WHY YULE-WALKER SHOULD NOT BE USED FOR AUTOREGRESSIVE MODELLING

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Abstract - Autoregressive modelling of noise data is widely used for system identification, surveillance, malfunctioning detection and diagnosis. Several methods are available to estimate an autoregressive model. Usually, the so-called Yule-Walker method is employed. The various estimation methods generally yield comparable parameter estimates. In some special cases however, involving nearly periodic signals, the Yule-Walker approach may lead to incorrect parameter estimates. Burg's method offers the best alternative to Yule-Walker. In this paper a theoretical explanation of this phenomenon is given, while the 1994 IAEA Benchmark test is presented as a practical example of Yule-Walker yielding poor parameter estimates.

I. INTRODUCTION

Autoregressive modelling of noise data was introduced in nuclear engineering in the mid seventies and gained popularity during the decades thereafter. A historical survey of the gradual acceptation and the diversity of its applications can be found in the so-called SMORN-proceedings (SMORN-III, SMORN-IV, SMORN-V). Nowadays, autoregressive modelling is a widely used means for performing system identification, surveillance, malfunctioning detection and diagnosis. Its attractiveness stems, among others, from the fact that the numerical algorithms involved are rather simple.

An autoregressive model depends on a limited number of parameters, which are estimated from measured noise data. Several methods exist to estimate the autoregressive parameters, such as least-squares, Yule-Walker and Burg's method. It can be shown that for large data samples these estimation techniques should lead to approximately the same parameter estimates. Mainly for historical reasons, most people use either the Yule-Walker or the least-squares method. This paper will show, however, that in some special cases the Yule-Walker estimation method leads to poor parameter estimates, even for moderately sized data samples. Least squares should not be used either, as it may lead to an unstable model. Burg's method is preferable.

In section II, we will present an overview of the basics of autoregressive modelling. The mathematical circumstances causing poor parameter estimates in case of the Yule-Walker technique are described in section III, while some simulations of autoregressive processes are discussed that support our hypotheses. Finally, in section IV, we will illuminate our findings with the application of autoregressive modelling for anomaly detection in the 1994 IAEA Benchmark noise data (Journeau, 1994).

II. THEORY OF AUTOREGRESSIVE MODELLING

The successive samples y_t of an autoregressive process linearly depend on their predecessors:

$$y_t + a_1 y_{t-1} + a_2 y_{t-2} + \cdots + a_p y_{t-p} = \eta_t , \tag{1}$$

in which a_i are the autoregressive parameters and the innovations η_t are a stationary purely random process with zero mean. It can be shown that the autocovariance function R_τ for delays 0 to *p* is related to the autoregressive parameters a_i through the Yule-Walker equation for the autoregressive process (Priestley, 1994):

$$\begin{pmatrix} R_0 & R_1 & \cdots & R_{p-1} \\ R_1 & R_0 & \cdots & R_{p-2} \\ \vdots & \vdots & \ddots & \vdots \\ R_{p-1} & R_{p-2} & \cdots & R_0 \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_p \end{pmatrix} = - \begin{pmatrix} R_1 \\ R_2 \\ \vdots \\ R_p \end{pmatrix} . \tag{2}$$

An estimated autoregressive model of the same order *p* can be written as

$$y_t + \hat{a}_1 y_{t-1} + \hat{a}_2 y_{t-2} + \cdots + \hat{a}_p y_{t-p} = \hat{\eta}_t , \tag{3}$$

in which $\hat{a}_i$ are the autoregressive-parameter estimates and $\hat{\eta}_t$ are the estimated innovations. A clear distinction should be made between the autoregressive process (Eq. (1)) and the corresponding autoregressive model (Eq. (3)) (Broersen and Wensink, 1993). Using Eq. (3), each data sample can be predicted from its predecessors:

$$\hat{y}_t = - \sum_{i=1}^{p} \hat{a}_i y_{t-i} . \tag{4}$$

As the samples y_t cannot be predicted exactly, a residue is introduced, which is defined as the difference between the measured value and the estimated value:

$$\text{residue} \equiv y_t - \hat{y}_t = \hat{\eta}_t , \tag{5}$$

which means that the residue is equal to the estimated innovation, as introduced in Eq. (3).

It is assumed in these equations that the autoregressive model order *p* is known. In practice, the model order has to be estimated as well, which is usually done using Akaike's criterion (Priestley, 1994).

Suppose that the *estimation* realisation *y* consists of *N* data points (an estimation realisation contains those data points that are used for parameter estimation). Three methods of autoregressive-parameter estimation from these data samples shall be considered here, being the least-squares approach (LS), the Yule-Walker approach (YW) and Burg's method (Burg):

- LS: The total squared residue over the data samples *p* + 1 to *N* is minimised, leading to a system of linear equations:

$$\begin{pmatrix} c_{11} & c_{12} & \cdots & c_{1p} \\ c_{21} & c_{22} & \cdots & c_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ c_{p1} & c_{p2} & \cdots & c_{pp} \end{pmatrix} \begin{pmatrix} \hat{a}_1 \\ \hat{a}_2 \\ \vdots \\ \hat{a}_p \end{pmatrix} = - \begin{pmatrix} c_{01} \\ c_{02} \\ \vdots \\ c_{0p} \end{pmatrix} , \tag{6}$$

in which the matrix elements

$$c_{ij} \equiv \frac{1}{N-p} \sum_{t=p+1}^{N} y_{t-i} y_{t-j} \tag{7}$$

form an unbiased estimate of the autocovariance function for delay *i* - *j*.

