# Spatial Temporal Models for Retail Credit

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# Outline

- Introduction
- Exploratory analysis
  - Trends and maps
- Measuring spatial association
  - Nonparametric clustering using SVDs
- Models
  - Spatial, temporal and spatio-temporal
- Next steps

- Collaborators
  - Sathyanarayan Anand
  - Chris Henderson and friends

# Key Points

- Exploratory analysis
  - Finds spatial association in various types of default (mortgage, installment, revolving)
- Analysis of spatial patterns
  - Correlation risk
  - Three spatio-temporal patterns
  - Nonstationarity motivates simple models.
- Models
  - Models with broad correlations predict better than those more narrowly defined
  - Correlations in data impact claims of precision

Mortgages

Cards

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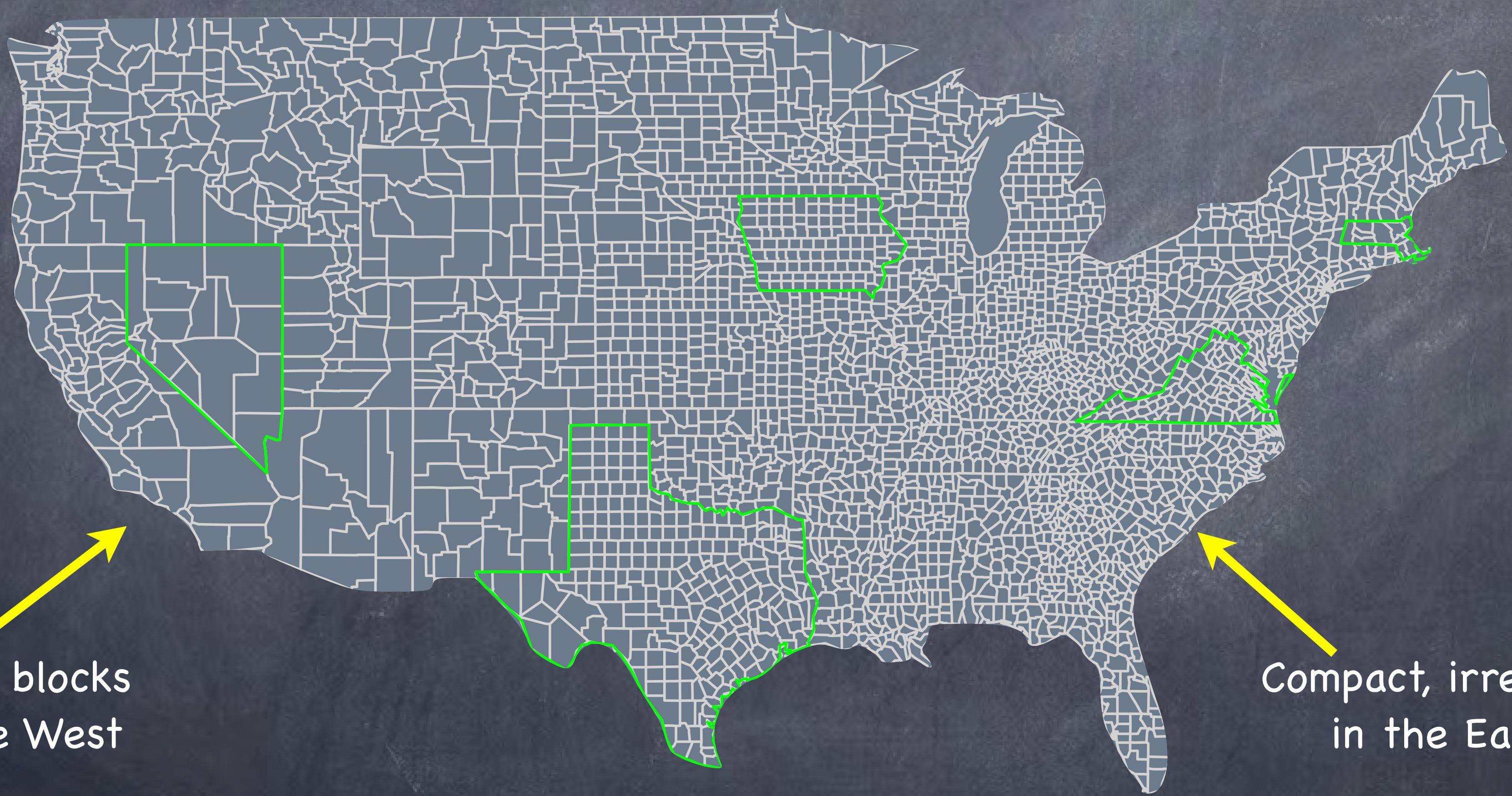
# Featured Data

- County-level
  - Default rates from Trend Data (TransUnion)
    - National coverage
    - Default rates based on quarterly samples, 1993-2010
  - Economic characteristics (Census)
  - Spatial locations
  - Small: 3,000 counties x 80 quarters = 240,000
- Multi-level inference
  - Individual -> Tract -> <u>County</u> -> State -> Nation
- Gaps in data...
  - Lender proprietary data (eg, vintage)
  - Individual loan characteristics
  - Housing data is incomplete

# Featured Data

- County-level data
  - County = political subdivision of state in US
  - 3,000 counties within continental US



Large blocks in the West

Compact, irregular in the East

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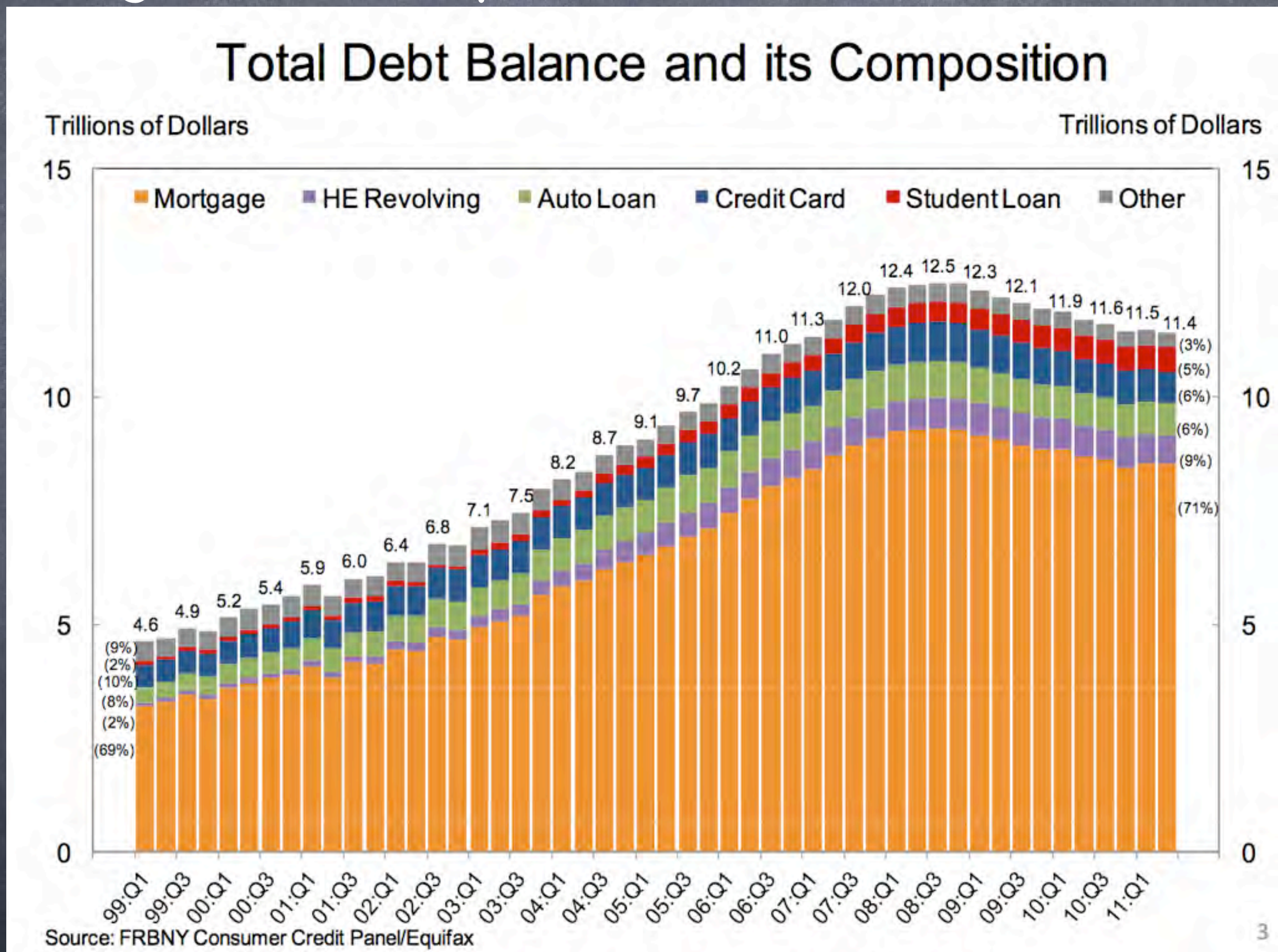
# National Trends

# Trends: Consumer Debt

- August 2011 report from US Federal Reserve

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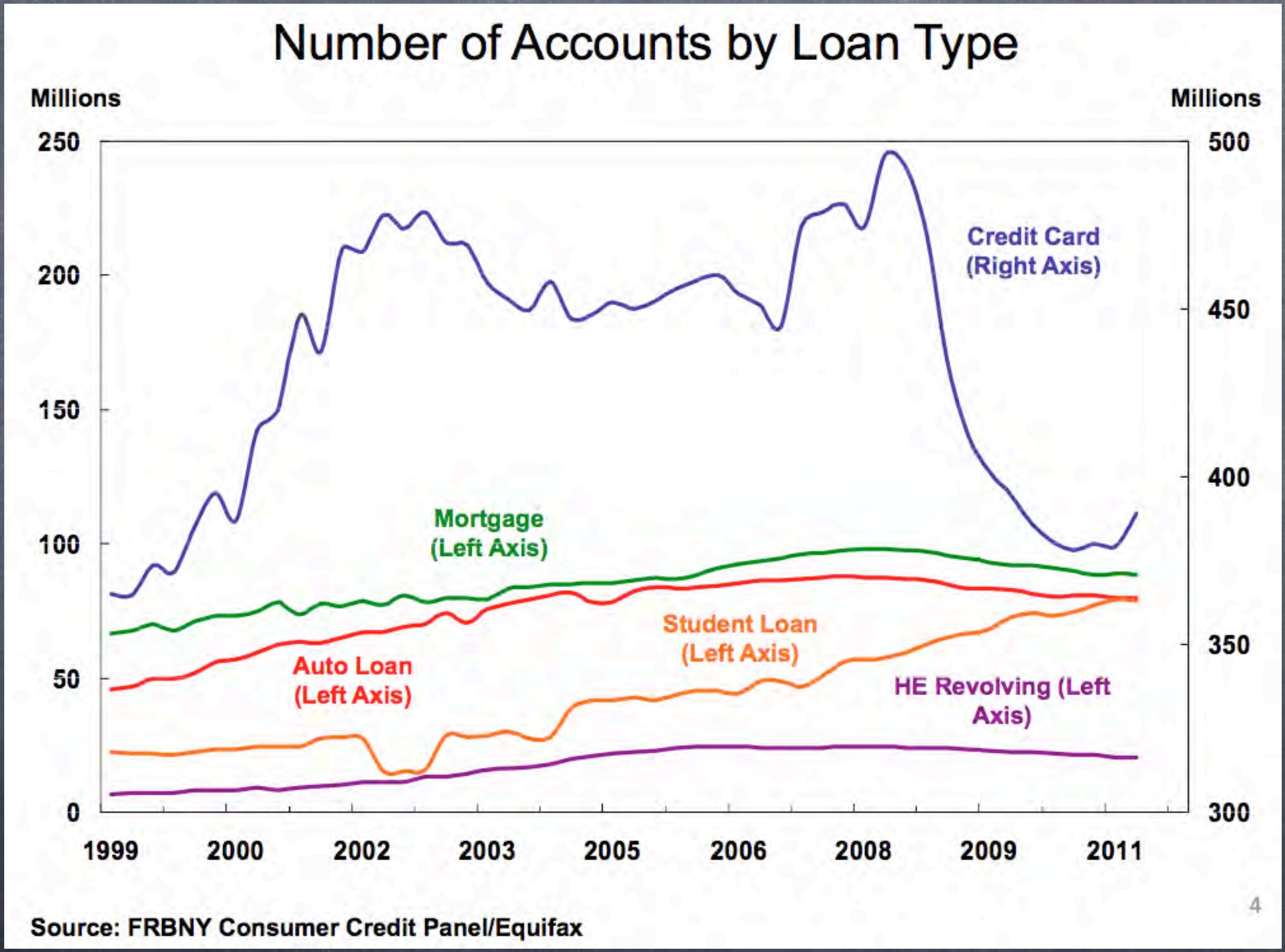
# Trends: Loan Volumes

- "Flight to quality"

