic to determine (Sprenkle, 1961: 204, 212–13).

The same difficulty hit the most sophisticated theoretical analysis of warrants from this period, by Paul Samuelson in collaboration with the MIT mathematician Henry P. McKean, Jr. McKean was a world-class specialist in stochastic calculus, the theory of stochastic processes in continuous time, which in the years after Bachelier's work had burgeoned into a key domain of modern probability theory. Even with McKean's help, however, Samuelson's model (which space constraints prevent me describing in detail) also depended, like Sprenkle's, on parameters that seemed to have no straightforward empirical referents: r_α, the expected rate of return on the underlying stock, and r_β, the expected return on the warrant (McKean, 1965; Samuelson, 1965). A similar problem was encountered in the somewhat simpler work of University of Chicago PhD student, A. James Boness. He made the simplifying assumption that option traders are risk-neutral, but his formula also involved r_α, which he could estimate only indirectly by finding the value that minimized the difference between predicted and observed option prices (Boness, 1964).

'The Greatest Gambling Game on Earth'

Theoretical analysis of warrant and option prices thus seemed always to lead to formulae involving parameters that were difficult or impossible to estimate. An alternative approach was to eschew a priori models and to study the relationship between warrant and stock prices empirically. The most influential work of this kind was conducted by Sheen Kassouf. After a mathematics degree from Columbia University, Kassouf set up a successful technical illustration firm. He was fascinated by the stock market and a keen, if not always successful, investor. In 1961, he wanted to invest in the defence company Textron, but could not decide between buying its stock or its warrants [Kassouf interview]. He started to examine the relationship between stock and warrant prices, finding empirically that a simple hyperbolic formula

$$w = \sqrt{c^2 + x^2} - c$$

seemed roughly to fit observed curvilinear relationships between warrant price, stock price and strike price (Kassouf, 1962: 26).

In 1962, Kassouf returned to Columbia to study warrant pricing for a PhD in economics. His earlier simple curve fitting was replaced by econometric techniques, especially regression analysis, and he posited a more complex relationship determining warrant prices:

$$w/c = [(x/c)^z + 1]^{1/z} - 1 \qquad (3)$$

where z was an empirically-determined function of the stock price, exercise price, stock price 'trend',[11] time to expiration, stock dividend, and the extent of the dilution of existing shares that would occur if all warrants were exercised (Kassouf, 1965).

Kassouf's interest in warrants was not simply academic: he wanted 'to make money' trading them [Kassouf interview]. He had rediscovered, even before starting his PhD, an old form of securities arbitrage† (see Weinstein, 1931: 84, 142–45). Warrants and the corresponding stock tended to move together: if the stock price rose, then so did the warrant price; if the stock fell, so did the warrant. So one could be used to offset the risk of the other. If, for example, warrants seemed overpriced relative to the corresponding stock, one could short sell† them, hedging the risk by buying some of the stock. Trading of this sort, conducted by Kassouf in parallel with his PhD research, enabled him 'to more than double $100,000 in just four years' (Thorp and Kassouf, 1967: 32).

In 1965, fresh from his PhD, Kassouf was appointed to the faculty of the newly established Irvine campus of the University of California. There, he was introduced to mathematician Edward O. Thorp. Alongside research in functional analysis and probability theory, Thorp had a long-standing interest in casino games. While at MIT in 1959–61 he had collaborated with the celebrated information theorist Claude Shannon on a tiny, wearable, analog computer system to predict where the ball would be deposited on a roulette wheel [Thorp interview]. Thorp went on to devise the first effective methods for beating the casino at blackjack, by keeping track of cards that had already been dealt and thus identifying situations favourable to the player (Thorp, 1961; Tudball, 2002).

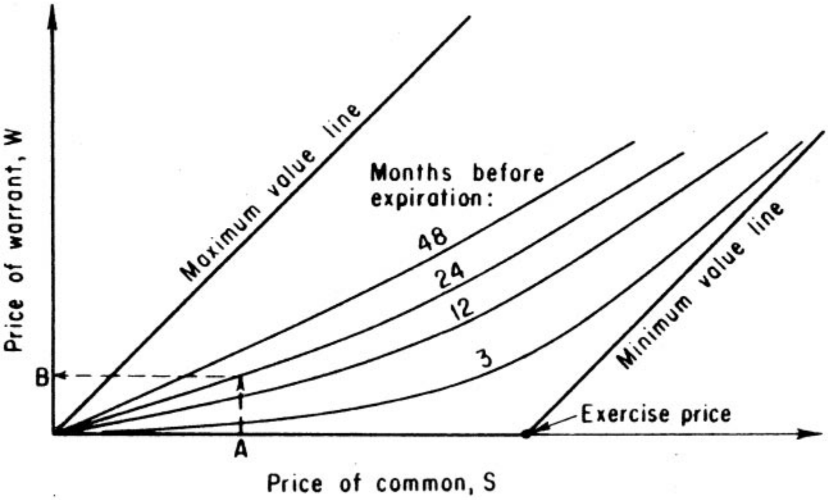
Thorp and Shannon's use of their wearable roulette computer was limited by frequently broken wires, but card-counting was highly profitable. In the MIT spring recess in 1961, Thorp travelled to Nevada equipped with a hundred US$100 bills provided by two millionaires with an interest in gambling. After 30 hours of blackjack, Thorp's US$10,000 had become US$21,000. He went on to devise, with computer scientist William E. Walden of the nuclear weapons laboratory at Los Alamos, a method for identifying favourable side bets in the version of baccarat played in Nevada. Thorp found, however, that beating the casino had disadvantages as a way of making money. At a time when US casinos were controlled largely by organized criminals, there were physical risks: while Thorp was playing baccarat in 1964, he was rendered almost unconscious by knock-out drops added to his coffee. The need to travel to places where gambling was legal was a further disadvantage to an academic with a family [Thorp interview].

Increasingly, Thorp's attention switched to the financial markets. 'The greatest gambling game on earth is the one played daily through the brokerage houses across the country', Thorp told the readers of the hugely successful book describing his card-counting methods (Thorp, 1966: 182). But could the biggest of casinos succumb to Thorp's mathematical skills? Predicting stock prices seemed too daunting: 'there is an extremely large number of variables, many of which I can't get any fix on'. However, he realized that 'I can eliminate most of the variables if I think about warrants versus common stock' [Thorp interview]. Thorp began to sketch graphs of the observed relationships between stock and warrant prices, and meeting Kassouf provided him with a formula (equation 3 above) for these curves.

Their book, *Beat the Market* (Thorp and Kassouf, 1967), explained graphically the relationship between the price of a warrant, w, and of the underlying common stock, x (see Figure 1). No warrant should ever cost more than the underlying stock, since it is simply an option to buy the latter, and this constraint yielded a 'maximum value line'. At expiration, as Sprenkle had noted, a warrant would be worthless if the stock price, x, was less than the strike price, c; otherwise it would be worth the difference ($x - c$). If, at any time, $w < x - c$, an instant arbitrage profit could be made by buying the warrant and exercising it (at a cost of $w + c$) and selling the stock thus acquired for x. So the warrant's value at expiration was also a minimum value for it at any time. As expiration approached, the 'normal price curves' expressing the value of a warrant dropped closer to its value at expiration.

These 'normal price curves' could then be used to identify overpriced and underpriced warrants.[12] The former could be sold short, and the latter bought, with the resultant risks hedged by taking a position in the stock (buying stock if warrants had been sold short; selling stock short if warrants had been bought). The appropriate size of hedge, Thorp and Kassouf explained (1967: 82), was determined by 'the slope of the normal price curve at our starting position'. If that slope were, say, 1:3, as it

FIGURE 1



'Normal price curves' for a warrant. From Edward O. Thorp and Sheen T. Kassouf, *Beat the Market: A Scientific Stock Market System* (New York: Random House, 1967), 31. © Edward O. Thorp and Sheen T. Kassouf. Used by permission of Random House, Inc. S is Thorp and Kassouf's notation for the price of the common stock.

roughly is at point (A,B) in Figure 1, the appropriate hedge ratio was to buy one unit of stock for every three warrants sold short. Any movements along the normal price curve caused by small stock price fluctuations would then have little effect on the value of the overall position, because the loss or gain on the warrants would be balanced by a nearly equivalent gain or loss on the stock. Larger stock price movements could of course lead to a shift to a region of the curve in which the slope differed from 1:3, and in their investment practice both Thorp and Kassouf adjusted their hedges when that happened (Thorp, 2002; Kassouf interview).