- YW: The first and last *p* data points are also included in the summation of Eq. (7), resulting in

$$\begin{pmatrix} \hat{R}_0 & \hat{R}_1 & \cdots & \hat{R}_{p-1} \\ \hat{R}_1 & \hat{R}_0 & \cdots & \hat{R}_{p-2} \\ \vdots & \vdots & \ddots & \vdots \\ \hat{R}_{p-1} & \hat{R}_{p-2} & \cdots & \hat{R}_0 \end{pmatrix} \begin{pmatrix} \hat{a}_1 \\ \hat{a}_2 \\ \vdots \\ \hat{a}_p \end{pmatrix} = -\begin{pmatrix} \hat{R}_1 \\ \hat{R}_2 \\ \vdots \\ \hat{R}_p \end{pmatrix}, \tag{8}$$

in which the matrix elements $\hat{R}_\tau$ constitute a biased estimate of the autocovariance function (Parzen, 1961):

$$\hat{R}_\tau \equiv \frac{1}{N} \sum_{t=\tau+1}^{N} y_t y_{t-\tau} \ . \tag{9}$$

The Levinson-Durbin algorithm provides a fast solution of a system of linear equations containing a Toeplitz-style matrix as in Eq. (8). Both Eqs. (6) and (8) are in fact approximations to the Yule-Walker process equation (Eq. (2)).

- Burg: The parameter estimation approach that is nowadays regarded as the most appropriate, is known as *Burg's* method. In contrast to the least-squares and Yule-Walker method, which estimate the autoregressive parameters directly, Burg's method first estimates the reflection coefficients, which are defined as the last autoregressive-parameter estimate for each model order *p*. From these, the parameter estimates are determined using the Levinson-Durbin algorithm. The reflection coefficients constitute unbiased estimates of the partial correlation coefficients.

Usually, these estimation methods lead to approximately the same results for the autoregressive parameters. Once these have been estimated from the time series *y*, the autoregressive model can be applied to an independent *prediction* realisation *x* of the same stochastic process. In terms of *x*, the autoregressive process (Eq. (1)) can be written as

$$x_t + a_1 x_{t-1} + a_2 x_{t-2} + \cdots + a_p x_{t-p} = \varepsilon_t, \tag{10}$$

in which the innovation process ε_t is statistically identical to the innovation process η_t. The corresponding autoregressive model can be written as in Eq. (3):

$$x_t + \hat{a}_1 x_{t-1} + \hat{a}_2 x_{t-2} + \cdots + \hat{a}_p x_{t-p} = \hat{\varepsilon}_t, \tag{11}$$

in which $\hat{a}_i$ are the autoregressive parameters estimated from realisation *y* and $\hat{\varepsilon}_t$ are the estimated innovations. As in Eq. (4), each data sample can be estimated from its predecessors:

$$\hat{x}_t = -\sum_{i=1}^{p} \hat{a}_i x_{t-i} \ . \tag{12}$$

The difference between the measured value and the estimated value is now defined as the prediction error:

$$\text{prediction error} \equiv x_t - \hat{x}_t = \hat{\varepsilon}_t \ . \tag{13}$$

The prediction error is therefore equal to the estimated innovation, as introduced in Eq. (11). Each prediction error can be calculated once the actual value of the data point is measured.

A clear distinction should be made between the residue and the prediction error and their variances (Broersen and Wensink, 1993). The residual variance $\text{var}(\hat{\eta}_t)$ is a measure for the fit of the autoregressive model to those data that have been used for the estimation of the autoregressive parameters, and can be estimated from the realisation *y*, which is used for the parameter estimation:

$$\hat{\text{var}}(\hat{\eta}_t) = \frac{1}{N-p} \sum_{t=p+1}^{N} \left(y_t - \hat{y}_t\right)^2 . \tag{14}$$

For the prediction of future data, instead of the residual variance, the variance of the prediction error $\text{var}(\hat{\varepsilon}_t)$ is essential. If the independent prediction realisation *x* contains N' data samples, the prediction error variance can be estimated from the sample variance:

$$\hat{\mathrm{var}}\left(\hat{\varepsilon}_t\right) = \frac{1}{N'-p}\sum_{t=p+1}^{N'}\left(x_t - \hat{x}_t\right)^2 . \tag{15}$$

The LS parameter estimation is based on the minimisation of the residual variance. Such a minimisation however does not imply that the variance of the prediction error is minimised as well. As usually the minimisation of the prediction error variance is our goal, the LS estimation of the autoregressive parameters is not necessarily superior to YW or Burg's method.

III. STABILITY, POLE LOCATION AND PARAMETER ESTIMATION

For large data samples, the difference between the estimates obtained by the various methods will be small (Priestley, 1994). In some special cases however, substantial differences may arise between these approaches even for data samples of moderate size. In the present paper it will be shown that YW should always be avoided. The behaviour of the autoregressive process as described by its pole locations is essential in this respect.