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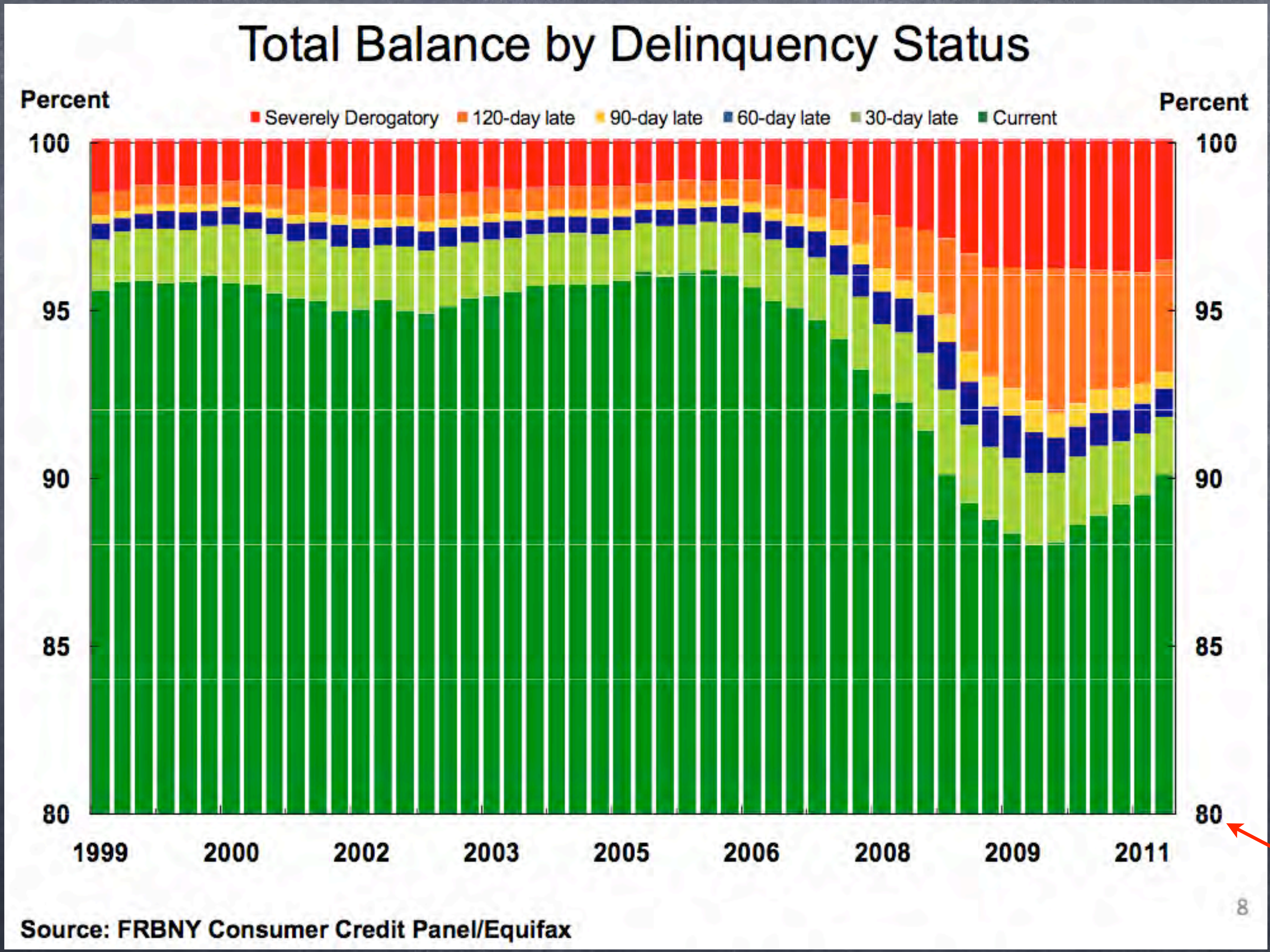
# Trends: Default Balances

- Balance primarily composed of mortgages.




careful!

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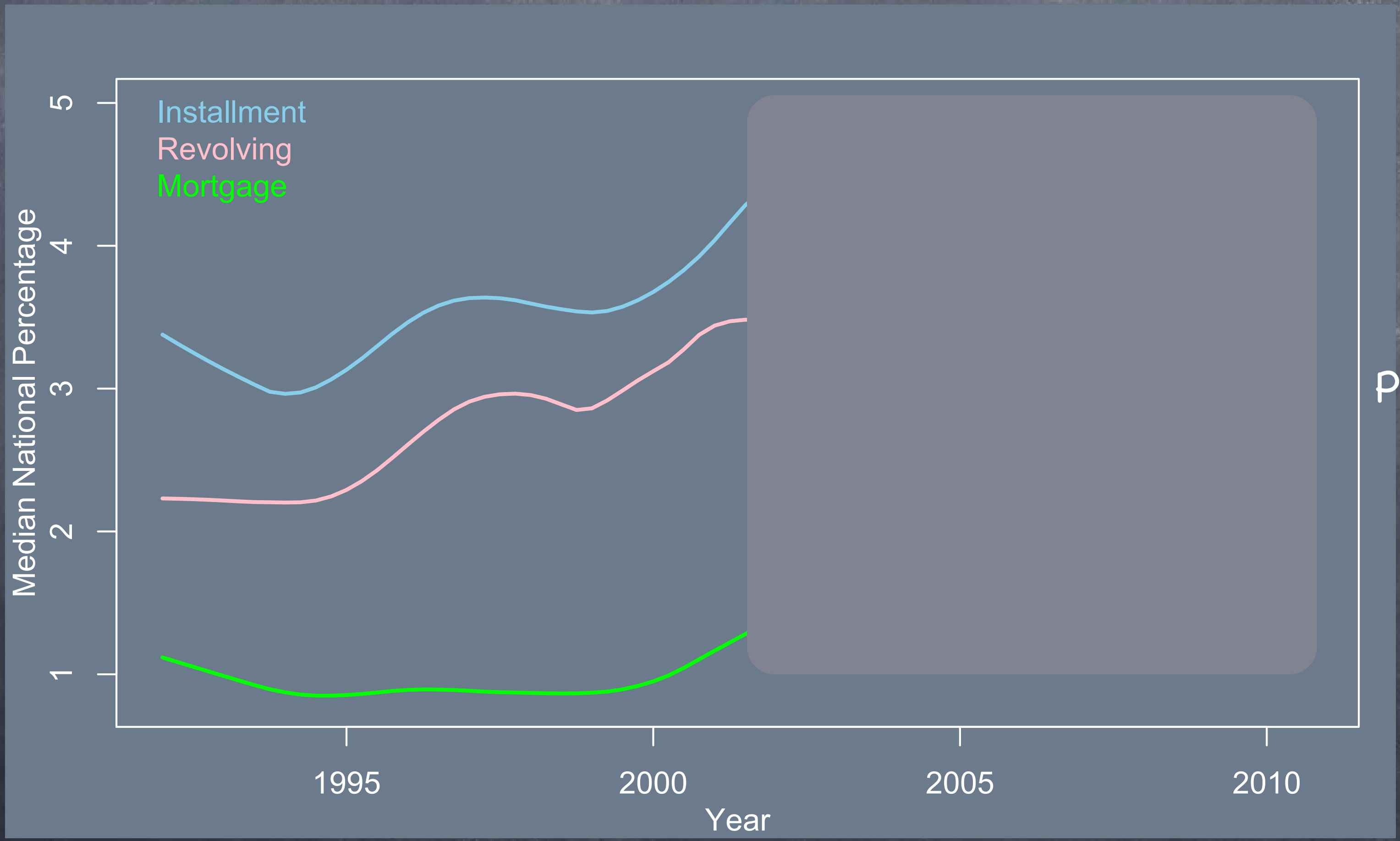
# Default Rates

- Median county-level quarterly default rates, 60 days past due and slightly smoothed
- Changing association among rates




Predict?

# Default Rates

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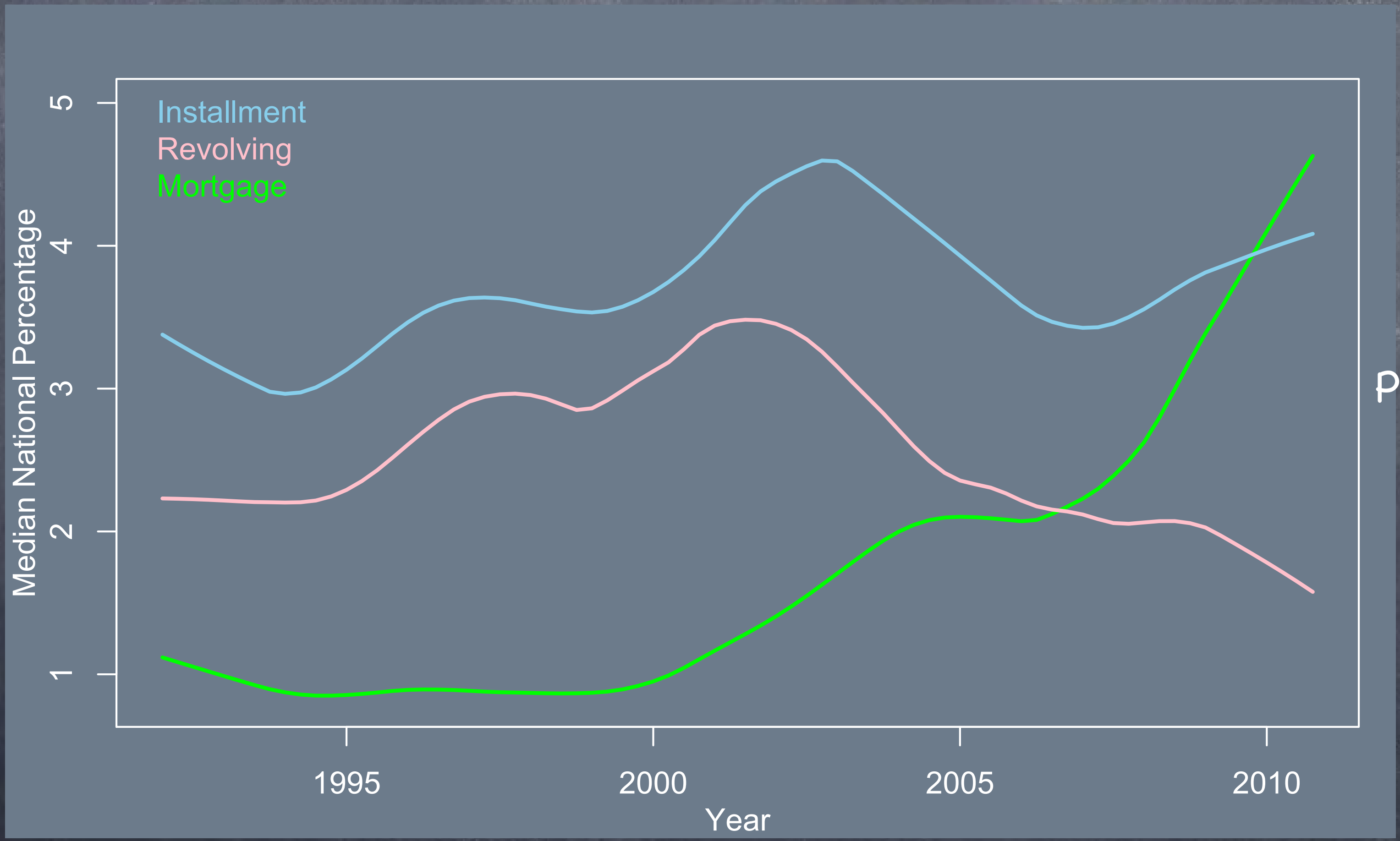


Predict?

