Statistics 955

Stochastic Calculus and Financial Applications

Professor J. Michael Steele

Prerequisites: This course is designed for students who want to develop professional skill in stochastic calculus and its application to problems in finance. Students are expected to have had some graduate-level experience with probability and real analysis. The course tries to attend to modeling issues, but much of our efforts are focused on mathematical foundations. Students who get the most out of the course are those with an appreciation — or even a appetite — for mathematical proofs and problem solving.

Course Plan: The course begins with simple random walk and the analysis of gambling games. This material is used to motivate the theory of martingales, and, after reaching a decent level of confidence with discrete processes, the course takes up the more demanding development of continuous time stochastic process, especially Brownian motion. The construction of Brownian motion is given in detail, and enough material on the subtle properties of Brownian paths is developed so that the student should evolve a good sense of when intuition can be trusted and when it cannot. The course then takes up the Itô integral and aims to provide a development that is honest and complete without being pedantic.

With the Itô integral in hand, the course focuses more on models. Stochastic processes of importance in Finance and Economics are developed in concert with the tools of stochastic calculus that are needed in order to solve problems of practical importance. The financial notion of replication is developed, and the Black-Scholes PDE is derived by three different methods. The course then introduces enough of the theory of the diffusion equation to be able to solve the Black-Scholes PDE and prove the uniqueness of the solution. The foundations for the martingale theory of arbitrage pricing are then prefaced by a well motivated development of the martingale representation theorems and Girsanov theory. Arbitrage pricing is then revisited and the notions of attainability and completeness are developed in order to give a clear view of the fundamental formula for the pricing of contingent claims.

Texts: *Stochastic Calculus and Financial Applications*, J.M. Steele, (Springer, 2003).

Homework: Regular homework will be assigned.

Grading: Grades are based on homework (10 %) in-class midterm (20%) and take-home final exam (70%)

Auditors: Auditors are welcome.

Office Hours: Monday 3:00-4:00 Wednesday 3:00-4:00