Using the backward shift operator z^{-1}, which is defined as $z^{-1}x_t = x_{t-1}$, and defining $a_0 \equiv 1$, a realisation of an autoregressive process can be expressed in terms of the innovations sequence ε_t as:

$$x_t = z^p\left(\sum_{i=0}^{p} a_i z^{p-i}\right)^{-1}\varepsilon_t, \tag{16}$$

ignoring the so-called complementary function (Priestley, 1994). The autoregressive operator $z^p\left(\sum_{i=0}^{p} a_i z^{p-i}\right)^{-1}$ obviously contains a p-fold zero at $z = 0$, as well as p poles determined by the characteristic equation of the autoregressive process

$$\sum_{i=0}^{p} a_i z^{p-i} = 0. \tag{17}$$

The roots of characteristic equation should lie inside the unit circle to ensure the autoregressive process to be stable. If the roots lie on the unit circle, the autoregressive process will only be stationary in case of ε_t being identical to zero. In that case a harmonic process will result, consisting of a sum of cosine functions.

One might wonder what will happen if the autoregressive process has poles in the neighbourhood of the unit circle. As poles on the unit circle represent a harmonic process, an autoregressive process with poles near the unit circle can be expected to demonstrate some kind of pseudo-periodic behaviour (Priestley, 1994). In this case the autocovariance function can be described as a sum of weakly damped periodic functions. Furthermore, as the noise terms ε_t are still present, the autoregressive process may exhibit a kind of almost non-stationary behaviour. Finally, the partial autocorrelation coefficients will be close to unity in absolute value. In the context of linear filtering theory, this means that the transfer function relating x_t to ε_t will be close to instability in the filtering sense (Oppenheim, 1978).

The pole locations will also affect the reliability of the various parameter estimation techniques. It is claimed by Priestley (1994) that YW may lead to poor parameter estimates, even for moderately large data samples, if the autoregressive operator has a pole near the unit circle. This is the more remarkable since LS and YW only differ in their treatment of the first and last p data points. Since this limited number of data points is relatively small compared to the total number of data points used for parameter estimation, it would be expected that LS and YW lead to almost the same results.

In this paper, instead of the pole locations, the poor condition of the autocovariance matrix in Eq. (2) is regarded as the cause of poor YW estimates. Side-effects of the poor autocovariance matrix condition are an almost non-stationary, pseudo-periodic behaviour of the autoregressive process as well as poles located closely to the unit circle and partial autocorrelation coefficients close to unity. These features can be used to detect the possibility of poor YW estimates, but should not be regarded as its cause. It should be noted

that autoregressive processes are possible having poles near the unit circle while the autocovariance matrix is well conditioned. In those cases YW will still provide correct results for the autoregressive parameters.

To introduce the concept of matrix conditioning, we consider a general system of linear equations:

$$Ax = B, \tag{18}$$

in which A is a matrix of order p and B is a vector of dimension p. A well-known result from linear algebra states that Eq. (18) cannot have one single solution if the matrix A is singular:

$$\det(A) = 0. \tag{19}$$

In cases in which matrix A is almost singular, the solution of Eq. (18) will be extremely sensitive to perturbations in either matrix A or vector B. The sensitivity to these perturbations can be measured with the so-called condition number, which is defined as

$$\kappa(A) = \|A\| \cdot \|A^{-1}\|, \tag{20}$$

where $\|\cdot\|$ is some matrix norm (Cybenko, 1980). In our case the 1-norm will be considered:

$$\|A\| = \max\left(\sum_{i=1}^{p} |A_{ij}| : j \in \{1,2,\ldots,p\} \right), \tag{21}$$

in which A_{ij} denote the matrix elements. The larger the condition number, the more sensitive the solution of Eq. (18) will be to perturbations. Roughly speaking, if $\kappa(A) = 10^p$, we may expect to lose about p significant figures in inverting an approximation to A. It should be noted that in order to calculate the autoregressive parameters from Eq. (2), an inversion of the matrix on the left hand side is required. A detailed treatment of matrix computations, norms and condition numbers is given by Stewart (1973).

The poor coefficient estimates in case of YW can be explained in terms of the condition of the autocovariance matrix in Eq. (2) (Cybenko, 1980). If the autocovariance matrix is poorly conditioned, the solution of Eq. (2) will strongly depend on perturbations in the coefficients R_τ. The bias in the YW autocovariance estimates $\hat{R}_\tau$ is one of these perturbations. Although this bias is usually too small to jeopardise the parameter estimation, in case of a poorly conditioned autocovariance matrix it will be magnified, as a result of which the YW parameter estimates will be useless.

In case of a first-order autoregressive process, the condition number reduces to unity for all pole locations. Therefore, YW will always yield a correct result for the parameter estimate in case of a first-order process. In case of second-order autoregressive processes, poor YW estimates may occur depending on the exact pole locations. Two second-order autoregressive processes are considered here, one having has its poles on the positive real axis (I), while the second one (II) has its poles on the positive imaginary axis. Simulations were made using LS, YW and Burg for poles approaching the unit circle. The condition number of the auto-covariance matrix can be calculated theoretically as a function of the distance to the unit circle, which is shown in Fig. 1. As the condition number in case of autoregressive process (I) increases dramatically, it is expected that YW will perform poorly if the poles are located near the unit circle. In case of autoregressive process (II), the condition