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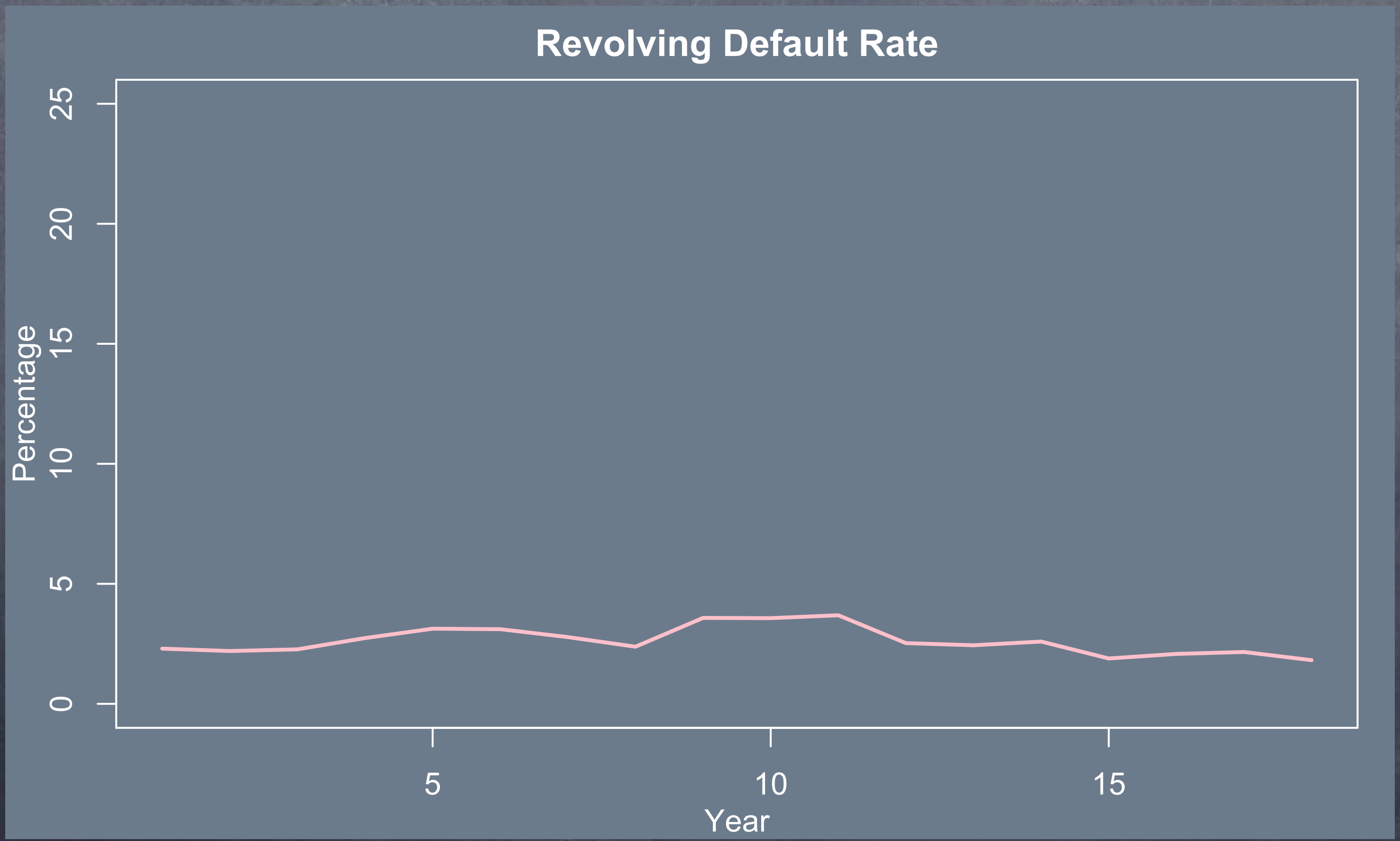
# Maps

# Enormous Heterogeneity

- Revolving default rates
  - Smooth national series
  - Huge regional variation in US:
    Near zero in some counties, 25% in others.



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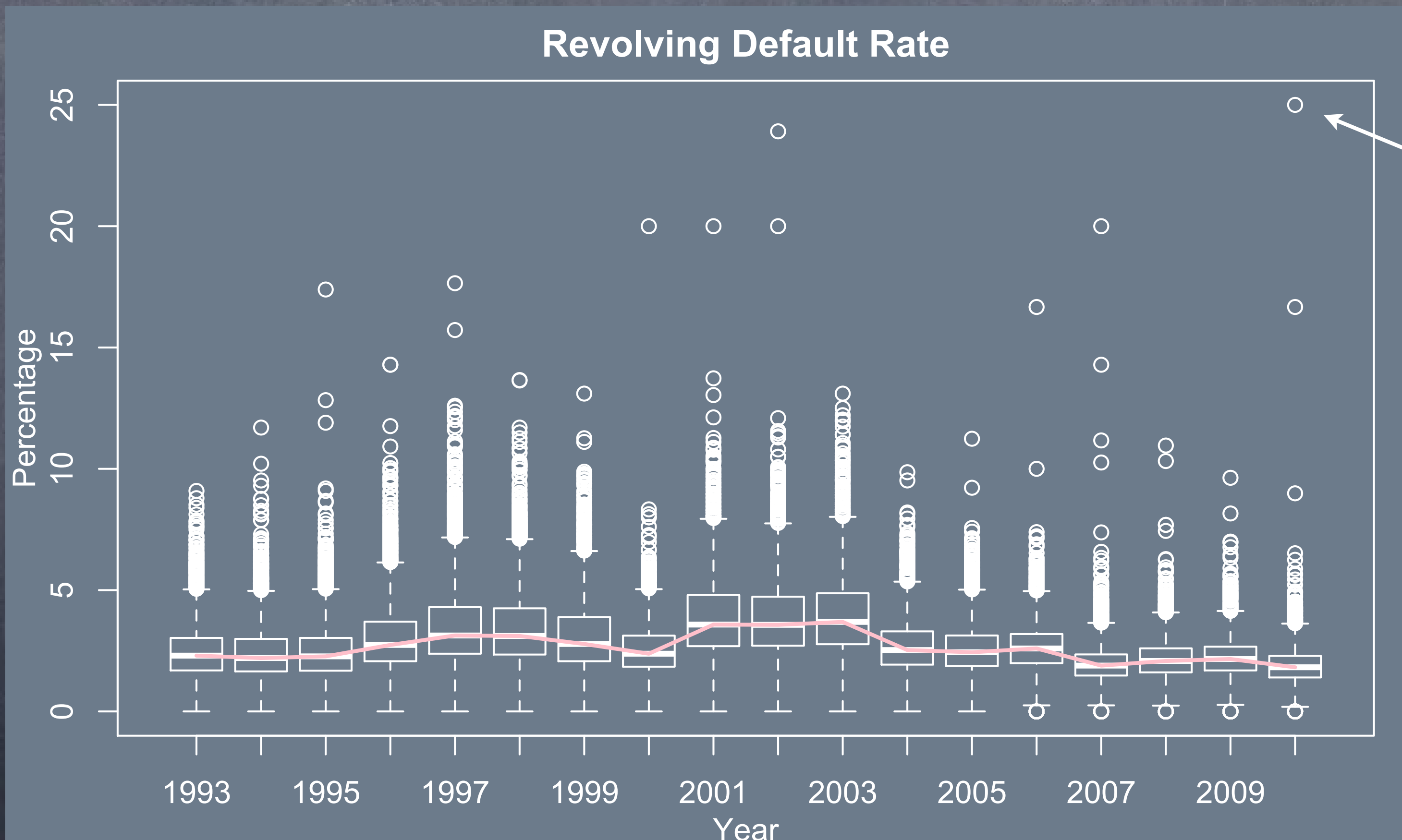


Small populations

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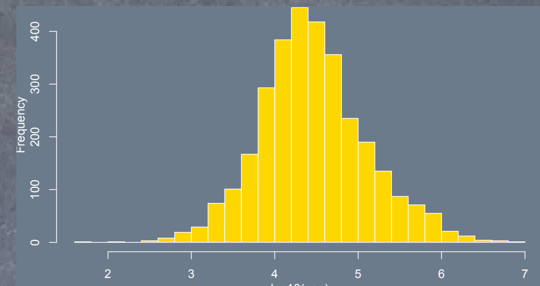
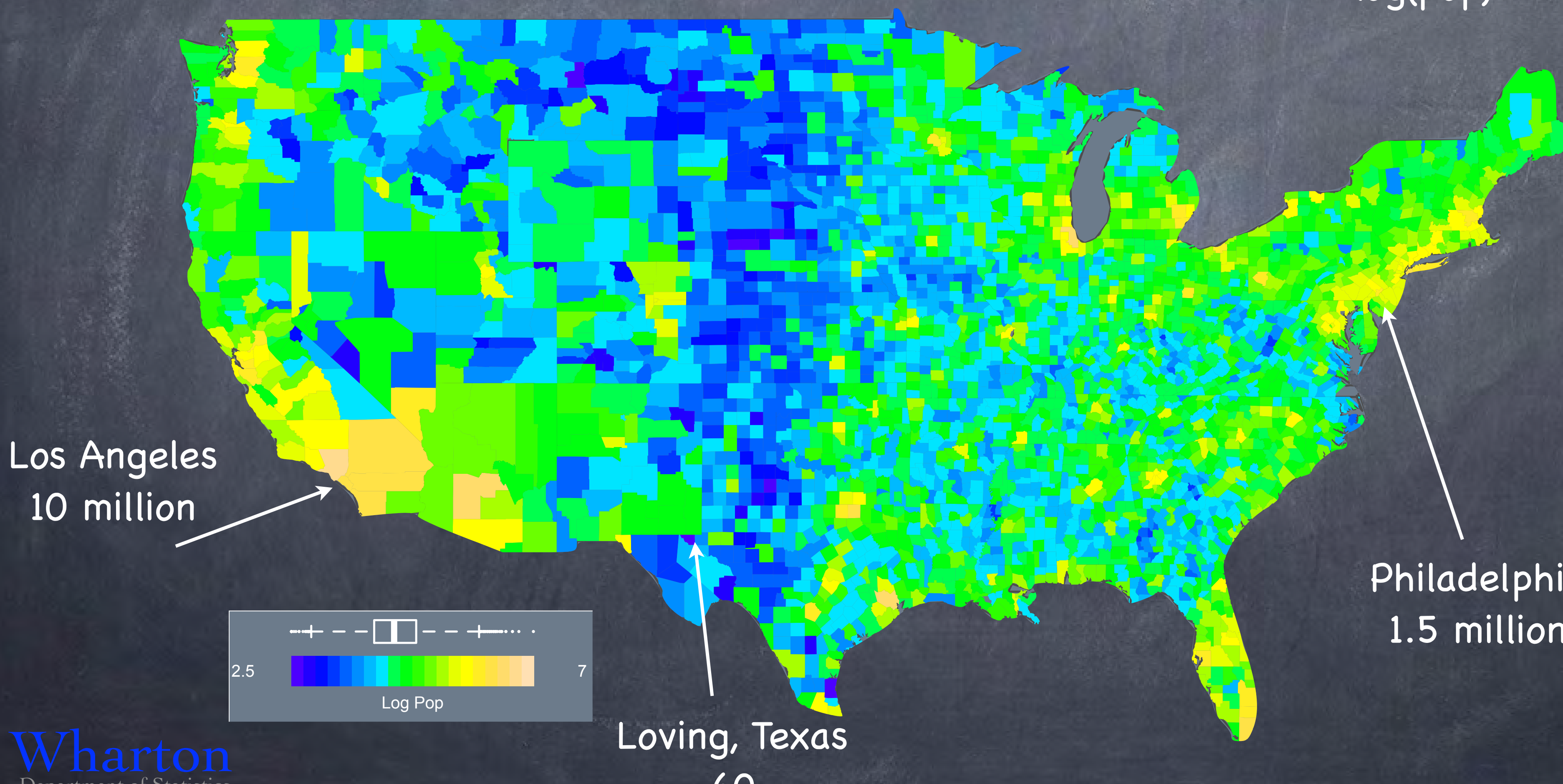
# Variation in Population

- Some counties have a hundreds, others have millions (lognormal)

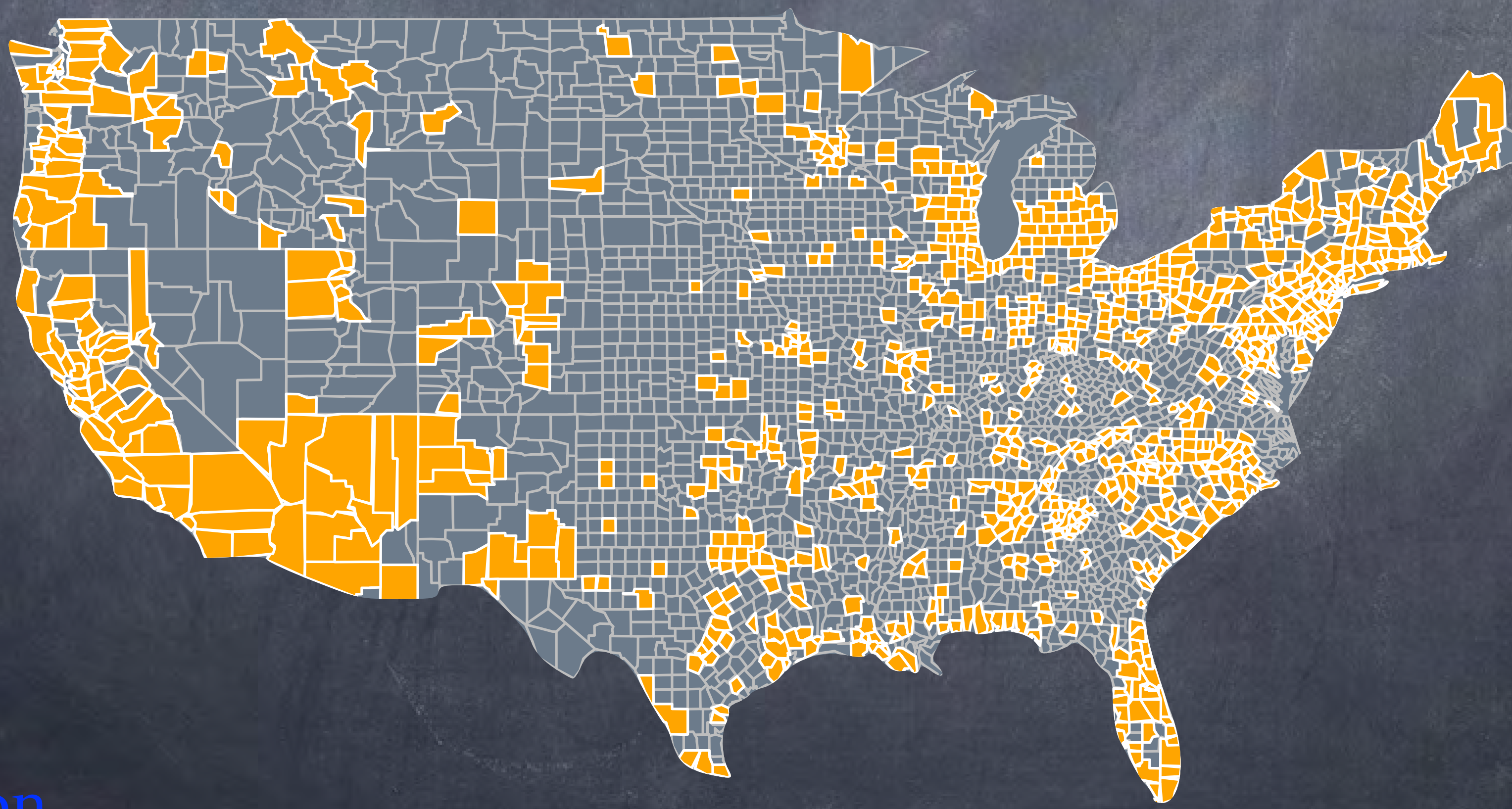



log(pop)