Initially, Thorp relied upon Kassouf's empirical formula for warrant prices (equation 3 above): as he says, 'it produced . . . curves qualitatively like the actual warrant curves'. Yet he was not entirely satisfied with it: 'quantitatively, I think we both knew that there was something more that had to happen' [Thorp interview]. He began his investigation of that 'something' in the same way as Sprenkle – applying the log-normal distribution to work out the expected value of a warrant at expiration – reaching a formula equivalent to Sprenkle's (equation 1 above).

Like Sprenkle's, Thorp's formula (Thorp, 1969: 281) for the expected value of a warrant involved the expected increase in the stock price, which there was no straightforward way to estimate. He decided to approximate it by assuming that the expected value of the stock rose at the riskless† rate of interest: he had no better estimate, and he 'didn't think that enormous errors would necessarily be introduced' by the approximation. Thorp found that the resultant formula was plausible – 'I couldn't find anything wrong with its qualitative behavior and with the actual forecast it was making' – and in 1967 he started to use it to identify grossly overpriced options to sell [Thorp interview]. It was formally equivalent to the Black-Scholes formula for a call option (equation 5 below), except for one feature: unlike Black and Scholes, Thorp did not discount† the expected value of the option at expiration back to the present. In the warrant markets he was used to, the proceeds of the short sale of a warrant were retained in their entirety by the broker, and were not available immediately to the seller as Black and Scholes assumed.[13] It was a relatively minor difference: when Thorp read Black and Scholes, he was able quickly to see why the two formulae differed and to add to his formula the necessary discount factor to make them identical (Thorp, 2002). In the background, however, lay more profound differences of approach.

Black and Scholes

In 1965, Fischer Black, with a Harvard PhD (Black, 1964) in what was in effect artificial intelligence, joined the operations research group of the consultancy firm Arthur D. Little, Inc. There, Black met Jack Treynor, a financial specialist at Little [Treynor interview]. Treynor had developed, though had not published, what later became known as the Capital Asset Pricing Model (also developed, independently, by academics William Sharpe, John Lintner, and Jan Mossin).[14] It was Black's (and also Scholes's) use of this model that decisively differentiated their work from the earlier research on option pricing.

The Capital Asset Pricing Model provided a systematic account of the 'risk premium': the additional return that investors demand for holding risky assets. That premium, Treynor pointed out, could not depend simply on the 'sheer magnitude of the risk', because some risks were 'insurable': they could be minimized by diversification, by spreading one's investments over a broad range of companies (Treynor, 1962: 13–14; 1999: 20). What could not be diversified away, however, was the risk of general market fluctuations. By reasoning of this kind, Treynor showed – and the other developers of the model also demonstrated – that a capital asset's risk premium should be proportional to its β (its covariance with the general level of the market, divided by the variance of the market). An asset whose β was zero, in other words an asset the price of which was uncorrelated with the overall level of the market, had no risk premium (any specific risks involved in holding it could be diversified away), and investors in it should earn only r, the riskless rate of interest. As the asset's β increased, so should its risk premium.

The Capital Asset Pricing Model was an elegant piece of theoretical reasoning. Its co-developer Treynor became Black's mentor in what was for Black the new field of finance, so it is not surprising that when Black began his own work in finance it was by trying to apply the model to a range of assets other than stock (which had been its main initial field of application). Also important as a resource for Black's research was a specific piece of joint work with Treynor on how companies should value cash flows in making their investment decisions. This was the problem that had most directly inspired Treynor's development of the Capital Asset Pricing Model, and the aspect of it on which Black and Treynor collaborated had involved Treynor writing an expression for the change in the value of a cash flow in a short, finite time interval Δt; expanding the expression using the standard calculus technique of Taylor expansion; taking expected values; dropping the terms of order Δt^2 and higher; dividing by Δt; and letting Δt tend to zero so that the finite difference equation became a differential equation. Treynor's original version of the latter was in error because he had left out a second derivative that did not vanish, but Black and he worked out how to correct the differential equation by adding the corresponding term.[15]

Amongst the assets to which Black tried to apply the Capital Asset Pricing Model were warrants. His starting point was directly modelled on his joint work with Treynor, with w, the value of the warrant, taking the place of cash flow, and x, the stock price, replacing the stochastically time-dependent 'information variables' of the earlier problem. If Δw is the change in the value of the warrant in time interval $(t, t + \Delta t)$,

$$\Delta w = w(x + \Delta x, t + \Delta t) - w(x,t)$$

where Δx is the change in stock price over the interval. Black then expanded this expression in a Taylor series and took expected values:

$$\mathrm{E}(\Delta w) = \frac{\partial w}{\partial x}\mathrm{E}(\Delta x) + \frac{\partial w}{\partial t}\Delta t + \frac{1}{2}\frac{\partial^2 w}{\partial x^2}\mathrm{E}(\Delta x^2) + \frac{\partial^2 w}{\partial x \partial t}\Delta t\mathrm{E}(\Delta x) + \frac{1}{2}\frac{\partial^2 w}{\partial t^2}\Delta t^2$$

where E designates 'expected value' and higher order terms are dropped. Black then assumed that the Capital Asset Pricing Model applied both to the stock and warrant, so that $\mathrm{E}(\Delta x)$ and $\mathrm{E}(\Delta w)$ would depend on, respectively, the β of the stock and the β of the warrant. He also assumed that the stock price followed a log-normal random walk and that it was permissible 'to eliminate terms that are second order in Δt'. These assumptions, a little manipulation, and letting Δt tend to zero, yielded the differential equation:

$$\frac{\partial w}{\partial t} = rw - rx\frac{\partial w}{\partial x} - \frac{1}{2}\sigma^2 x^2\frac{\partial^2 w}{\partial x^2} \qquad (4)$$

where r is the riskless rate of interest and σ the volatility[†] of the stock price.[16]

'I spent many, many days trying to find the solution to that equation', Black later recalled: 'I ... had never spent much time on differential equations, so I didn't know the standard methods used to solve problems like that'. He was 'fascinated' that in the differential equation apparently key features of the problem (notably the stock's β and thus its expected return, a pervasive feature in earlier theoretical work on option pricing) no longer appeared. 'But I was still unable to come up with the formula. So I put the problem aside and worked on other things' (Black, 1989: 5–6).

In the autumn of 1968, however, Black (still working for Arthur D. Little in Cambridge, MA) met Myron Scholes, a young researcher who had just joined the finance group in MIT's Sloan School of Management. The pair teamed up with finance scholar Michael Jensen to conduct an empirical test of the Capital Asset Pricing Model, which was still largely a theoretical postulate. Scholes also became interested in warrant pricing, not, it seems, through Black's influence but through supervising an MIT Master's dissertation on the topic (Scholes, 1998). Scholes' PhD thesis (Scholes, 1970) involved the analysis of securities as potential substitutes for each other, with the potential for arbitrage ensuring that securities whose risks are alike will offer similar expected returns. Scholes' PhD adviser, Merton H. Miller, had introduced this form of theoretical argument – 'arbitrage proof' – in what by 1970 was already seen as classic work with Franco Modigliani (Modigliani and Miller, 1958). Scholes started to investigate whether similar reasoning could be applied to warrant pricing, and began to consider the hedged portfolio formed by buying warrants and short selling the underlying stock (Scholes, 1998: 480).

The hedged portfolio had been the central idea of Thorp and Kassouf's *Beat the Market* (1967), though Scholes had not yet read the book [Scholes interview]. Scholes' goal, in any case, was different. Thorp and Kassouf's hedged portfolio was designed to earn high returns with low risk in real markets. Scholes' was a desired theoretical artifact. He wanted a portfolio with a β of zero: that is, with no correlation with the overall level of the market. If such a portfolio could be created, the Capital Asset Pricing Model implied that it would earn, not high returns, but only the riskless rate of interest, r. It would thus not be an unduly enticing investment, but knowing the rate of return on the hedged portfolio might solve the problem of warrant pricing.