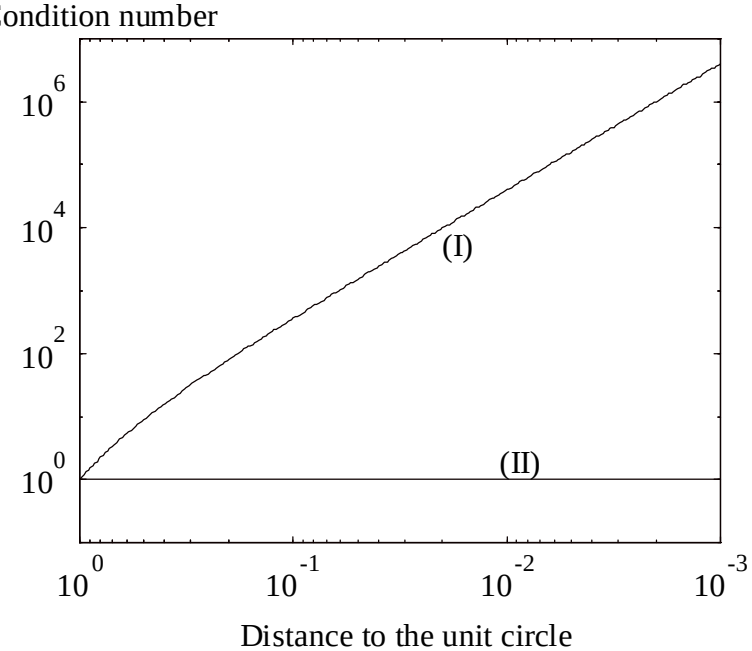

Fig. 1 Calculated condition number of the auto-covariance matrix in case of autoregressive processes (I) and (II).

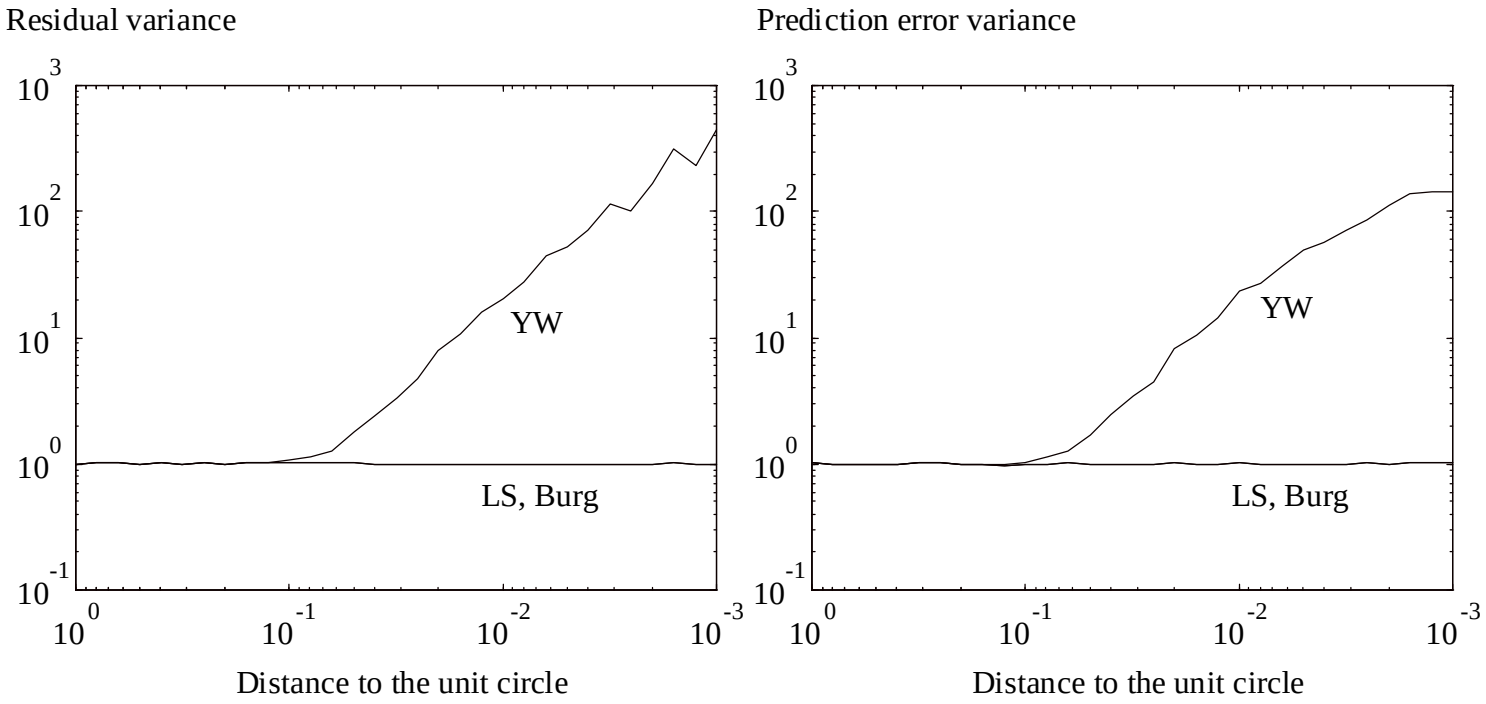

Fig. 2 Simulation results for the residual and the prediction error variance in case of autoregressive process (I), having poles on the positive real axis, using the various estimation techniques.

number equals unity for all pole distances to the unit circle. Therefore, we expect YW to perform well in this case.