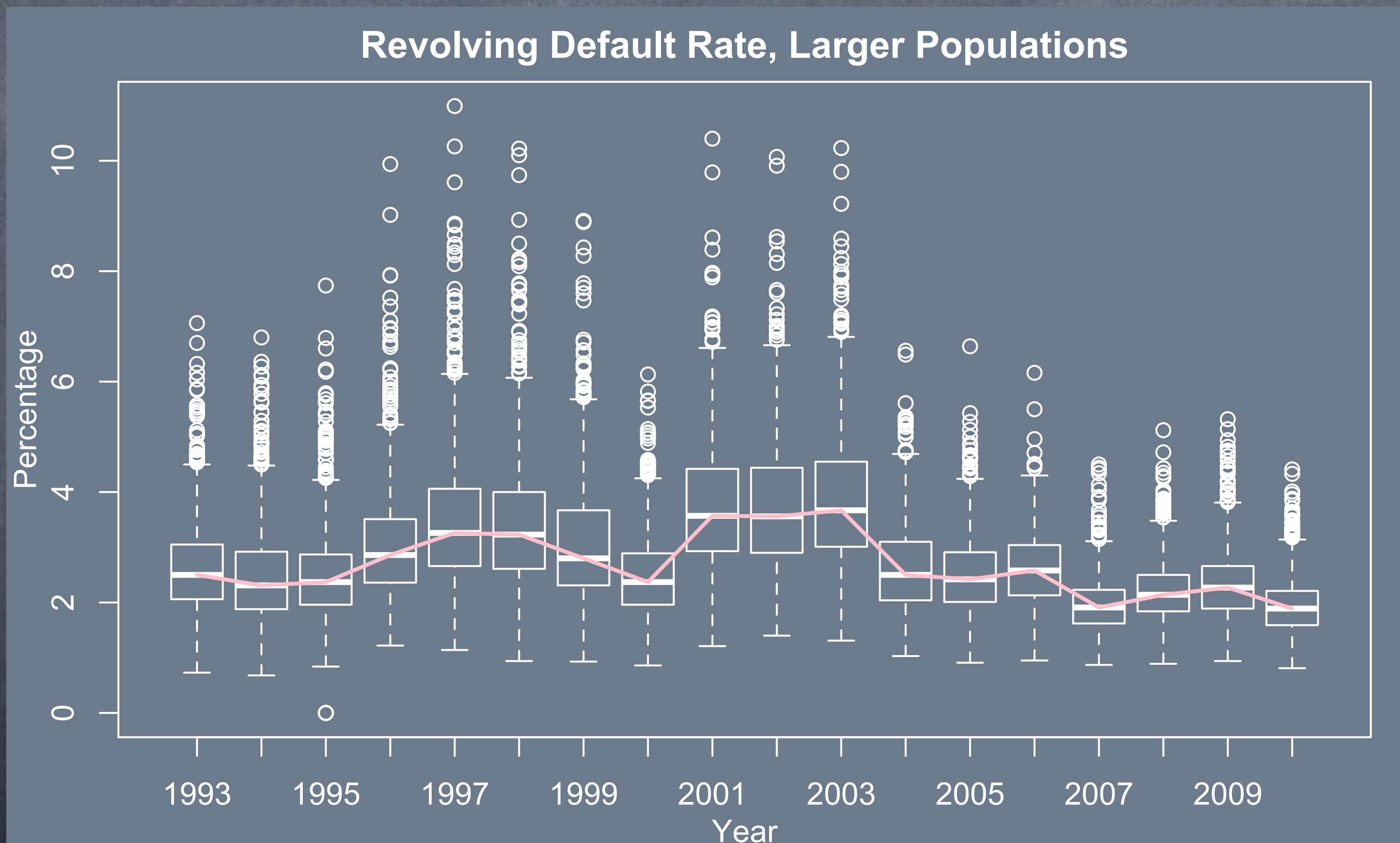
# Analysis Subset

- Default rates and demographics are unreliable in sparsely populated areas.
- Limit analysis to counties with 50,000 people
  - Covers 85% of population, 900+ counties



# Heterogeneity Persists

- Revolving default rates
  - Rates skewed, close to log normal
  - More reliable, fewer missing

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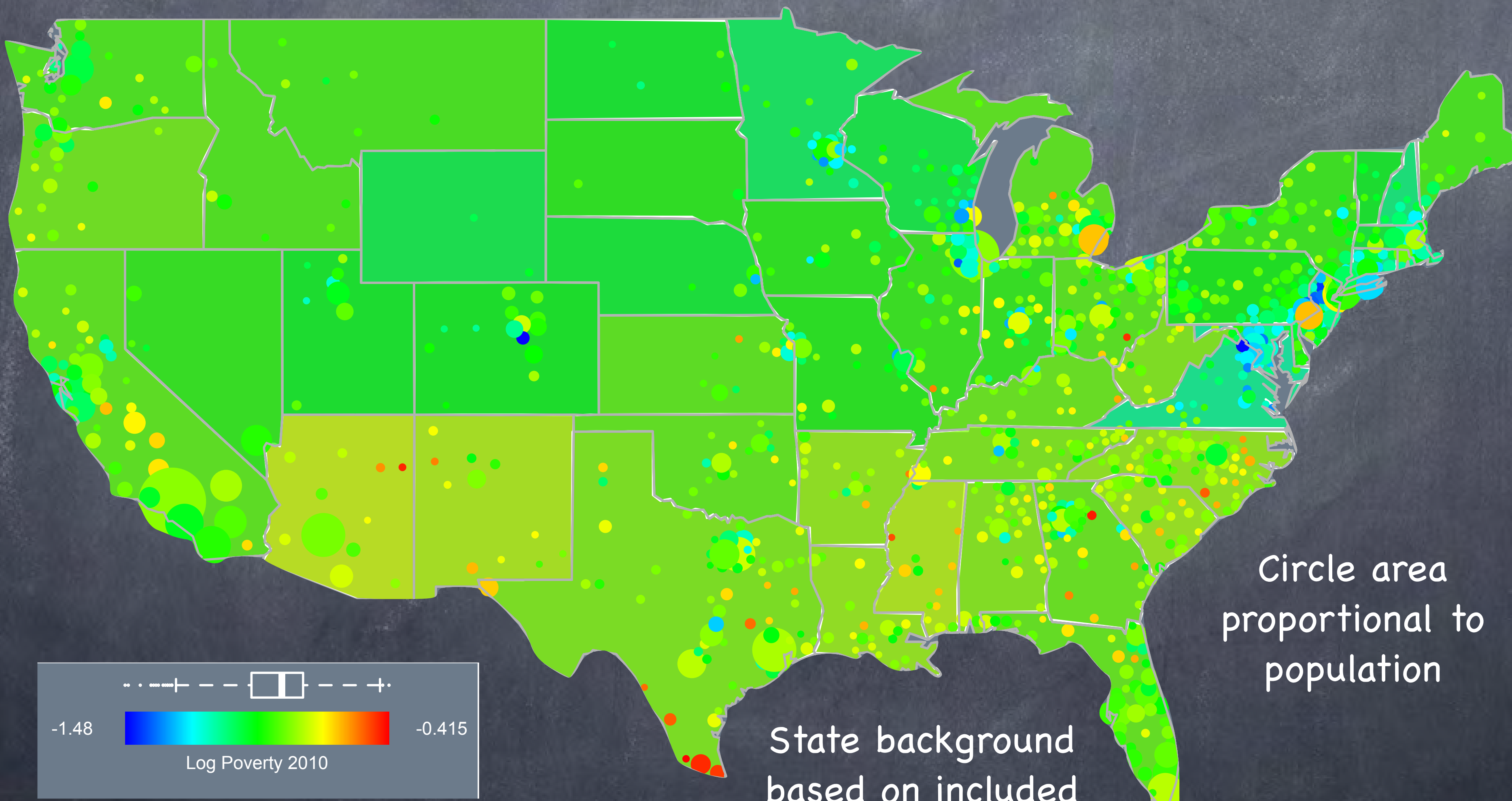
# Spatial Patterns

- Poverty rates
  - Wealth concentrates around urban cores




Circle area proportional to population

State background based on included counties.

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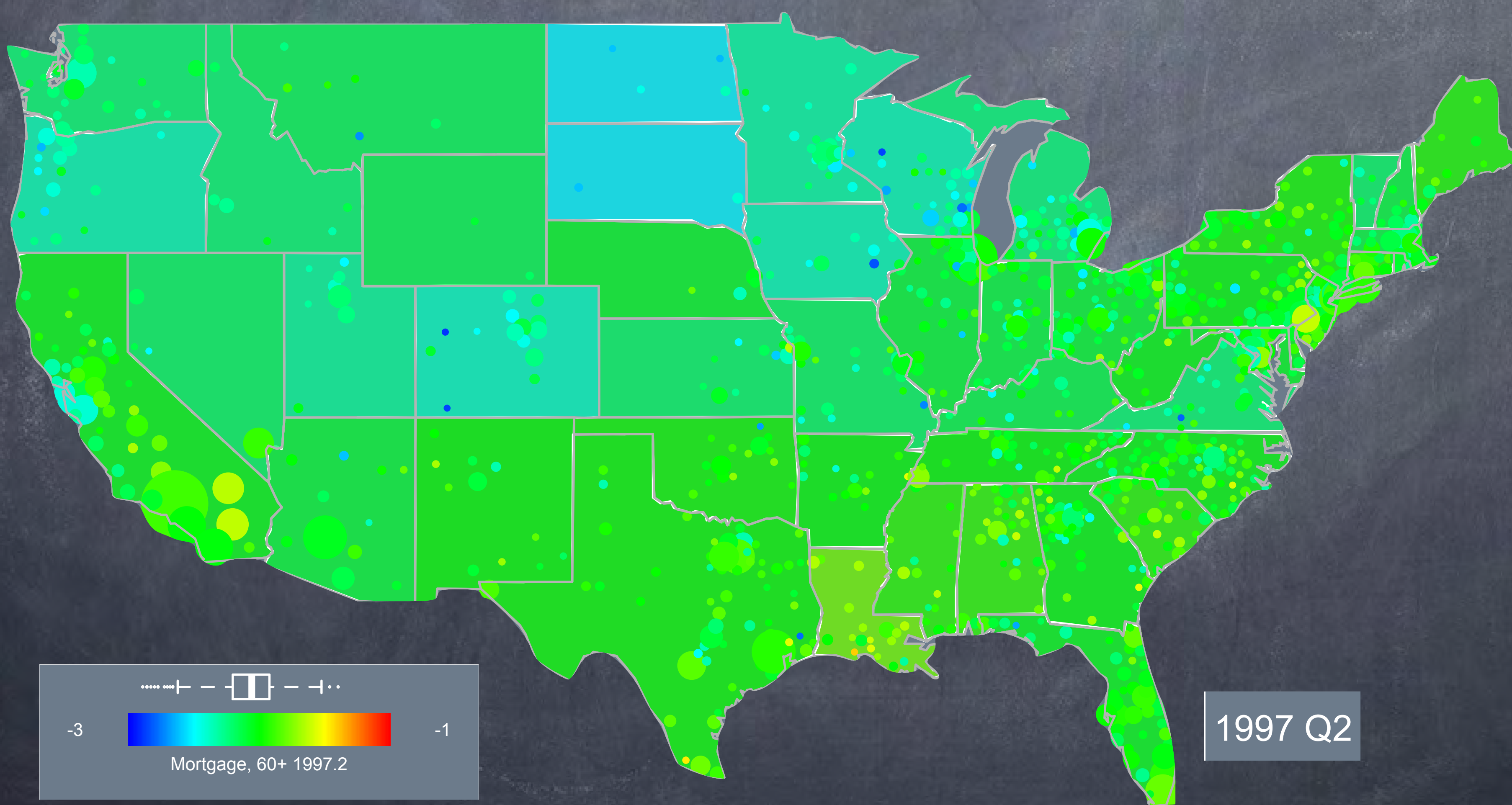
# Evolution of Defaults