What Scholes could not work out, however, was how to construct a zero-β portfolio. He could see that the quantity of shares that had to be sold short must change with time and with changes in the stock price, but he could not see how to determine that quantity. '[A]fter working on this concept, off and on, I still couldn't figure out analytically how many shares of stock to sell short to create a zero-beta portfolio' (Scholes, 1998: 480). Like Black, Scholes was stymied. Then, in 'the summer or early fall of 1969', Scholes told Black of his efforts, and Black described the different approach he had taken, in particular showing Scholes the Taylor series expansion of the warrant price (Scholes, 1998: 480). The two men then found how to construct a zero-β portfolio. If the stock price changed by the small amount Δx, the option price would alter by $\frac{\partial w}{\partial x}\Delta x$. So the necessary hedge was to short sell a quantity $\frac{\partial w}{\partial x}$ of stock for every warrant held. This was the same conclusion Thorp and Kassouf had arrived at: $\frac{\partial w}{\partial x}$ is their hedging ratio, the slope of the curve of w plotted against x as in Figure 1.

While the result was in that sense equivalent, it was embedded in quite a different chain of reasoning. Although the precise way in which Black and Scholes argued the point evolved as they wrote successive versions of their paper,[17] the crux of their mathematical analysis was that the hedged portfolio must earn the riskless rate of interest. The hedged portfolio was not entirely free from risk, they argued in August 1970, because the hedging would not be exact if the stock price altered significantly and because the value of an option altered as expiration became closer. The change in value of the hedged portfolio resulting from stock price movements would, however, depend only on the magnitude of those movements not on their sign. It was, therefore, the kind of risk that could be diversified away. So, according to the Capital Asset Pricing Model, the hedged portfolio could earn only the riskless rate of interest (Black and Scholes, 1970a: 6). In other words, the expected return on the hedged portfolio in the short time interval $(t, t + \Delta t)$ is just its price at time t multiplied by $r\Delta t$. Simple manipulation of the Taylor expansion of $w(x + \Delta x, t + \Delta t)$ led to a finite difference equation that could be transformed into a differential equation by letting Δt tend to zero, and to equation 4 above: the Black-Scholes option pricing equation, as it was soon to be called.

As noted above, Black had been unable to solve equation 4, but he and Scholes now returned to the problem. It was, however, still not obvious how to proceed. Like Black, Scholes was 'amazed that the expected rate of return on the underlying stock did not appear in [equation 4]' (Scholes, 1998: 481). This prompted Black and Scholes to experiment, as Thorp had done, with setting the expected return on the stock as the riskless rate, r. They substituted r for k in Sprenkle's formula for the expected value of a warrant at expiration (equation 1 above). To get the warrant price, they then had to discount[†] that terminal value back to the present. How could they do that? 'Rather suddenly, it came to us', Black later recalled. 'If the stock had an expected return equal to the [riskless] interest rate, so would the option. After all, if all the stock's risk could be diversified away, so could all the option's risk. If the beta of the stock were zero, the beta of the option would have to be zero too. . . . [T]he discount rate that would take us from the option's expected future value to its present value would always be the [riskless] interest rate' (Black, 1989: 6). These modifications to Sprenkle's formula led to the following formula for the value of a warrant or call option:

$$\begin{aligned} w = xN[\frac{\ln(x/c) + (r + \frac{1}{2}\sigma^2)(t^\star - t)}{\sigma\sqrt{t^\star - t}}] \\ - c[\exp\{r(t - t^\star)\}]N[\frac{\ln(x/c) + (r - \frac{1}{2}\sigma^2)(t^\star - t)}{\sigma\sqrt{t^\star - t}}] \end{aligned} \tag{5}$$

where c is the strike price, σ the volatility of the stock, $t^\star$ the expiration of the option, and N the Gaussian distribution function. Instead of facing the difficult task of directly solving equation 4, all Black and Scholes had now to do was show by differentiating equation 5 that it (the Black-Scholes call option or warrant formula) was a solution of equation 4.

Merton

Black's and Scholes' tinkering with Sprenkle's expected value formula (equation 1 above) was in one sense no different from Boness' or Thorp's. However, Boness' justification for his choice of expected rate of return was empirical – he chose 'the rate of appreciation most consistent with market prices of puts and calls' (Boness, 1964: 170) – and Thorp freely admits he 'guessed' that the right thing to do was to set the stock's rate of return equal to the riskless rate: it was 'guesswork not proof' [Thorp interview]. Black and Scholes, on the other hand, could prove mathematically that their call option formula (equation 5) was a solution to their differential equation (equation 4), and the latter had a clear theoretical justification.

It was a justification apparently intimately bound up with the Capital Asset Pricing Model. Not only was the model drawn on explicitly in both the equation's derivations, but it also made Black's and Scholes' entire mathematical approach seem permissible. Like all others working on the problem in the 1950s and 1960s (with the exception of Samuelson, McKean, and Merton), Black and Scholes used ordinary calculus – Taylor series expansion, and so on – but in a context in which x, the stock price, was known to vary stochastically. Neither Black nor Scholes knew the mathematical theory needed to do calculus rigorously in a stochastic environment, but the Capital Asset Pricing Model provided an economic justification for what might otherwise have seemed dangerously unrigorous mathematics. 'We did not know whether our formulation was exact', says Scholes, 'but intuitively we thought investors could diversify away any residual risk that was left' (Scholes, 1998: 483).

As noted above, Black had been a close colleague of the Capital Asset Pricing Model's co-developer, Treynor, while Scholes had done his graduate work at the University of Chicago, one of the two leading sites of financial economics, where the model was seen as an exemplary contribution to the field. However, at the other main site, MIT, the original version of the Capital Asset Pricing Model was regarded much less positively. The model rested upon the 'mean-variance' view of portfolio selection: that investors could be modelled as guided only by their expectations of the returns on investments and their risks as measured by the expected standard deviation or variance of returns. Unless returns followed a joint normal distribution (which was regarded as ruled out, because it would imply, as noted above, a non-zero probability of negative prices), mean-variance analysis seemed to rest upon a specific form of 'utility function' (the function that characterizes the relationship between the return on an investor's portfolio, y, and his or her preferences). Mean-variance analysis seemed to imply that investors' utility functions were quadratic: that is, they contained only terms in y and y^2.

For MIT's Paul Samuelson, the assumption of quadratic utility was over-specific – one of his earliest contributions to economics (Samuelson, 1938) had been his 'revealed preference' theory, designed to eliminate the non-empirical aspects of utility analysis – and a 'bad . . . representation of human behaviour' [Samuelson interview].[18] Seen from Chicago, Samuelson's objections were 'quibbles' [Fama interview] when set against the virtues of the Capital Asset Pricing Model: 'he's got to remember what Milton Friedman said – "Never mind about assumptions. What counts is, how good are the predictions?" ' [Miller interview; see Friedman, 1953]. Nevertheless, they were objections that weighed heavily with Robert C. Merton. Son of the social theorist and sociologist of science Robert K. Merton, he switched in autumn 1967 from graduate work in applied mathematics at the California Institute of Technology to study economics at MIT. He had been an amateur investor since aged 10 or 11, had graduated from stocks to options and warrants, and came to realize 'that I had a much better intuition and "feel" into economic matters than physical ones'. In spring 1968, Samuelson appointed the mathematically-talented young Merton as his research assistant, even allocating him a desk inside his MIT office (Merton interview; Merton, 1998: 15–16).

It was not simply a matter of Merton finding the assumptions underpinning the standard Capital Asset Pricing Model 'objectionable' (Merton, 1970: 2). At the centre of Merton's work was the effort to replace simple 'one-period' models of that kind with more sophisticated 'continuous-time' models. In the latter, not only did the returns on assets vary in a continuous stochastic fashion, but individuals took decisions about portfolio selection (and also consumption) continuously, not just at discrete points in time. In any time interval, however short, individuals could change the composition of their investment portfolios. Compared with 'discrete-time' models, 'the continuous-time models are mathematically more complex', says Merton. He quickly became convinced, however, that 'the derived results of the continuous-time models were often more precise and easier to interpret than their discrete-time counterparts' (Merton, 1998: 18–19). His 'intertemporal' capital asset pricing model (Merton, 1973), for example, did not necessitate the 'quadratic utility' assumption of the original.