Each simulation consisted of 1024 data points, using a normally distributed purely random innovation process having unit variance. To prevent the occurrence of close to non-stationary behaviour, the recorded simulations were preceded with 10240 dummy simulations. Each simulation was carried out 25 times. In each simulation, the residual and the prediction error variance as well as the first and second autoregressive parameter were estimated, which were thereupon averaged over the number of simulations. The residual variance was estimated from Eq. (14). The prediction error variance was estimated using a second series of 1024 data samples. The first series served as the estimation realisation, while the second series provided a prediction realisation in order to estimate the prediction error variance from Eq. (15).

The simulation results for the residual variance as well as the prediction error variance in case of autoregressive process (I) are shown in Fig. 2. While LS and Burg still yield a residual variance close to the actual value (being unity) as the poles approach the unit circle, YW is no longer able to describe the autoregressive process correctly. Even for poles located at 0.01 from the unit circle, the residual variance in case of YW is almost twenty times as large as in case of LS. Furthermore, it was found that the autoregressive-parameter estimates were not accurate in case of YW. The first and second autoregressive-parameter estimates and their actual values for autoregressive process (I) are plotted in Fig. 3. The Yule-Walker technique actually estimates a first-order autoregressive model, since the second autoregressive-parameter estimate approaches zero, while the first autoregressive-parameter is estimated to be its value in a first-order model.

In case of autoregressive process (II), no such effects were found. All of the estimation techniques, including YW, provided correct results for the residual and prediction error variance as well as for the estimated parameters, due to the matrix condition remaining bounded. These results agree with our expectations.

We can conclude that YW should not be used to estimate autoregressive parameters if the autocovariance matrix is almost singular. LS should not be used either for reasons of stability of the estimated model. The stability of an estimated autoregressive model can be verified by substituting the estimated autoregressive parameters into Eq. (17). If there turns out to be a root lying outside the unit circle, the estimated autoregressive model becomes invalid as the theory of autoregressive modelling is applicable to stationary stochastic processes only. Fortunately, YW as well as Burg guarantees the estimated model to be stable, in contrary to LS. It can furthermore be shown that slight deviations in the

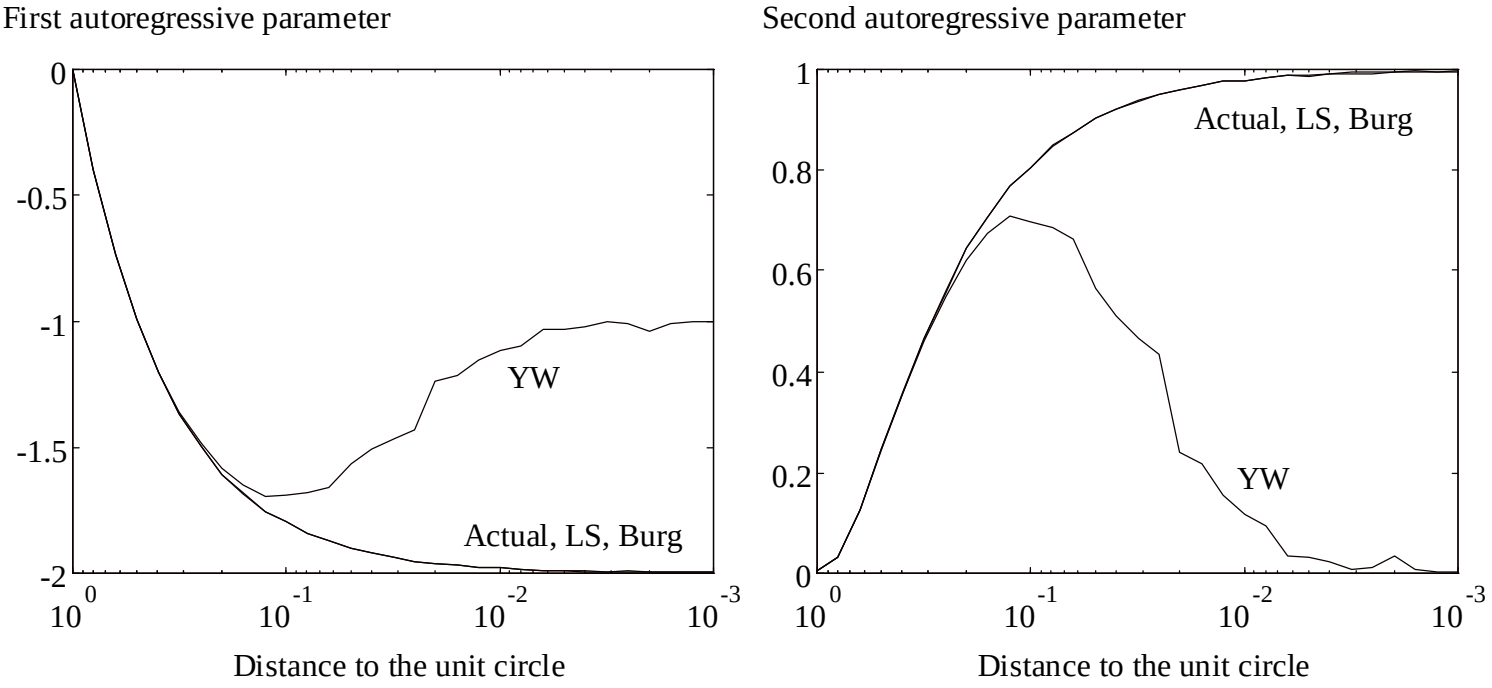

Fig. 3 Simulation results for the first and second autoregressive-parameter estimate in case of autoregressive process (I), having poles on the positive real axis, using the various estimation techniques (Actual = the actual value of the autoregressive parameter).

autoregressive-parameter estimates can result in large deviations in the estimated pole locations if the poles of an autoregressive process are located near the unit circle (Tretter, 1976). Therefore, each slight deviation is the parameter estimates can result in an unstable autoregressive model if the LS approach is employed. Burg is the only reliable autoregressive-parameter estimation technique, yielding accurate parameter estimates as well as an autoregressive model guaranteed to be stable.