- Mortgage rates
  - Rates on log scale

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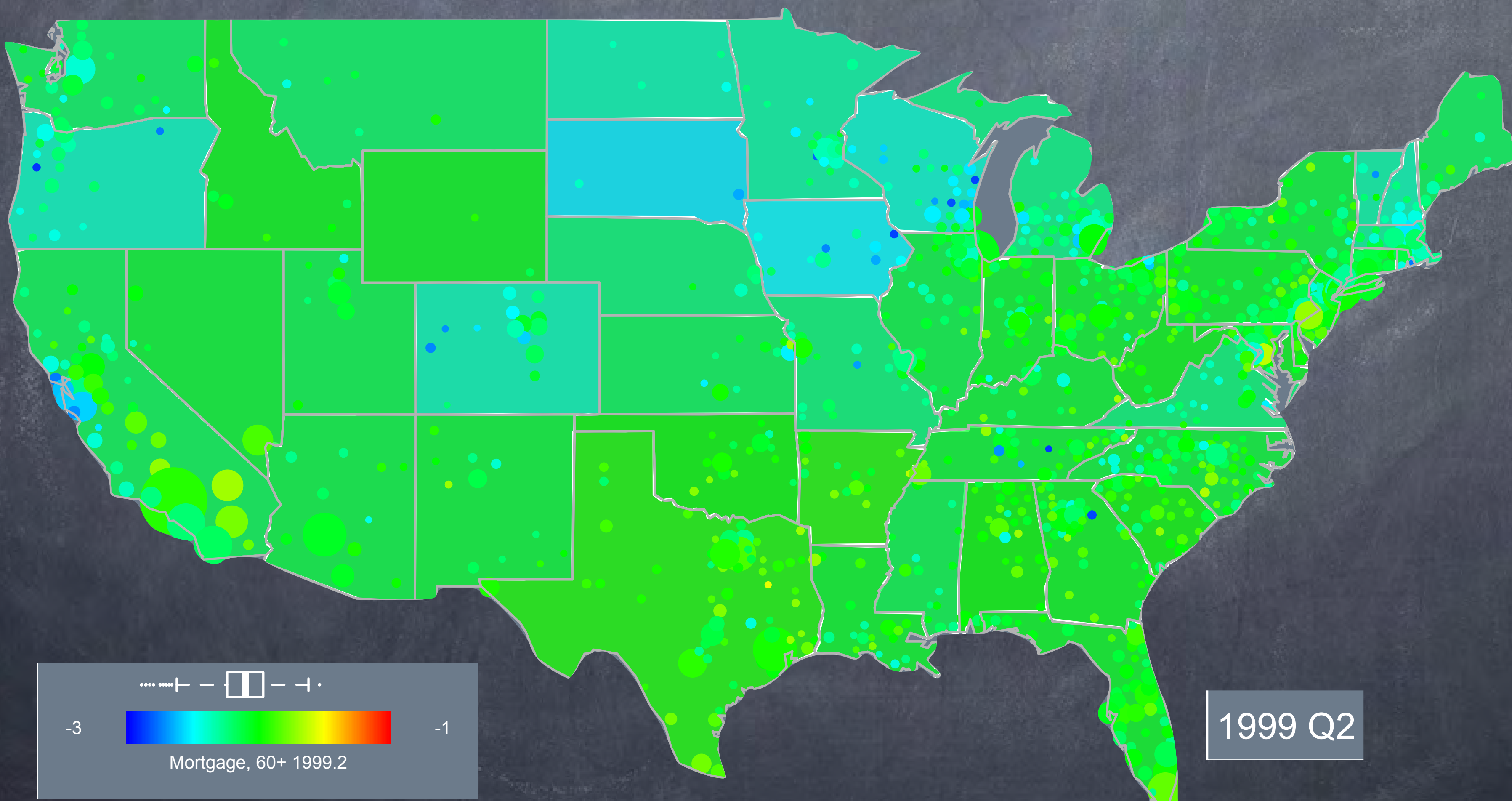


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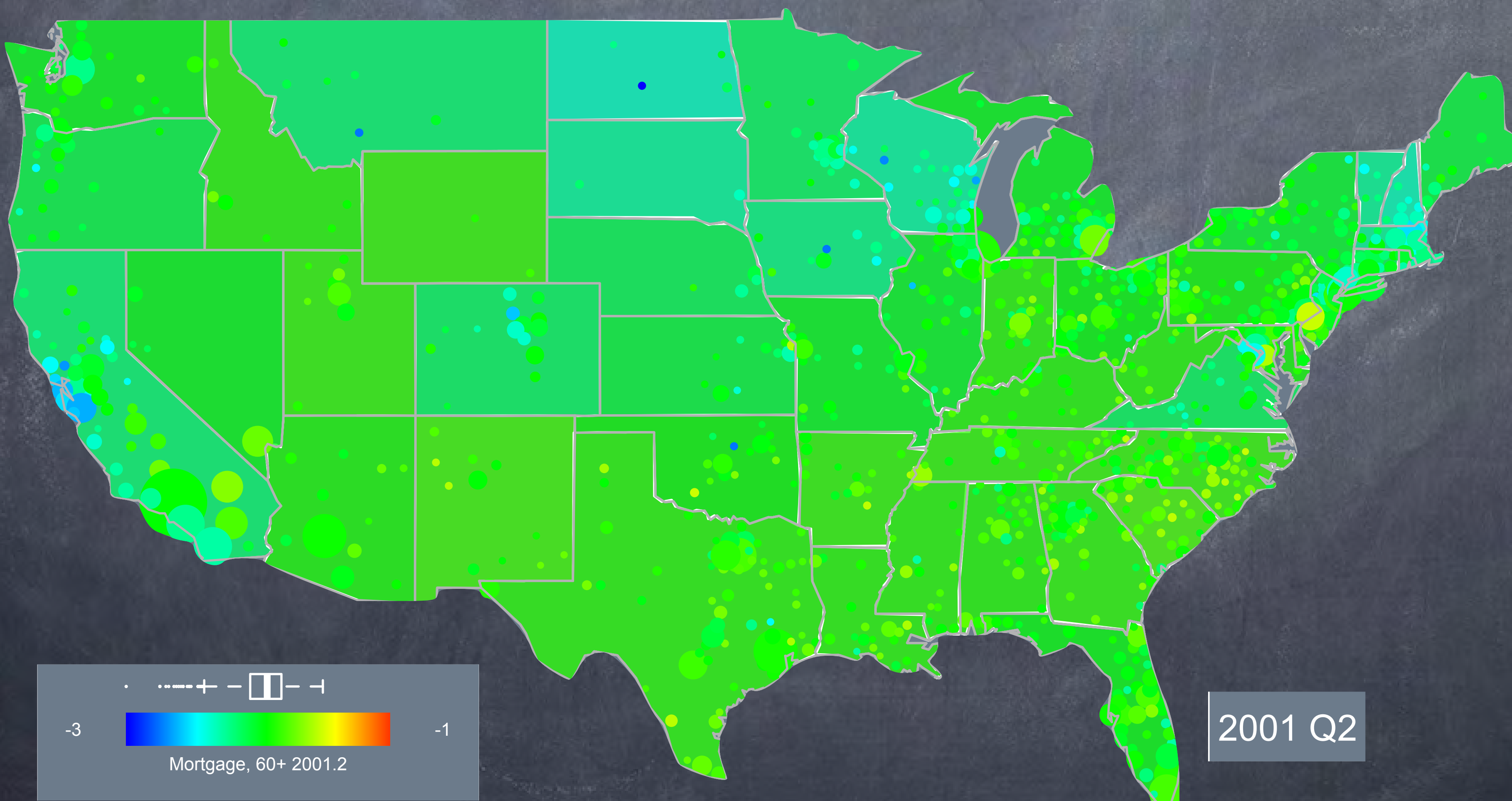


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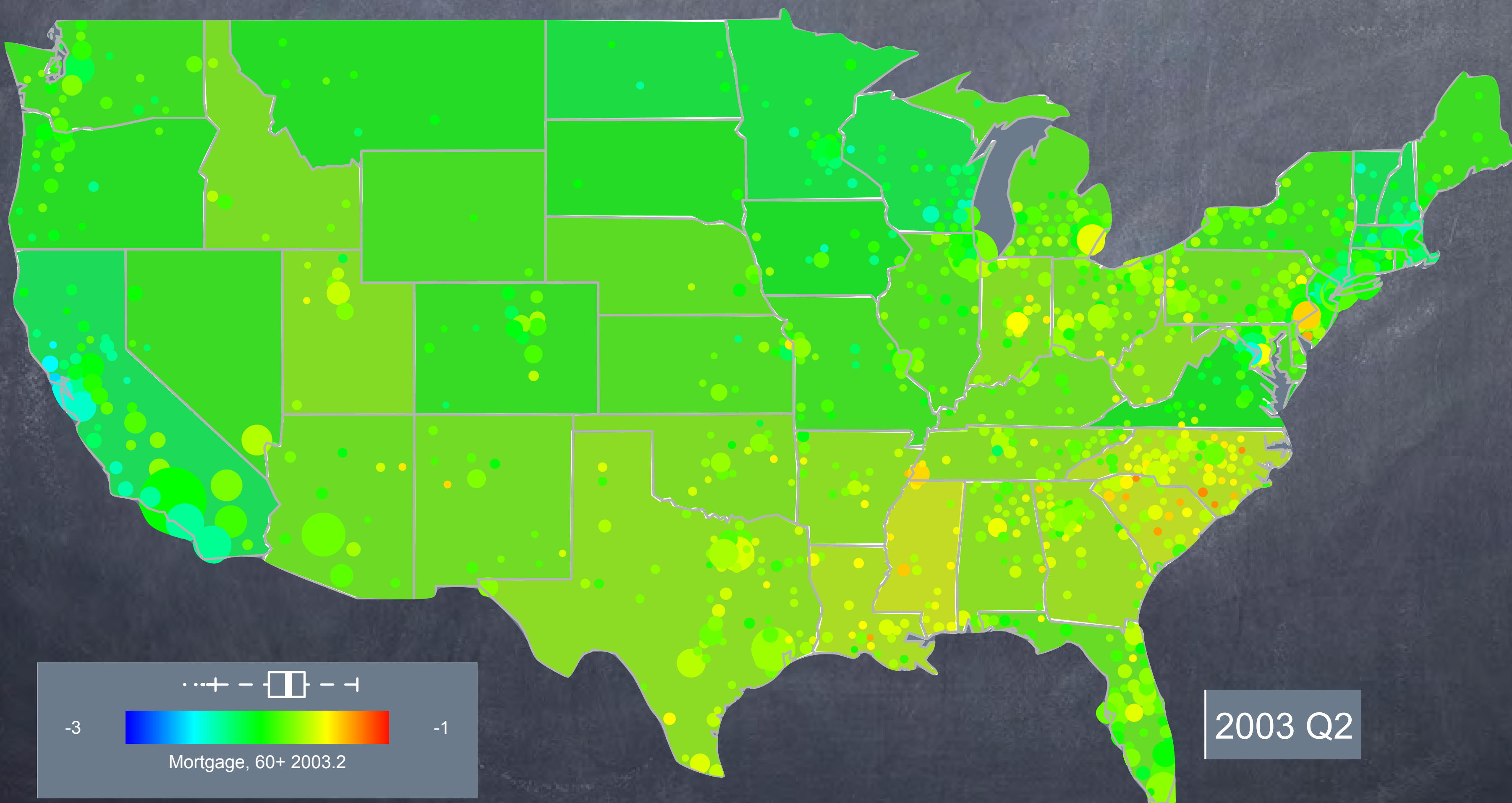


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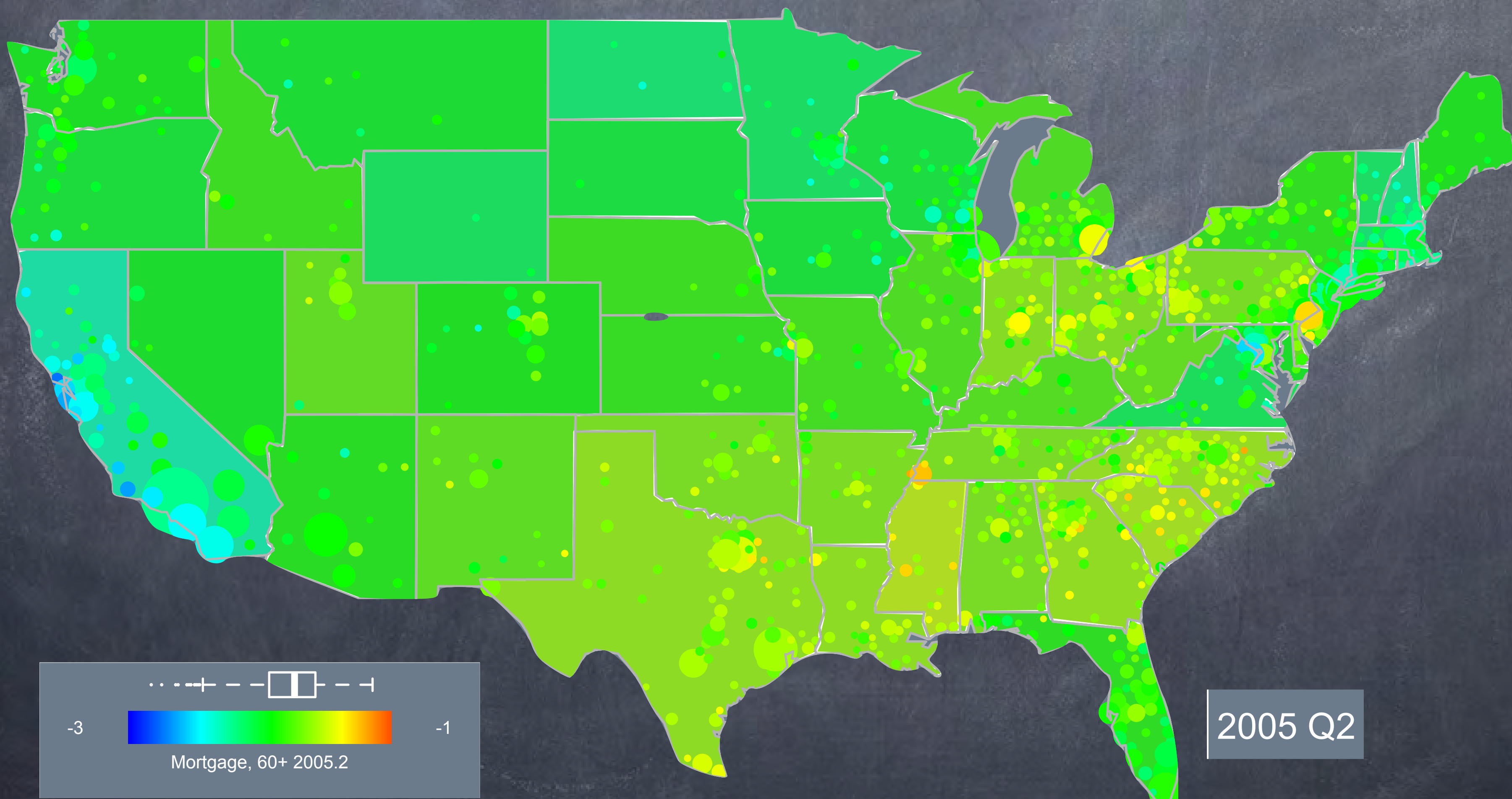