With continuous-time stochastic processes at the centre of his work, Merton felt the need not just to make ad hoc adjustments to standard calculus but to learn stochastic calculus. It was not yet part of economists' mathematical repertoire (it was above all Merton who introduced it), but by the late 1960s a number of textbook treatments by mathematicians (such as Cox and Miller, 1965 and Kushner, 1967) had been published, and Merton used these to teach himself the subject [Merton interview]. He rejected as unsuitable the 'symmetrized' formulation of stochastic integration by R.L. Stratonovich (1966): it was easier to use for those with experience only of ordinary calculus, but when applied to prices it in effect allowed investors an illegitimate peek into the future. Merton chose instead the original 1940s' definition of the stochastic integral by the Japanese mathematician, Kiyosi Itô, and Itô's associated apparatus for handling stochastic differential equations (Stroock and Varadhan, 1987).

Amongst the problems on which Merton worked, both with Samuelson and independently, was warrant pricing, and the resultant work formed two of the five chapters of his September 1970 PhD thesis (Samuelson and Merton, 1969; Merton, 1970: chapters 4 and 5). Black and Scholes read the 1969 paper in which Samuelson and Merton described their joint work, but did not immediately tell them of the progress they had made: there was 'friendly rivalry between the two teams', says Scholes (1998: 483). In the early autumn of 1970, however, Scholes did discuss with Merton his work with Black. Merton immediately appreciated that this work was a 'significant "break-through" ' (Merton, 1973: 142), and it was Merton, for example, who christened equation 4 the 'Black-Scholes' equation. Given Merton's critical attitude to the Capital Asset Pricing Model, however, it is also not surprising that he also believed that 'such an important result deserves a rigorous derivation', not just the 'intuitively appealing' one Black and Scholes had provided (Merton, 1973: 161–62). 'What I sort of argued with them [Black and Scholes]', says Merton, 'was, if it depended on the [Capital] Asset Pricing Model, why is it when you look at the final formula [equation 4] nothing about risk appears at all? In fact, it's perfectly consistent with a risk-neutral world' [Merton interview].

So Merton set to work applying his continuous-time model and Itô calculus to the Black-Scholes hedged portfolio. 'I looked at this thing', says Merton, 'and I realized that if you did ... dynamic trading ... if you actually [traded] literally continuously, then in fact, yeah, you could get rid of the risk, but not just the systematic risk, all the risk'. Not only did the hedged portfolio have zero β in the continuous-time limit (Merton's initial doubts on this point were assuaged),[19] 'but you actually get a zero sigma': that is, no variance of return on the hedged portfolio. So the hedged portfolio can earn only the riskless rate of interest, 'not for the reason of [the Capital] Asset Pricing Model but ... to avoid arbitrage, or money machine': a way of generating certain profits with no net investment [Merton interview]. For Merton, then, the 'key to the Black-Scholes analysis' was an assumption Black and Scholes did *not* initially make: continuous trading, the capacity to adjust a portfolio at all times and instantaneously. '[O]nly in the instantaneous limit are the warrant price and stock price perfectly correlated, which is what is required to form the "perfect" hedge' (Merton, 1972: 38).

Black and Scholes were not initially convinced of the correctness of Merton's approach. Merton's additional assumption – his world of continuous-time trading – was a radical abstraction, and in a January 1971 draft of their paper on option pricing Black and Scholes even claimed that equilibrium prices in capital markets could not have characteristics assumed by Merton's analysis (Black and Scholes, 1971: 20). Merton, in turn, told Fischer Black in a 1972 letter that 'I ... do not understand your reluctance to accept that the standard form of CAPM [Capital Asset

Pricing Model] just does not work' (Merton, 1972). Despite this disagreement, Black and Scholes used what was essentially Merton's revised form of their derivation in the final, published version of their paper (Black and Scholes, 1973), though they also presented Black's original derivation, which drew directly on the Capital Asset Pricing Model. Black, however, remained ambivalent about Merton's derivation, telling a 1989 interviewer that 'I'm still more fond' of the Capital Asset Pricing Model derivation: '[T]here may be reasons why arbitrage is not practical, for example trading costs'. (If trading incurs even tiny transaction costs, continuous adjustment of a portfolio is infeasible.) Merton's derivation 'is more intellectual[ly] elegant but it relies on stricter assumptions, so I don't think it's really as robust'.[20]

Black, indeed, came to express doubts even about the central intuition of orthodox financial economics, that modern capital markets were efficient (in other words that prices in them incorporate all known information). Efficiency held, he suggested, only in a diluted sense: 'we might define an efficient market as one in which price is within a factor of 2 of value'. Black noted that this position was intermediate between that of Merton, who defended the efficient market hypothesis, and that of 'behavioural' finance theorist Robert Shiller: 'Deviations from efficiency seem more significant in my world than in Merton's, but much less significant in my world than in Shiller's' (Black, 1986: 533; see Merton, 1987 and Shiller, 1989).

The Equation and the World

It was not immediately obvious to all that what Black, Scholes and Merton had done was a fundamental breakthrough. The *Journal of Political Economy* originally rejected Black and Scholes' paper because, its editor told Black, option pricing was too specialized a topic to merit publication in a general economic journal (Gordon, 1970), and the paper was also rejected by the *Review of Economics and Statistics* (Scholes, 1997: 484). True, the emerging new breed of financial economists quickly saw the elegance of the Black-Scholes solution. All the parameters in equations 4 and 5 seemed readily observable empirically: there were none of the intractable estimation problems of earlier theoretical solutions. That alone, however, does not account for the wider impact of the Black-Scholes-Merton work. It does not explain, for example, how a paper originally rejected by an economic journal as too specialized should win a Nobel prize in economics (Scholes and Merton were awarded the prize in 1997; Black died in 1995).

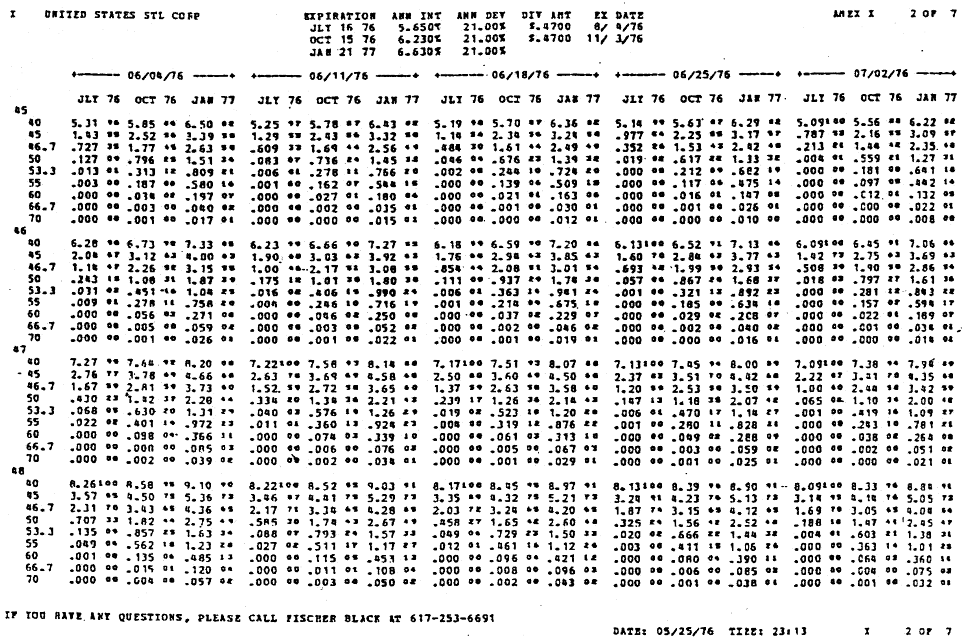
That the world came to embrace the Black-Scholes equation was in part because the world was changing – see the remarks at the start of the paper on the transformation of academic finance and the professionalization of US business schools – and in part because the equation (unlike, for example, Bachelier's work) changed the world.[21] The latter was the case in four senses. First, the Black–Scholes equation seems to have altered patterns of option prices. After constructing their call-option pricing formula (equation 5 above), Black and Scholes tested its empirical validity for the ad hoc New York options market, using a broker's diaries in which were 'recorded all option contracts written for his customers'. They found only an approximate fit: 'in general writers [the sellers of options] obtain favorable prices, and . . . there tends to be a systematic mispricing of options as a function of the variance of returns of the stock' (Black and Scholes, 1972: 403, 413). A more organized, continuous options exchange was established in Chicago in 1973, but Scholes' student Dan Galai also found that prices there initially differed from the Black–Scholes model, indeed to a greater extent than in the New York market (Galai, 1977).