We will now turn to an actual autoregressive analysis in which poor YW parameter estimation occurred, having detrimental effects on its conclusions.

IV. AUTOREGRESSIVE ANALYSIS OF THE 1994 IAEA BENCHMARK

One of the applications of autoregressive modelling in nuclear reactor analysis is the detection of anomalies during the reactor operation. The basic idea is to determine an autoregressive model of measured signals in a nuclear reactor during normal operation. It is assumed that the autoregressive model thus found will no longer be applicable in case of a disturbance of the reactor operation. This will lead to a large prediction error variance during the anomaly, which can then be detected.

In this section, we will discuss the autoregressive analysis of the 1994 IAEA Benchmark test data aimed at the detection of anomalies. A detailed description of this Benchmark test is provided by Journeau (1994). Noise measurements during normal reactor operation were available, as well as synthesised noise data that contained an anomaly during the reactor operation. As the sampling interval is not relevant for our discussion on the performance of YW, we will use a discrete time axis. Hoogenboom and Schoonewelle employed autoregressive analysis as described previously to determine the onset of the anomaly (1994a, 1994b). The noise data during normal operation were used for determining the autoregressive model of the steady-state process, while the synthesised noise data were used for anomaly detection by spotting sudden increases in the prediction error variance. Hoogenboom and Schoonewelle (1994b) concluded that increases in the prediction error variance due to anomalies occurred only if the autoregressive parameters were estimated using LS. Employing YW, there is hardly any increase in the prediction error variance during the anomaly.

These results are due to the nature of the noise data used for parameter estimation. Figure 4 shows, in arbitrary units, a section of 300 consecutive data points of the noise signal during normal reactor operation

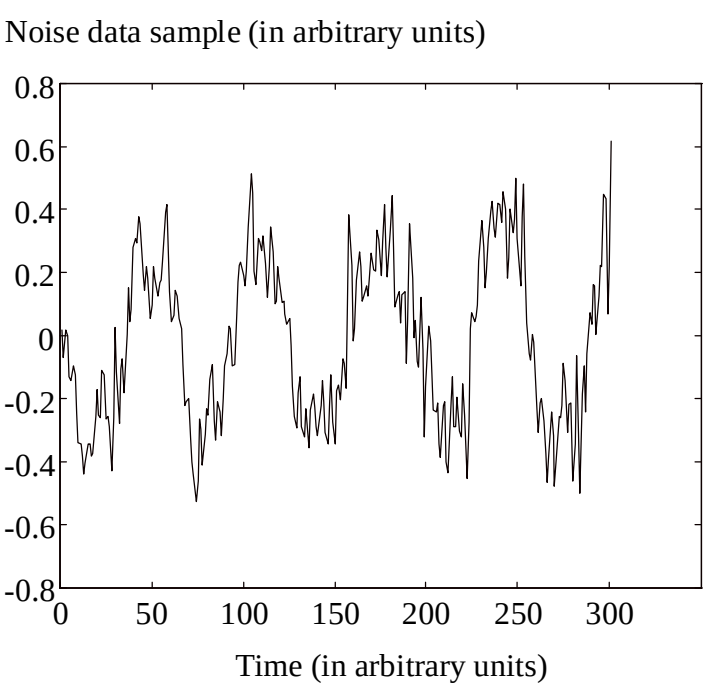

Fig. 4 Section of the noise signal during normal reactor operation containing 300 consecutive noise data points in arbitrary units.

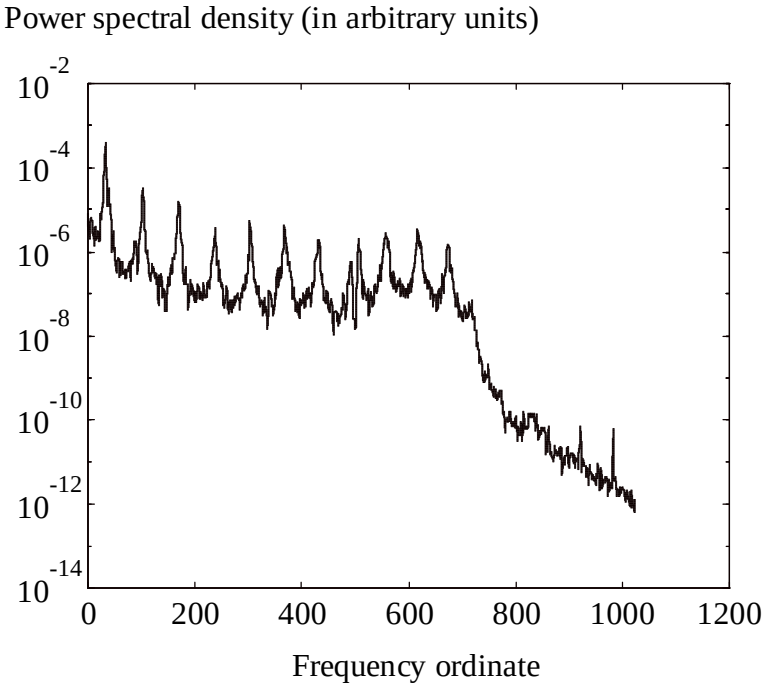

Fig. 5 Estimated spectrum of the noise data in arbitrary units as a function of the discrete frequency ordinates.

to illustrate its almost periodic behaviour. It shows that a strong cyclical component is present, having a period of about 60 times the sampling interval, as well as periodic components at higher frequencies.