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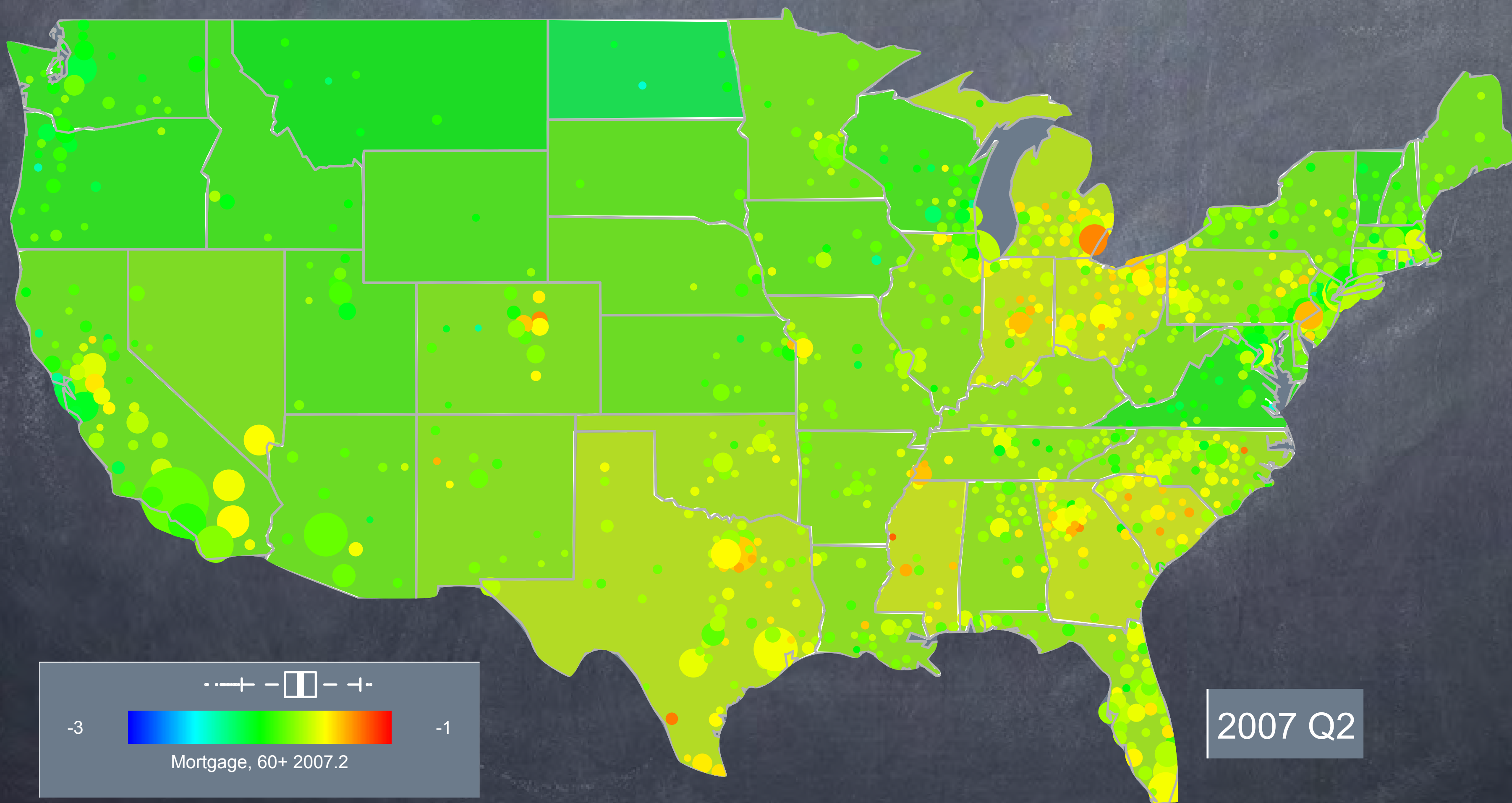


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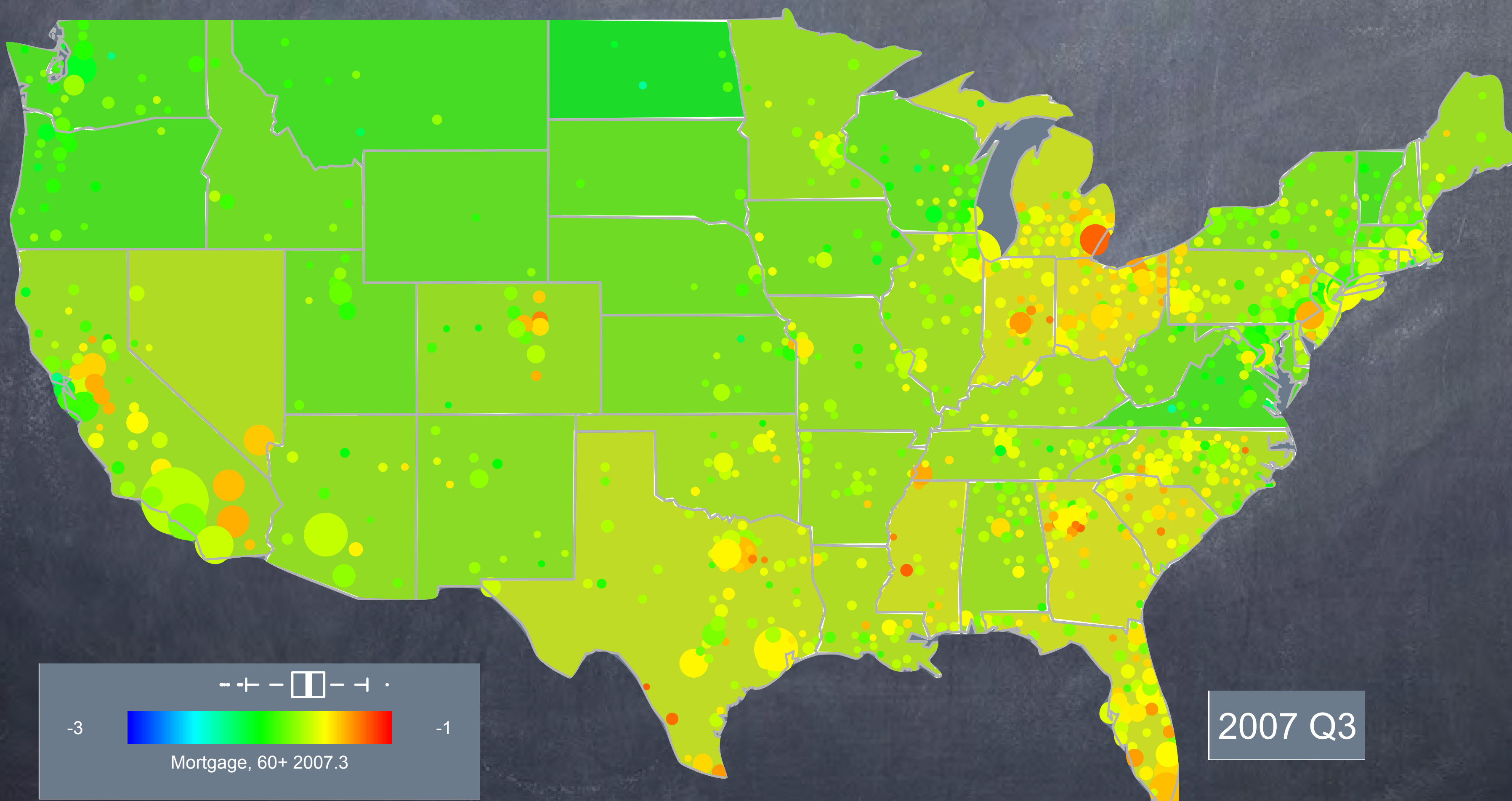

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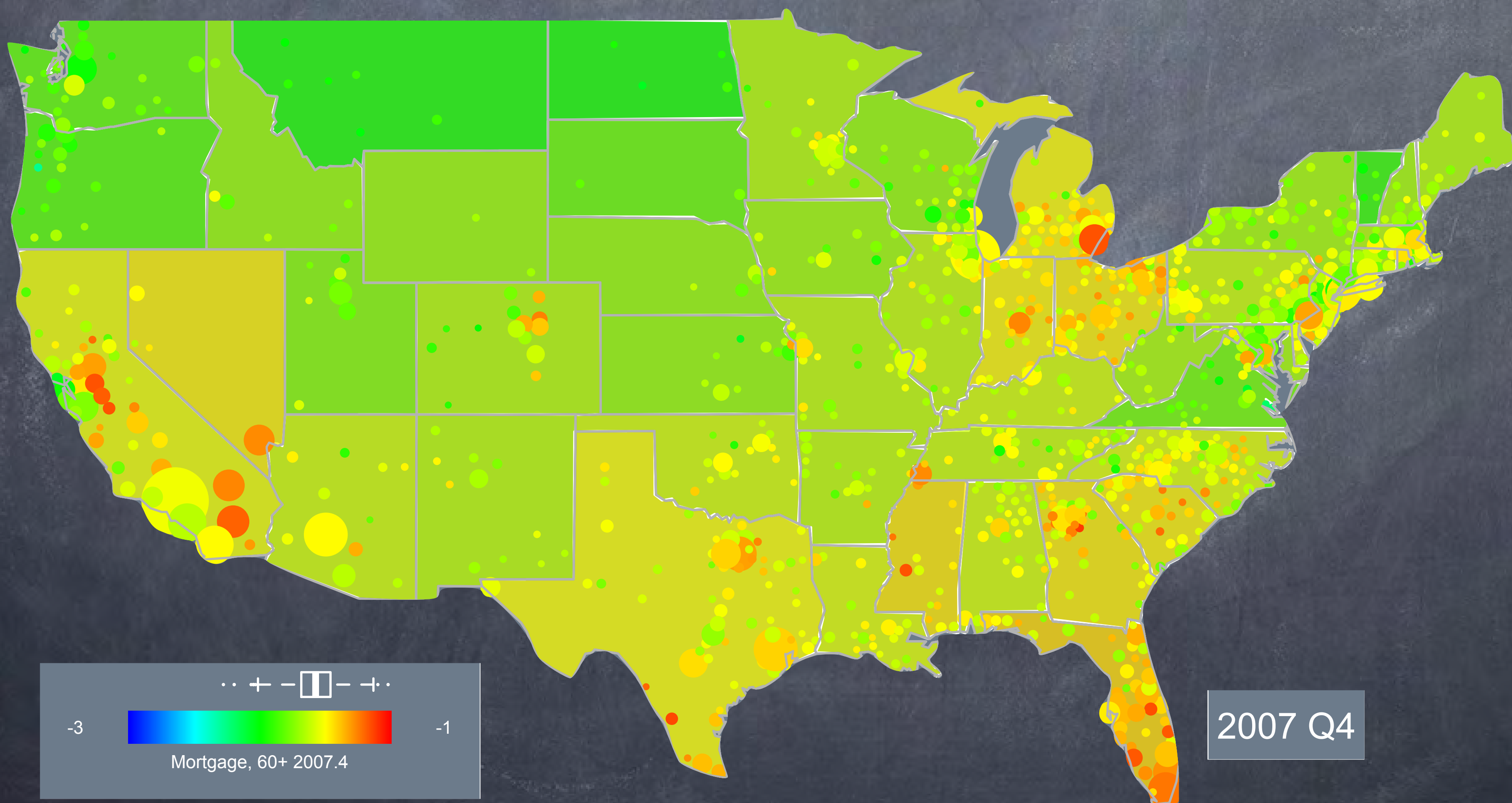