By the second half of the 1970s, however, discrepancies between patterns of option pricing in Chicago and the Black–Scholes model diminished to the point of economic insignificance (the ad hoc New York market quickly withered after Chicago and other organized options exchanges opened). The reasons are various, but they include the use of the Black-Scholes model as a guide to arbitrage. Black set up a service selling sheets of theoretical option prices to market participants (see Figure 2). Options market makers† used those sheets and other material exemplifications of the Black–Scholes model to identify relatively over-priced and under-priced options on the same stock, sold the former and hedged their risk by buying the latter. In so doing, they altered patterns of pricing in a way that increased the validity of the model's predictions, in particular helping the model to pass its key econometric test: that the implied volatility† of all options on the same stock with the same expiration should be identical (MacKenzie and Millo, forthcoming).

The second world-changing, performative aspect of the Black–Scholes–Merton work was deeper than its use in arbitrage. In its mathematical assumptions, the equation embodied a world, so to speak. (From this viewpoint, the differences between the Black–Scholes world and Merton's world are less important than their commonalities.) In the final published version of their option pricing paper in 1973, Black and Scholes spelled out these assumptions, which included not just the basic assumption that the 'stock price follows a [lognormal] random walk in continuous time', but also assumptions about market conditions: that there are 'no transaction costs in buying or selling the stock or the option'; that it is 'possible to borrow any fraction of the price of a security to buy it or to hold it', at the riskless rate of interest; and that these are 'no penalties to short selling' (Black and Scholes, 1973: 640).

In 1973, these assumptions about market conditions were wildly unrealistic. Commissions (a key transaction cost) were high everywhere. Investors could not purchase stock entirely on credit – in the USA this was banned by the Federal Reserve's famous 'Regulation T' – and such loans would be at a rate of interest in excess of the riskless rate. Short selling was legally constrained and financially penalized: stock lenders retained the proceeds of a short sale as collateral for the loan, and refused to pass on all

FIGURE 2



I UNITED STATES STL CORP

EXPIRATION	ANN INT	ANN DEV	DIV AMT	EX DATE
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JAN 21 77	6.630%	21.00%		

AMEX I 2 OF 7

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45																
	40	5.31	5.85	6.50	5.25	5.78	6.43	5.19	5.70	6.36	5.14	5.63	6.29	5.09	5.56	6.22
	45	1.43	2.52	3.39	1.29	2.43	3.32	1.14	2.34	3.24	.977	2.25	3.17	.787	2.16	3.09
	46.7	.727	1.77	2.63	.609	1.69	2.56	.484	1.61	2.49	.352	1.53	2.42	.213	1.44	2.35
	50	.127	.796	1.51	.083	.736	1.45	.046	.676	1.39	.019	.617	1.33	.004	.559	1.27
	53.3	.013	.313	.809	.006	.278	.766	.002	.244	.724	.000	.212	.682	.000	.181	.641
	55	.003	.187	.580	.001	.162	.544	.000	.139	.509	.000	.117	.475	.000	.097	.442
	60	.000	.034	.197	.000	.027	.180	.000	.021	.163	.000	.016	.147	.000	.012	.132
	66.7	.000	.003	.040	.000	.002	.035	.000	.001	.030	.000	.001	.026	.000	.000	.022
	70	.000	.001	.017	.000	.000	.015	.000	.000	.012	.000	.000	.010	.000	.000	.008
46																
	40	6.28	6.73	7.33	6.23	6.66	7.27	6.18	6.59	7.20	6.13	6.52	7.13	6.09	6.45	7.06
	45	2.08	3.12	4.00	1.90	3.03	3.92	1.76	2.98	3.85	1.60	2.84	3.77	1.42	2.75	3.69
	46.7	1.18	2.26	3.15	1.00	2.17	3.08	.854	2.08	3.01	.693	1.99	2.93	.508	1.90	2.86
	50	.243	1.08	1.87	.175	1.01	1.80	.111	.937	1.74	.057	.867	1.68	.018	.797	1.61
	53.3	.031	.451	1.04	.016	.406	.990	.006	.363	.941	.001	.321	.892	.000	.281	.843
	55	.009	.278	.758	.004	.246	.716	.001	.214	.675	.000	.185	.634	.000	.157	.594
	60	.000	.056	.271	.000	.046	.250	.000	.037	.229	.000	.029	.208	.000	.022	.189
	66.7	.000	.005	.059	.000	.003	.052	.000	.002	.046	.000	.002	.040	.000	.001	.034
	70	.000	.001	.026	.000	.001	.022	.000	.001	.019	.000	.000	.016	.000	.000	.014
47																
	40	7.27	7.64	8.20	7.22	7.58	8.14	7.17	7.51	8.07	7.13	7.45	8.00	7.09	7.38	7.96
	45	2.76	3.78	4.66	2.63	3.69	4.58	2.50	3.60	4.50	2.37	3.51	4.42	2.22	3.41	4.35
	46.7	1.67	2.81	3.73	1.52	2.72	3.65	1.37	2.63	3.58	1.20	2.53	3.50	1.00	2.44	3.42
	50	.430	1.42	2.28	.334	1.34	2.21	.239	1.26	2.14	.147	1.18	2.07	.065	1.10	2.00
	53.3	.068	.630	1.31	.040	.576	1.26	.019	.523	1.20	.006	.470	1.14	.001	.419	1.09
	55	.022	.401	.972	.011	.360	.924	.008	.319	.876	.001	.280	.828	.000	.243	.781
	60	.000	.098	.366	.000	.074	.339	.000	.061	.313	.000	.049	.288	.000	.038	.264
	66.7	.000	.008	.085	.000	.006	.076	.000	.005	.067	.000	.003	.059	.000	.002	.051
	70	.000	.002	.039	.000	.002	.034	.000	.001	.029	.000	.001	.025	.000	.000	.021
48																
	40	8.26	8.58	9.10	8.22	8.52	9.03	8.17	8.45	8.97	8.13	8.39	8.90	8.09	8.33	8.88
	45	3.57	4.50	5.36	3.46	4.41	5.29	3.35	4.32	5.21	3.24	4.23	5.13	3.14	4.18	5.05
	46.7	2.31	3.43	4.36	2.17	3.34	4.28	2.03	3.24	4.20	1.87	3.15	4.12	1.69	3.05	4.08
	50	.707	1.82	2.75	.585	1.74	2.67	.458	1.65	2.60	.325	1.56	2.52	.188	1.47	2.45
	53.3	.135	.857	1.63	.088	.793	1.57	.049	.729	1.50	.020	.666	1.44	.004	.603	1.38
	55	.049	.562	1.23	.027	.511	1.17	.012	.461	1.12	.003	.411	1.06	.000	.363	1.01
	60	.001	.135	.485	.000	.115	.453	.000	.096	.421	.000	.080	.390	.000	.064	.360
	66.7	.000	.015	.120	.000	.011	.108	.000	.008	.096	.000	.006	.085	.000	.004	.075
	70	.000	.004	.057	.000	.003	.050	.000	.002	.043	.000	.001	.038	.000	.001	.032

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One of Black's sheets (courtesy Mark Rubinstein). The numbers on the extreme left hand side of the table are stock prices, the next set of numbers are strike prices, and the large numbers in the body of the table are the Black-Scholes values for call options with given expiry dates (e.g. 16 July 1976) at particular points in time (e.g. 4 June 1976). The smaller numbers in the body of the table are the option 'deltas' ($\frac{\partial w}{\partial x}$ multiplied by 100). A delta of 96, for example, implies that the value of the option changes by $0.96 for a one dollar move in the stock price. The data at the head of the table are interest rates, Black's assumption about stock volatility, and details of the stock dividends.

(or sometimes any) of the interest earned on those proceeds [Thorp interview].

Since 1973, however, the Black–Scholes–Merton assumptions have become, while still not completely realistic, a great deal more so (see MacKenzie and Millo, forthcoming). In listing these assumptions, Black and Scholes wrote: 'we will assume "ideal conditions" in the market for the stock and for the option' (Black and Scholes, 1973: 640). Of course, 'ideal' here means simplified and thus mathematically tractable, like the physicist's frictionless surface: non-zero transaction costs and constraints on borrowing and short selling hugely complicate the option pricing problem. 'Ideal', however, also connotes the way things *ought* to be. This was not Black and Scholes' intended implication: neither was an activist in relation to the politics of markets. From the early 1970s onwards, however, an increasingly influential number of economists and others *were* activists for the 'free market' ideal.