The almost periodic behaviour of the background noise can also be demonstrated by estimating its power spectral density, which is shown semi-logarithmically in Fig. 5. Since the spectrum contains large peaks at specific frequency ordinates, it can be concluded that strong harmonic components are present.

Finally, the pseudo-periodic behaviour of the noise data is shown by the estimated pole locations (Fig. 6). Since the poles are all located closely to the unit circle, the autoregressive process will behave pseudo-periodically.

In this case, the pseudo-periodic behaviour of the noise data leads to a poor condition of the autocovariance matrix. Because the autocovariance function is not available theoretically as in the previous chapter, the matrix condition can only be estimated. The best estimate can be calculated from the LS autocovariance matrix, because all of its estimates are unbiased. This results in

$$\kappa = 9.7620 \cdot 10^{6}. \qquad (22)$$

This condition number is extremely large. The matrix condition is extremely poor (compare to Fig. 1), as a result of which poor YW parameter estimates can be expected.

The autoregressive model was estimated from the first 2048 data points applying LS, YW and Burg. Using Akaike's Information Criterion (*AIC*), model order p = 32 was selected. The autoregressive-parameter estimates are given in the Appendix. LS and Burg lead to comparable parameter estimates, while those estimated by YW deviate strongly. The poles of the estimated models were always located inside the unit circle, thereby fulfilling the condition for stability.

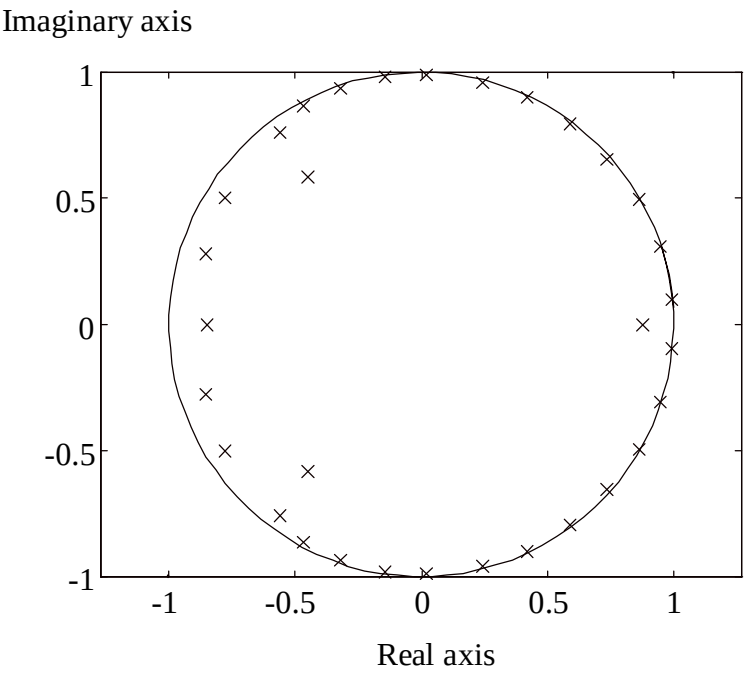

Fig. 6 Poles of the estimated autoregressive model (LS).

The residual variance $\mathrm{var}(\hat{\eta})$ was estimated for each estimation procedure from the estimation realisation using Eq. (14), while the prediction error variance $\mathrm{var}(\hat{\varepsilon})$ was estimated using Eq. (15) from a prediction realisation containing 2048 data points. Table I shows the results. As the residual variance and the prediction error variance in case of YW are about twice as large as the respective figures for LS and Burg, it can be concluded that YW does not yield a correct autoregressive model.

Table I Residual and prediction error variance in case of the various estimation procedures

	Residual variance	Prediction error variance
LS	$7.3039 \cdot 10^{-4}$	$8.7667 \cdot 10^{-4}$
YW	$1.4799 \cdot 10^{-3}$	$1.9120 \cdot 10^{-3}$
Burg	$7.3100 \cdot 10^{-4}$	$8.7720 \cdot 10^{-4}$

Applying the autoregressive model to the anomaly noise data, the anomaly can be detected from the prediction error variance only if LS or Burg's parameter estimates are employed. In case of YW, the anomaly cannot be detected from the prediction error variance due to the poor parameter estimates. Fig. 7 shows the prediction error for the noise section containing the anomaly in each of the cases LS, YW and Burg. In case of LS and Burg, visual inspection of Fig. 7 enables us to locate the start of the anomaly at the increase the prediction error variance, as indicated by the arrow. No such conclusion can be made if the YW parameter estimates are used, which means that YW is unusable for anomaly detection.