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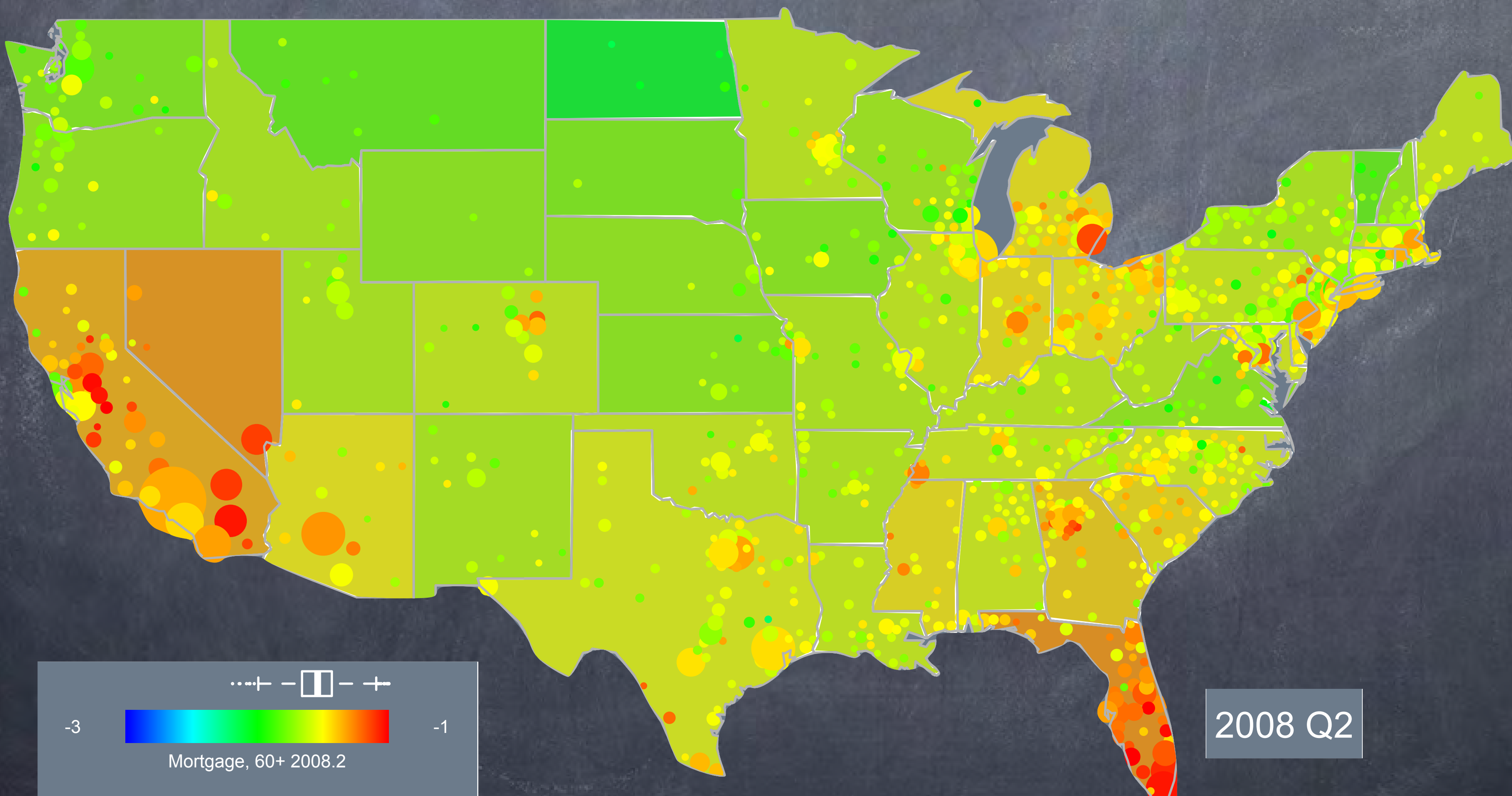


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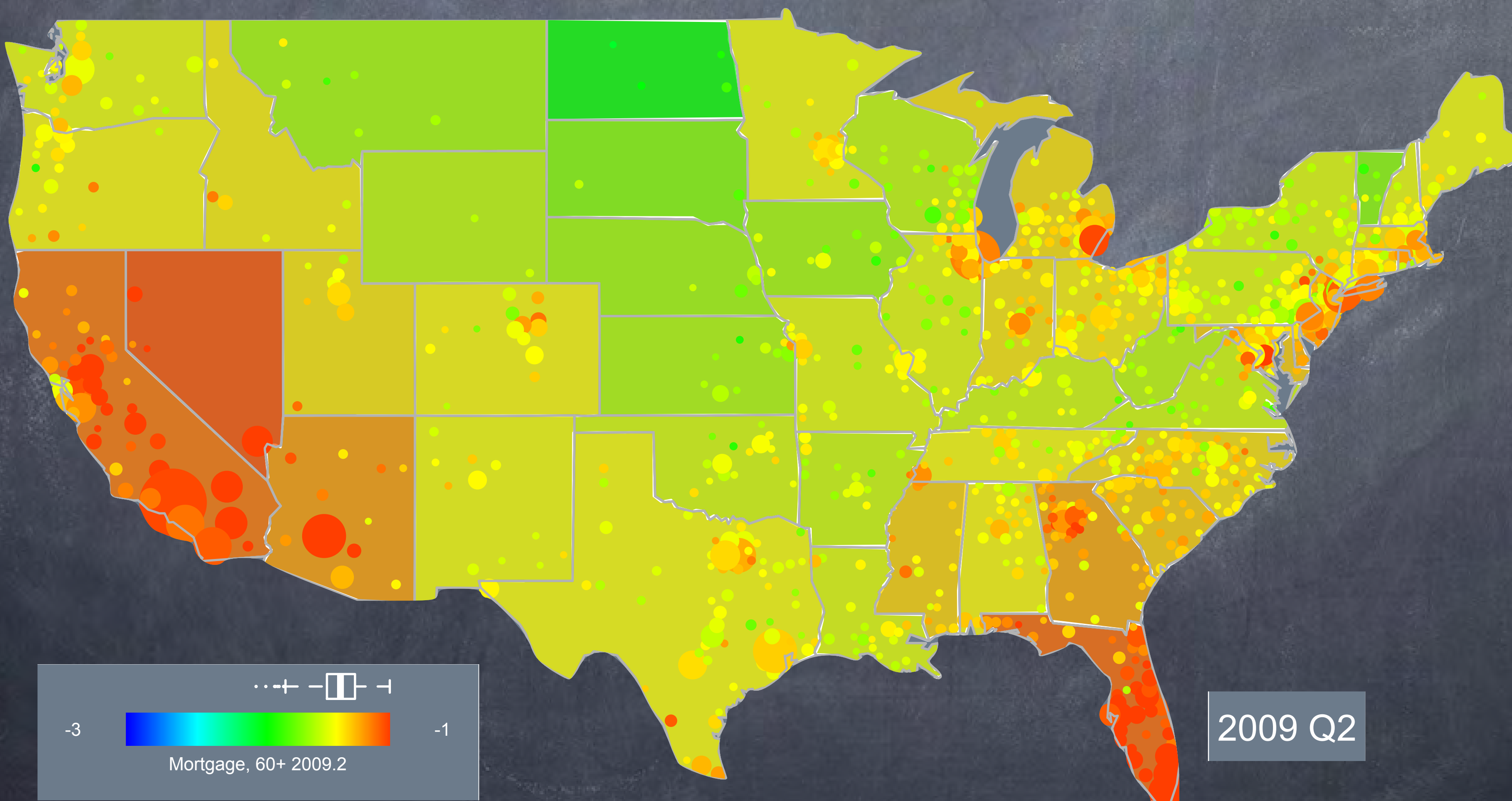


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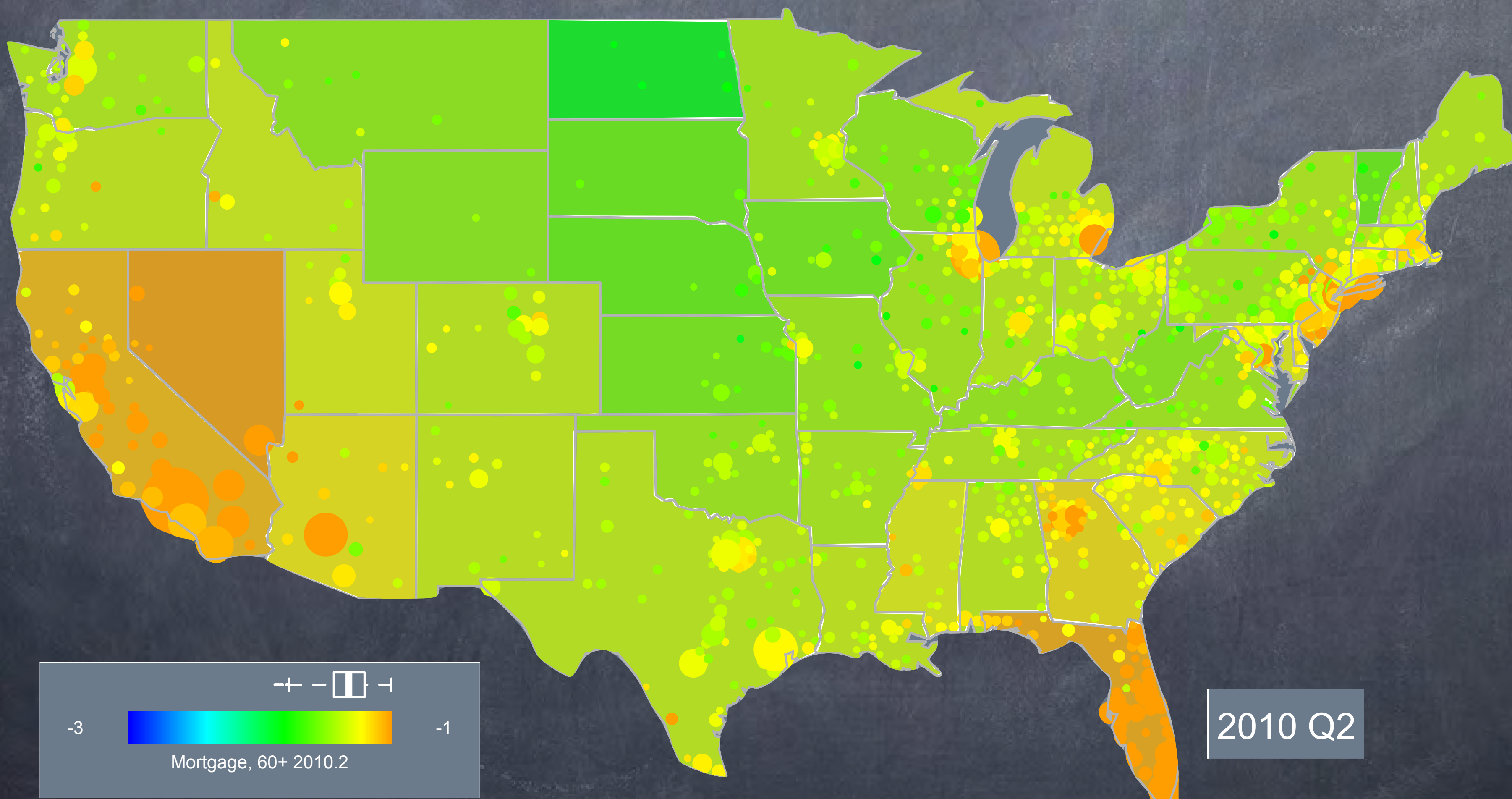


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# Spatial Correlations

- Standard measure of spatial correlation

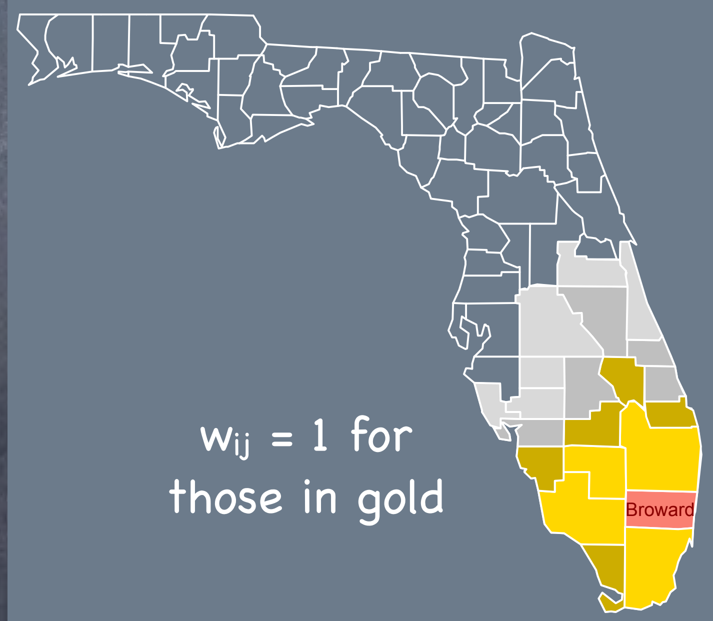
$$\text{Moran's I} = \frac{\sum w_{ij}\,(X_i-\overline{X})(X_j-\overline{X})}{\sum w_{ij} \quad s_x^2}$$

where $w_{ij}$ identify 'neighbors'.

- Example

$w_{ij}$ = 1 if within two layers of the target county.