Their activities (along with other factors, such as the role of technological change in reducing transaction costs) helped make the world embodied in the Black–Scholes–Merton assumptions about market conditions more real. The Black–Scholes–Merton analysis itself assisted this process by helping to legitimize options trading and thus helping to create the efficient, liquid markets posited by the model. The Chicago Board Options Exchange's counsel recalls:

> Black–Scholes was really what enabled the exchange to thrive. . . . [I]t gave a lot of legitimacy to the whole notions of hedging and efficient pricing, whereas we were faced, in the late 60s–early 70s with the issue of gambling. That issue fell away, and I think Black–Scholes made it fall away. It wasn't speculation or gambling, it was efficient pricing. I think the SEC [Securities and Exchange Commission] very quickly thought of options as a useful mechanism in the securities markets and it's probably – that's my judgement – the effects of Black–Scholes. I never heard the word 'gambling' again in relation to options. [Rissman interview]

The Black–Scholes–Merton model also had more specific impacts on the nature of the markets it analysed. Earlier upsurges of options trading had typically been reversed, arguably because option prices had usually been 'too high' in the sense that they made options a poor purchase: options could too seldom be exercised profitably (Kairys and Valerio, 1997). The availability of the Black–Scholes formula, and its associated hedging techniques, gave participants the confidence to write options at lower prices, again helping options exchanges to grow and to prosper, becoming more like the markets posited by the theory. The Black–Scholes analysis was also used to free hedging by options market makers† from the constraints of Regulation T. So long as their stock positions were close to the theoretical hedging ratio $(\frac{\partial w}{\partial x})$, they were allowed to construct such hedges using entirely borrowed funds (Millo, forthcoming). It was a delightfully direct loop of performativity: the model being used to make one of its key assumptions a reality.

Third, the Black–Scholes–Merton solution to the problem of option pricing became paradigmatic in the deeper Kuhnian sense of 'exemplary solution' (Kuhn, 1970: 175), indeed more deeply so than the Capital Asset Pricing Model. The Black–Scholes–Merton analysis provided a range of intellectual resources for those tackling problems of pricing derivatives[†] of all kinds. Amongst those resources were the idea of perfect hedging (or of a 'replicating portfolio', a portfolio whose returns would exactly match those of the derivative in all states of the world); no-arbitrage pricing (deriving prices from the argument that the only patterns of pricing that can be stable are those that give rise to no arbitrage opportunities); and a striking example of the use in economics of Itô's stochastic calculus, especially of the basic result known as 'Itô's lemma', the stochastic equivalent of Taylor expansion, which serves inter alia as a 'bridging result', allowing those trained only in ordinary calculus to perform at least some manipulations in Itô calculus. Open any textbook of modern mathematical finance (for example, Hull, 2000), and one finds multiple uses of these ideas. These uses are creative solutions to problems of sometimes great difficulty, not rote applications of these ideas – a paradigm is a resource, not a rule – but the family resemblance to the Black–Scholes–Merton solution is clear. In the words of option trader and theorist Nassim Taleb, far from an uncritical admirer of the Black-Scholes-Merton work, 'most everything that has been developed in modern finance since 1973 is but a footnote on the BSM [Black–Scholes–Merton] equation' (Taleb, 1998: 35).

The capacity to generate theoretical prices – not just for what soon came to be called the 'vanilla' options analysed by Black, Scholes and Merton but for a wide range of often exotic derivatives – played a vital role in the emergence of the modern derivatives markets, especially when, as was the case with the original Black–Scholes–Merton analysis, the theoretical argument that generated prices also generated rules for hedging the risk of involvement in such derivatives. I have already touched on the role played by theory in supporting the emergence and success of organized options exchanges, but it was at least equally important in the growth of what is known as the 'over-the-counter' (direct, institution-to-institution) derivatives market, the overall volume of which is now larger. (In December 2002, the over-the-counter market accounted for 85.6 percent of total notional value of derivatives contracts outstanding globally.[22]) Many of the instruments traded in this market are highly specialized, and sometimes no liquid market, or easily observable market price, exists for them. However, both the vendors of them (most usually investment banks) and at least the more sophisticated purchasers of them can often calculate theoretical prices, and thus have a benchmark 'fair' price. The Black–Scholes–Merton analysis and subsequent developments of it are also central to the capacity of an investment bank to operate at large scale in this market. They enable the risks involved in derivatives portfolios to be decomposed mathematically. Many of these risks are mutually offsetting, so the residual risk that requires to be hedged is often quite small in relation to the overall portfolio. Major investment banks can thus ‘operate on such a scale that they can provide liquidity as if they had no transaction costs’ (Taleb, 1998: 36).[23] So the Black–Scholes–Merton assumption of zero transaction costs is now close to true for major investment banks – in part because the use of that theory and its developments by those banks allow them to manage their portfolios in a way which minimizes transaction costs.

Fourth, option pricing theory allowed a reconceptualization of risk that is only beginning to be recognized in the burgeoning literature on ‘risk society’.[24] Since 1973, a wide range of situations involving uncertainty have been reconceptualized as involving implicit options. Closest to traditional finance is the application of option theory to corporate liabilities such as bonds. Black and Scholes (1973: 649–52) pointed out that when a corporation’s bonds mature its shareholders can either repay the principal (and own the corporation free of bond liabilities) or default (and thus pass the corporation’s assets to the bond holders). A corporation’s bond holders have thus in effect sold a call option to its shareholders. This kind of reasoning allows, for example, calculation of implicit probabilities of bankruptcy. More generally, many insurance contracts have at least some of the structure of put options, and this way of thinking has facilitated the growing integration of insurance and derivatives trading (such as the sale of ‘hurricane bonds’ as a marketized form of reinsurance). Even areas that at first sight seem unlikely candidates for rethinking as involving implicit options have been conceptualized in this way: for example, professorial tenure, pharmaceuticals innovation, and decisions about the production of film sequels (Merton, 1998). In the case of film sequels, for instance, it is cheaper to make a sequel at the same time as the original, but postponing the sequel grants a valuable option not to make it: option theory can be used to calculate which is better. Option pricing theory has altered how risk is conceptualized, by practitioners as well as by theorists.

Conclusion: Bricolage, Exemplars, Disunity and Performativity

The importance of bricolage in the history of option pricing theory, especially in Black and Scholes’ work, is clear. They followed no rules, no set methodology, but worked in a creatively ad hoc fashion. Their mathematical work can indeed be seen as Lynch’s ‘particular courses of action with materials at hand’ (Lynch, 1985: 5) – in this case, conceptual materials. Consider, for example, Black and Scholes’ use of Sprenkle’s work. The latter would rate scarcely a mention in a ‘Whig’ history of option pricing: his model is, for example, dismissed in a footnote in Sullivan and Weithers’ history as possessing ‘serious drawbacks’ (Sullivan and Weithers, 1994: 41). True, central to Sprenkle’s work was the hope that analysing option pricing would reveal investors’ attitudes to risk, a goal that in the Black–Scholes–Merton analysis (which implies that options are priced as if all investors are entirely risk-neutral) is not achievable. Yet, as we have seen, Black and Scholes' tinkering with Sprenkle's equation was the key step in their finding a solution to their differential equation, and 'tinkering' is indeed the right word.[25]

It was, however, tinkering inspired by an exemplar, the Capital Asset Pricing Model. Here, the contrast with Thorp is revealing. He was far better-trained mathematically than Black and Scholes were, and had extensive experience of trading options (especially warrants), when they had next to none. He and Kassouf also conceived of a hedged portfolio of stock and options (with the same hedging ratio, $\frac{\partial w}{\partial x}$), and they, unlike Black and Scholes, had implemented approximations to such hedged portfolios in their investment practice. Thorp had even tinkered in essentially the same way as Black and Scholes with an equation equivalent to Sprenkle's (equation 1 above). But while Black and Scholes were trying to solve the option pricing problem by applying the Capital Asset Pricing Model, Thorp had little interest in the latter: he was aware of it, but not 'at the expert level'.[26] Indeed, for him the proposition (central to the mathematics of Black and Scholes, and in a different way to Merton's analysis as well) that a properly hedged portfolio could earn only the riskless rate would have stood in direct contradiction to his empirical experience. He and Kassouf were regularly earning far more than that from their hedged portfolios.

