Note about Exercise 6.8 on page 101 and solution on page 249/250.

Show that for all $0 \le x, y, z \le 1$, one has

$$L(x,y,z) = \frac{x^2}{1+y} + \frac{y^2}{1+z} + \frac{z^2}{1+x+y} + x^2(y^2-1)(z^2-1) \le 2$$

In the list of errata it is pointed out that $L(x,y,z)$ is not convex in the z direction. I find that L is not convex in y, either (for example, $x = 1$, $y = 1$ and $z = 1/3$). However, we only need $L(x,y,z)$ strictly convex in the x direction. This is easy to show.

After that, L can only attain maximum at $x = 0$ or $x = 1$. If $x = 0$, since $0 \le y, z \le 1$,

$$L = \frac{y^2}{1+z} + \frac{z^2}{1+y} \le y^2 + z^2 \le 2$$

in which the equalities cannot both hold. If $x = 1$,

$$L = \frac{1}{1+y} + \frac{y^2}{1+z} + \frac{z^2}{2+y} + y^2z^2 - y^2 - z^2 + 1 \tag{1}$$

$$= \frac{1}{1+y} + \left(\frac{y^2}{1+z} - y^2\right) + \left(\frac{z^2}{2+y} - z^2\right) + y^2z^2 + 1 \tag{2}$$

$$\le 1 + 0 + 0 + 0 + 1 = 2 \tag{3}$$

equality only holds when $y = z = 0$. Done.