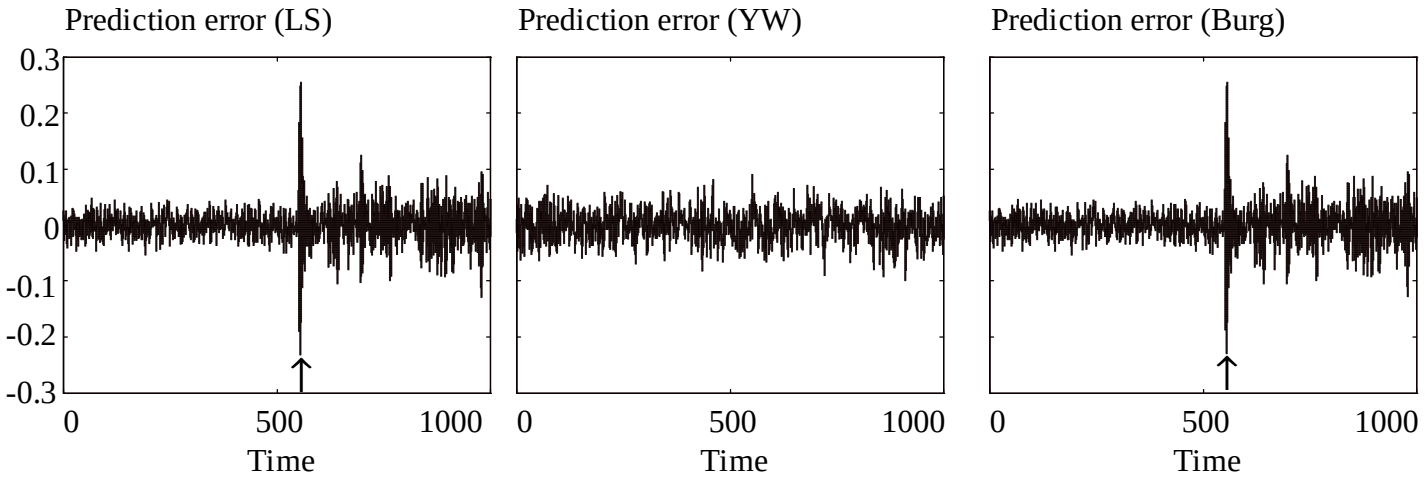

Fig. 7 Prediction error for the anomaly noise data using LS, YW and Burg respectively.

V. CONCLUSION

The Yule-Walker method should not be used as a means of autoregressive-parameter estimation if the autocovariance matrix is poorly conditioned. In that case the relatively small covariance estimate bias can lead to a large deviation in the estimated parameters, resulting in an invalid model.

A poor autocovariance matrix condition also involves pole locations near the unit circle, as a result of which the autoregressive process exhibits a kind of almost non-stationary, pseudo-periodic behaviour. The variance of the stochastic process will be large due to the innovation process not being identically zero, which is the case for a harmonic process.

The least-squares approach as well as Burg's method are still able to estimate the autoregressive model correctly. Least squares should be used with caution though, as it does not guarantee the estimated autoregressive model to be stable, as a result of which a small deviation in the parameter estimates may cause the estimated poles to move outside the unit circle. In that case the estimated autoregressive model will be invalid.

This leaves Burg's method as the most reliable estimation technique, as it provides reliable parameter estimates as well as an estimated model guaranteed to be stable.

The preceding conclusions were obtained for univariate autoregressive analysis only. However, due to the mathematical similarity between univariate and multivariate autoregressive analysis, we expect similar results for the multivariate case.

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APPENDIX: AUTOREGRESSIVE-PARAMETER ESTIMATES

The following table contains the autoregressive-parameter estimates for LS, YW and Burg. The parameters were estimated using 2048 data samples from the 1994 IAEA benchmark (section IV).

Autoregressive parameter	LS	YW	Burg	Autoregressive parameter	LS	YW	Burg
1	-2.5239	-1.6270	-2.5241	17	-0.8539	0.0613	-0.9391
2	4.4626	1.8549	4.4635	18	0.5770	0.1235	0.6612
3	-6.8731	-2.0611	-6.8729	19	-0.5165	-0.2018	-0.5888
4	8.8541	1.5264	8.8488	20	0.5163	0.0929	0.5718
5	-10.0568	-0.7230	-10.0468	21	-0.4882	0.0461	-0.5266
6	10.1852	-0.1009	10.1662	22	0.3595	-0.1229	0.3799
7	-9.3866	0.5498	-9.3616	23	-0.1024	0.1154	-0.1079
8	8.0237	-0.5359	7.9984	24	-0.2577	-0.0682	-0.2605
9	-6.4827	0.2793	-6.4615	25	0.6098	0.0009	0.6198
10	5.1613	0.0795	5.1489	26	-0.8109	0.1031	-0.8215
11	-4.2741	-0.3821	-4.2768	27	0.8691	-0.0916	0.8774
12	3.6496	0.3384	3.6721	28	-0.7647	0.0246	-0.7709
13	-3.1153	-0.0799	-3.1567	29	0.6449	0.1507	0.6481
14	2.5708	-0.1456	2.6278	30	-0.2553	-0.0077	-0.2580
15	-1.9859	0.2132	-2.0543	31	0.1978	0.2802	0.1989
16	1.3793	-0.1694	1.4607	32	-0.2009	-0.3362	-0.2011