For Thorp, then, to have put forward Black and Scholes' or Merton's central argument would have involved overriding what he knew of empirical reality. For Scholes (trained as he was in Chicago economics), and even for Black (despite his doubts as to the precise extent to which markets were efficient), it was reasonable to postulate that markets would not allow money-making opportunities like a zero-β (or, in Merton's version, zero-risk) portfolio that earned more than the riskless rate. Thorp, however, was equally convinced that such opportunities *could* be found in the capital markets. The 'conventional wisdom' had been that 'you couldn't beat the casino': in the terminology of economics, that 'the casino markets were efficient'. Thorp had showed this was not true, 'so why should I believe these people who are saying the financial markets are efficient?' [Thorp interview].

Theoretical commitment was thus important to the development of option pricing. It was not, however, commitment to the literal truth of economics' models. Black and Scholes, for example, knew (indeed, they showed: see Black et al., 1972) that the Capital Asset Pricing Model's empirical accuracy was questionable. That, however, did not stop them regarding the model as identifying an economic process of great importance. Nor, crucially, did it deter them from using the model as a resource with which to solve the option pricing problem. Similarly, neither they, nor Merton, mistook their option model for a representation of reality. Black, for example, delighted in pointing out 'The Holes in Black–Scholes' (Black, 1988): economically consequential ways in which the model's assumptions were unrealistic. For Black, Scholes and Merton –

like the economists studied by Yonay and Breslau (2001) – a model had to be simple enough to be mathematically tractable, yet rich enough to capture the economically most important aspects of the situations modelled. Models were resources, not (in any simple sense) representations: ways of understanding and reasoning about economic processes, not putative descriptions of reality. If the latter is the criterion of truth, all of the financial economists discussed here would agree with their colleague Eugene Fama that any model is 'surely false' (Fama, 1991: 1590).

Nor were the theoretical inspirations and commitments of option pricing theorists unitary. Black–Scholes–Merton option pricing theory is central to the 'orthodox' modern economic analysis of financial markets. But that does not mean that Black, Scholes and Merton adhered to the same theoretical viewpoint. They disagreed, for example, on the validity of the original form of the Capital Asset Pricing Model. As we have seen, Merton considered the original derivations of the Black–Scholes equation unrigorous; Black remained to a degree a sceptic as to the virtues of Merton's derivation. Nor did this kind of disagreement end in 1973. For example, to Michael Harrison, an operations researcher (and essentially an applied mathematician) at Stanford University, the entire body of work in option pricing theory prior to the mid-1970s was insufficiently rigorous. Harrison and his colleague David Kreps asked themselves, 'Is there a Black–Scholes theorem?' From the viewpoint of the 'theorem-proof culture . . . I [Harrison] was immersed in' [Harrison interview] there was not. So they set to work to formulate and prove such a theorem, a process that eventually brought to bear modern 'Strasbourg' martingale theory (an advanced and previously a rather 'pure' area of probability theory).[27]

Divergences of this kind might seem to be a source of weakness. In the case of option pricing theory, however, they are a source of strength, even more directly so than in the more general case discussed by Mirowski and Hands (1998). If the Black–Scholes equation could be derived in only one way, it would be a fragile piece of reasoning. But it can be derived in several: not just in the variety of ways described above, but also, for example, as a limit case of the later finite-time Cox–Ross–Rubinstein model (Cox et al., 1979). Plug the log-normal random walk and the specific features of option contracts into Harrison and Kreps' martingale model, and Black–Scholes again emerges. Diversity indeed yields robustness. For example, as Black pointed out, defending the virtues of the original derivation from the Capital Asset Pricing Model, that derivation 'might still go through' even if the assumptions of the arbitrage-based derivation failed.[28]

This rich diversity of ways of deriving the Black–Scholes equation may prompt in the reader a profoundly unsociological thought: perhaps the equation is simply true? This is where this article's final theme, performativity, is relevant. As an empirical description of patterns of option pricing, the equation started out as only a rough approximation, but then pricing patterns altered in a way that made it more true. In part, this was because the equation was used in arbitrage. In part, it was because the hypothetical world embedded in the equation (perhaps especially in Merton's continuous-time derivation of it) has been becoming more real, at least in the core markets of the Euro-American world. As Robert C. Merton, in this context appropriately the son of Robert K. Merton (with his sensitivity to the dialectic of the social world and knowledge of that world), puts it, 'reality will eventually imitate theory' (Merton, 1992: 470; see Merton, 1936, 1949).

Perhaps, though, the reader's suspicion remains: that this talk of performativity is just a fancy way of saying that the Black–Scholes equation is the correct way to price options, but market practitioners only gradually learned that. Not so. The phase of increasing empirical accuracy of the Black-Scholes equation has been followed by a phase, since 1987, in which the fit of the empirical prices to the model has again deteriorated (Rubinstein, 1994). One way of expressing this partial breakdown after 1987 of the performativity of classic option theory is to note that while, as noted above, some of its assumptions have become more true (in part because of feedback loops from the theory), this has not been the case for the assumption of the log-normality of the price movements of stocks or other underlying assets. The gigantic one-day fall of the US stock market on 19 October 1987 was a grotesquely unlikely event on the assumption of log-normality: for example, Jackwerth and Rubinstein (1996: 1612) calculate the probability on that assumption of the actual fall in S&P index futures as 10^{-160}. In addition, 19 October was far more than the disembodied rejection of the null hypothesis of log-normality. The fall in stock prices came close to setting off a chain of market-maker bankruptcies that would have threatened the very existence of organized derivatives exchanges in the USA. The subsequent systematic departure from Black–Scholes option pricing – the so-called 'volatility skew'[29] – is more than a mathematical adjustment to empirical departures from log-normality: it is too large fully to be accounted for in that way (Jackwerth, 2000). It can in a sense be seen as the options market's collective defence mechanism against systemic risk (MacKenzie and Millo, forthcoming).

More generally, market practitioners' adoption of financial economics has not rendered fully performative economics' pervasive, often implicit, underlying assumption of rational egoism. *Pace* Callon (1998), *homo œconomicus* has not in general been brought fully into being. What has not to date been grasped in the debate over economics' performativity (for example Miller, 2002) is that there exists a reasonably precise probe as to whether or not actors have been configured into *homines œconomici*: collective action, in other words action that advances the interests of an entire group but in regard to which the rational egoist will free-ride. (A classic example of collective action is blood donation in a country such as the UK where such donation is unremunerated [Titmus, 1970]. Well-stocked blood banks are in the collective interest of the entire population of the UK, but a rational egoist would nonetheless be unlikely to donate blood because the minor inconvenience and discomfort involved would almost certainly outweigh the miniscule probability of benefiting personally from his or her own donation.) As the analysis by Olson (1980) famously shows, if all actors are *homines œconomici* they will all free-ride in such a situation, and collective action will therefore be impossible.

However, participants in financial markets have, at least to some extent, retained the capacity for collective action. The very creation of the Chicago Board Options Exchange, which set in train the key processes that have rendered option theory performative, involved donations of unremunerated time that were structurally akin to blood donation (MacKenzie and Millo, forthcoming). The classic social network analysis of option pricing by Baker (1984) can, likewise, be read as showing the persistence, at least in CBOE's smaller trading crowds, of collective action, and, as noted above, the volatility skew can also be interpreted, at least tentatively, as collective action.

The analysis of economics' performativity does not point, therefore, to the smoothly performed world feared by Callon's critics such as Miller (2002). It points to contested terrain. When, in 1968, David Durand, a leading figure in the older form of the academic study of finance, inspected the mathematical models that were beginning to transform his field, he commented that 'The new finance men . . . have lost virtually all contact with terra firma' (Durand, 1968: 848). As we have seen, the decades since 1968 have seen the world of finance change in such a way that the apparently ungrounded models that horrified Durand have gained verisimilitude as they have become incorporated into the structures and practices of markets. However, the financial markets remain, and I suspect will always remain, an only partially configured world. The struggles to configure that world, and the forces opposing and undermining that configuring, are, and will remain, at the heart of the history of our times.

List of Interviews

Fama, Eugene, interviewed by author, Chicago, 5 November 1999.
Harrison, J. Michael, interviewed by author, Stanford, CA, 8 October 2001.
Kassouf, Sheen, interviewed by author, Newport Beach, CA, 3 October 2001.
Merton, Robert C., interviewed by author, Cambridge, MA, 2 November 1999.
Miller, Merton, interviewed by author, Chicago, 5 November 1999.
Rissman, Burton, interviewed by author, Chicago, 9 November 1999.
Samuelson, Paul A., interviewed by author, Cambridge, MA, 3 November 1999.
Scholes, Myron S., interviewed by author, San Francisco, CA, 15 June 2000.
Sprenkle, Case M., interviewed by author by telephone to Champaign, IL, 16 October 2002.
Thorp, Edward O., interviewed by author, Newport Beach, CA, 1 October 2001.
Treynor, Jack, interviewed by author, Palos Verdes Estates, CA, 3 October 2001.

Notes

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1. See, for example, Klaes (2000), Mirowski (1989 and 2002), Sent (1998), Weintraub (1991), Yonay (1994), Yonay and Breslau (2001).
2. Examples include Izquierdo (1998, 2001), Knorr Cetina and Bruegger (2002), Lépinay (2000), Lépinay and Rousseau (2000), MacKenzie (2001), Millo (forthcoming), Muniesa (2000), Preda (2002). This body of work interacts with a pre-existing tradition of the sociology and anthropology of financial markets, such as Abolafia (1996, 1998), Baker (1984), Hertz (1998), Smith (1999).
3. Data from Bank for International Settlement, www.bis.org. These figures are adjusted for the most obvious forms of double-counting, but still arguably exaggerate the economic significance of derivatives markets. Swaps, for example, are measured by notional principal, when this is not in fact exchanged. See also note 22 below.
4. Aside from the recollections of Black and Scholes themselves (Black, 1989; Scholes, 1998), the main existing history is Bernstein (1992: chapter 11), which eschews detailed mathematical exposition. More mathematical, but unfortunately somewhat Whiggish (see below), is Sullivan and Weithers (1994).
5. Bricoleur is French for odd-job person. Lévi-Strauss (1966) introduced the Anglo-Saxon social sciences to the metaphor. Its appropriateness to describe science is argued in Barnes (1974, chapter 3).
6. 'Contango' was the premium paid by the purchaser of a security to its seller in return for postponing payment from one settlement date to the next.
7. Bachelier (1900: 21, 35, 37); the quotations are from the English translation (Bachelier, 1964: 17, 28–29, 31). In the French market studied by Bachelier, option prices were fixed and strike prices variable (the reverse of the situation studied by the American authors discussed below), hence Bachelier's interest in the determination of strike prices rather than option prices.
8. Interviews by the author drawn on in this paper are listed as an appendix
9. Readers of Galison (1997) will not be surprised to discover there are deep issues here as to the meaning of 'random', in particular as to the precise nature of the stochastic dynamics of stock prices. Unfortunately, these cannot be discussed here.
10. To avoid confusion, I have made minor alterations (e.g. interchanging letters) to the notation used by the authors whose work is described, and have sometimes slightly rearranged the terms in equations. More substantial differences between their mathematical approaches are preserved.
11. Stock price 'trend' was measured by 'the ratio of the present price to the average of the year's high and low' (Kassouf, 1965: 50).
12. The curves are of course specific to an individual warrant, but as well as providing their readers with Kassouf's formula for calculating them, Thorp and Kassouf (1967: 78–79) provided standardized 'average' curves based on the prices of 1964–66.
13. As Thorp explained (Thorp, 1973: 526), 'to sell warrants short [and] buy stocks, and yet achieve the riskless rate of return requires a higher warrant short sale price than for the corresponding call [option]' under the Black-Scholes assumptions. Thorp had also been selling options in the New York market, where the seller did receive the sale price immediately (minus 'margin' retained by the broker), but the price discrepancies he was exploiting were gross (so gross he felt able to proceed without hedging in stock), and thus the requisite discount factor was not a salient consideration.
14. See Treynor (1962). The dating of this unpublished paper follows a private communication to the author from Jack Treynor, 4 March 2003. See also Lintner (1965), Mossin (1966), Sharpe (1964). Treynor's typescript draft was eventually published as Treynor (1999).

15. Treynor interview; Black (1989: 5). Treynor and Black did not publish their work immediately: it eventually appeared in 1976. The corrected differential equation is equation 2 of their paper (Treynor and Black, 1976: 323).
16. Unfortunately, I have been unable to locate any contemporaneous documentary record of this initial phase of Black's work on option pricing, and it may be that none survives. The earliest extant version appears to date from August 1970 (Black and Scholes, 1970a), and is in the personal files of Prof. Stewart Myers at MIT (I am grateful to Perry Mehrling for a copy of this paper). There is an October 1970 version in Fischer Black's papers (Black and Scholes, 1970b). Black's own account of the history of option formula (Black, 1989: 5) contains only a verbal description of the initial phase of his work. It seems clear, however, that what is being described is the 'alternative derivation' of the October paper (Black and Scholes, 1970b: 10–12): the main derivation in that paper and in Black and Scholes (1970a) is the hedged portfolio derivation described below, which was chronologically a later development.
17. Thus in Black and Scholes (1970b: 8–9) they show that the covariance of the hedged portfolio with the overall level of the market was zero, assuming that in small enough time intervals changes in stock price and in overall market level have a joint normal distribution. Using the Taylor expansion of *w*, Black and Scholes showed that the covariance of warrant price changes with market level changes is: $\frac{1}{2}\frac{\partial w}{\partial x}$ cov (Δx^2, Δm), where 'cov' indicates covariance and Δm is the change in market level. If Δx and Δm are jointly normally distributed over small time periods, cov (Δx^2, Δm) is the covariance of the square of a normal variable with a normal variable, which is always zero. With a zero covariance with the market, the hedged portfolio must, according to the Capital Asset Pricing Model, earn the riskless rate of interest.
18. A quadratic utility function has the form $U(y) = l + my + ny^2$, where l, m, and n are constant: n must be negative if, as will in general be the case, 'the investor prefers smaller standard deviation to larger standard deviation (expected return remaining the same)' (Markowitz, 1959: 288), and negative n implies that above a threshold value utility will diminish with increasing returns. Markowitz's position is that while quadratic utility cannot reasonably be assumed, a quadratic function centred on expected return is a good approximation to a wide range of utility functions: see Levy and Markowitz (1979).
19. See note 17 above for how Black and Scholes demonstrated $\beta = 0$ in the October 1970 version of their paper.
20. Fischer Black interviewed by Zvi Bodie, July 1989. I'm grateful to Prof. Bodie for a copy of the transcript of this unpublished interview.
21. See Jarrow (1999), though Jarrow has in mind a sense of 'changed the world' weaker than performativity.
22. Data from Bank for International Settlements, www.bis.org. Many over-the-counter derivatives positions are closed out by entering into offsetting derivatives contracts, so the comparison probably overstates the relative importance of the over-the-counter market, but it is nonetheless substantial.
23. See also Hull (2000: 54) on the extent to which typical assumptions of finance theory are true of major investment banks.
24. Beck (1992). For one of the few treatments bringing financial risk (but not option theory) into the discussion, see Green (2000).
25. It is used in a one-sentence summary of Black's own history (Black, 1989: 4), but the summary is probably an editorial addition, not Black's own.
26. Edward O. Thorp, email message to author, 19 October 2001.
27. See Harrison and Kreps (1979) and Harrison and Pliska (1981). The first derivation of the Black–Scholes formula that Harrison and Kreps would allow as reasonably rigorous is in Merton (1977). This latter paper explicitly responds to queries that had been raised about the original derivation. For example, Smith (1976: 23) had noted that the option price, *w*, is, in the original work, assumed but not proved 'to be twice differentiable everywhere'.

28. Fischer Black interviewed by Zvi Bodie, July 1989.
29. In the Black–Scholes–Merton model, the relationship of implied† volatility to strike† price is a flat line. Since October 1987, however, the relationship has become skewed, with options with low strike prices having higher implied volatilities than those with higher strike prices (Rubinstein, 1994). The option market has come to 'expect' crashes, in other words.

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