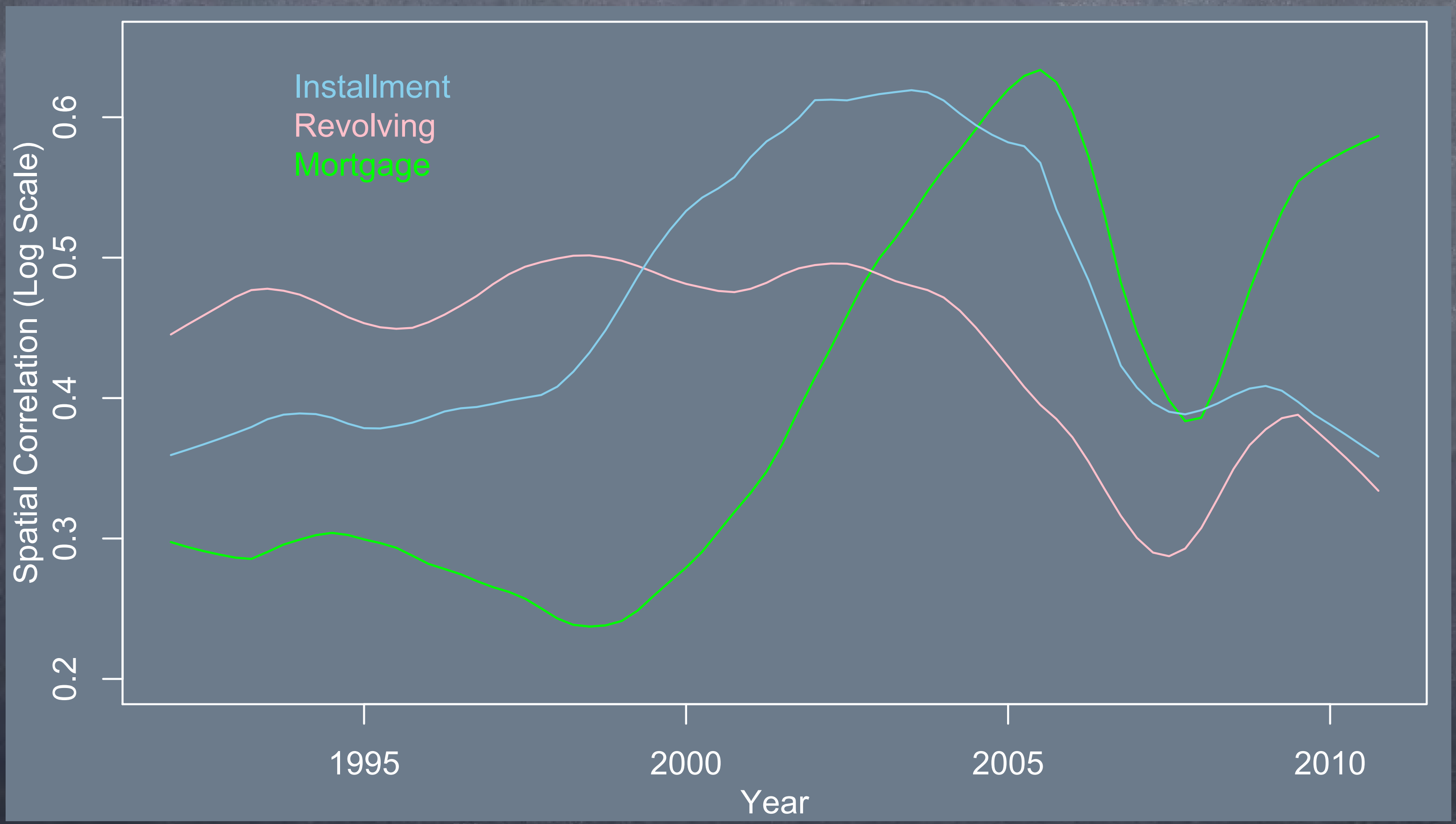
$w_{ij}$ = 0 otherwise.




$w_{ij}$ = 1 for those in gold

# Spatial Correlations

- Moran's I shows surprising correlation for various types of default.



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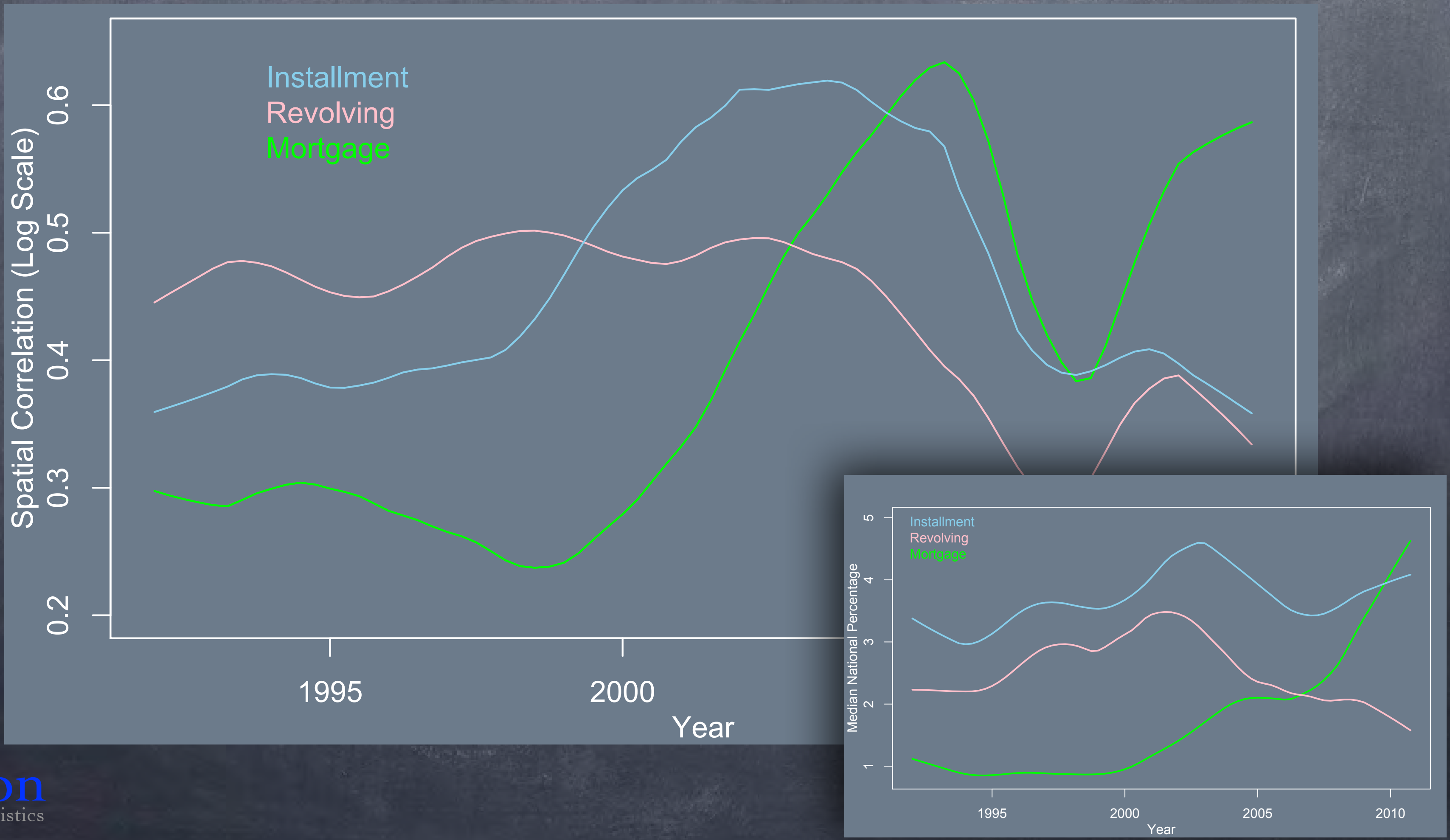


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# Spatial Patterns

# Correlation Risk

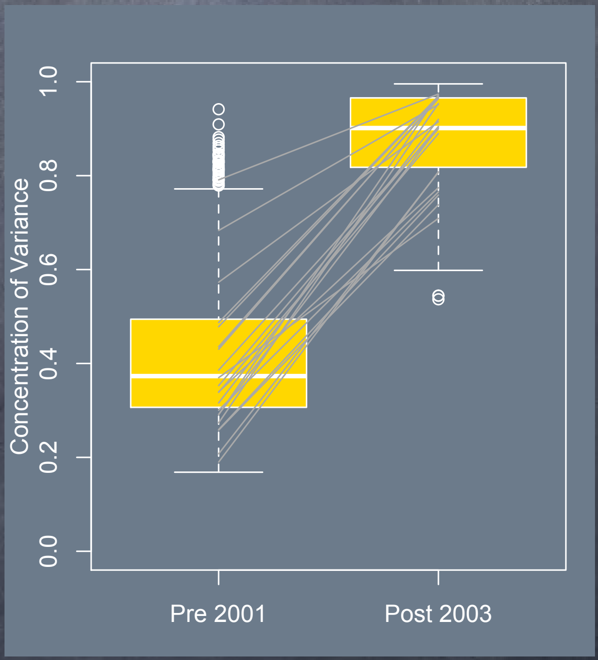
- Spatial association suggests correlation risk.
- Question
  - Pick a county c with neighbors N(c)
  - How much of the variation in default rates among neighbors of c can be described using a common trend?

$$D_{N(c),t} \approx u_c \, v_t$$

- Principal components
  - First principal component of the covariance matrix S among the neighbors of a county
  - Largest eigenvalue indicates amount of variation represented by common trend

# Correlation Risk

- Neighborhoods
  - Consider all neighborhoods among the 900+ counties in the analysis
  - Compare the percentage of variation in the first component using quarters before 2001 to the percentage in quarters after 2003
- Results
  - Mortgage default rates
  - Percentage of variation rises basically everywhere
  - Median increases from 0.4 up to 0.9.



# Patterns in Variation

- Borrow technique from climatology
  - Empirical orthogonal functions
  - Segmentation: Find locations that covary in time
- Singular value decomposition
  - Extend principal components
  - X holds default rates at 900 locations, 76 times
  - Approximation

$$X = UDV', \text{ or } X = \sum u_i (d_i v_i')$$

  - U captures spatial patterns, V holds time
- Orthogonal rotation
  - Rotate the orthogonal factors to clarify geographic clustering

# Low-Rank Approximation

Matrix of
default rates

$X = \boxed{d_{ct}}$

# Low-Rank Approximation

Matrix of default rates

Time trend

$v_{11} \quad v_{21} \quad v_{31} \quad \ldots \quad v_{T1}$

$$X = \boxed{d_{ct}} = \begin{array}{c} u_{11} \\ u_{21} \\ u_{31} \\ \vdots \\ \vdots \\ u_{n1} \end{array} \boxed{u_{c1}\ v_{t1}} + \text{more}$$

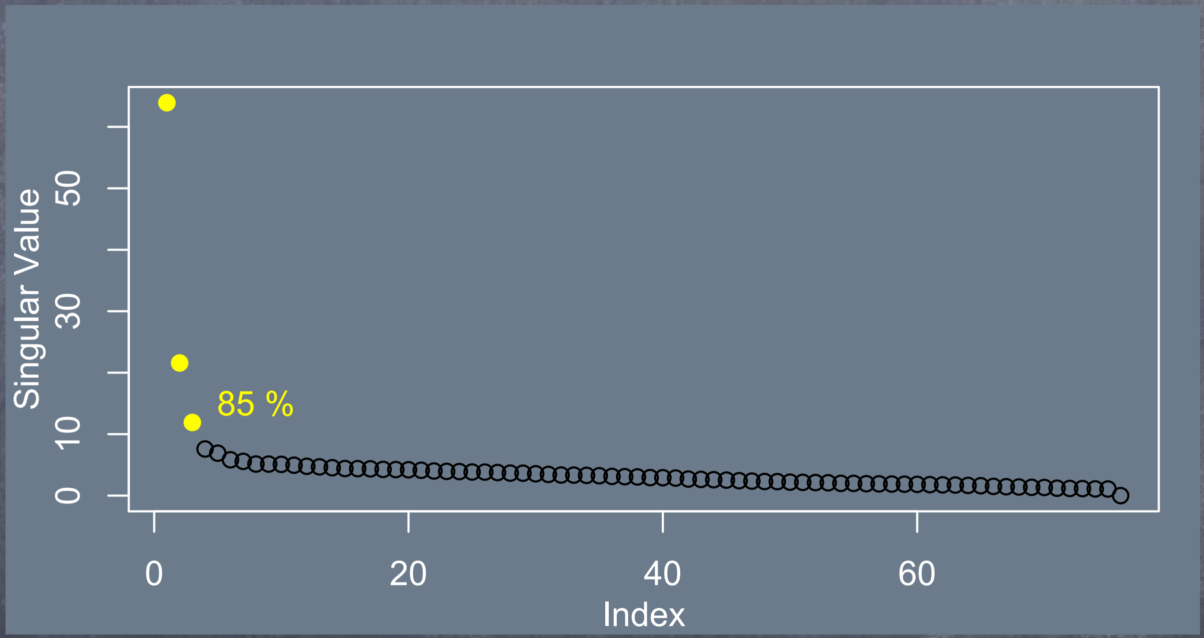
Spatial effects

Decomposition 'knows' nothing of time or space...

Are counties with common trends adjacent?

# Singular Value Decomposition

- How many terms
  - Singular values suggest need three terms to represent variation in mortgage defaults.

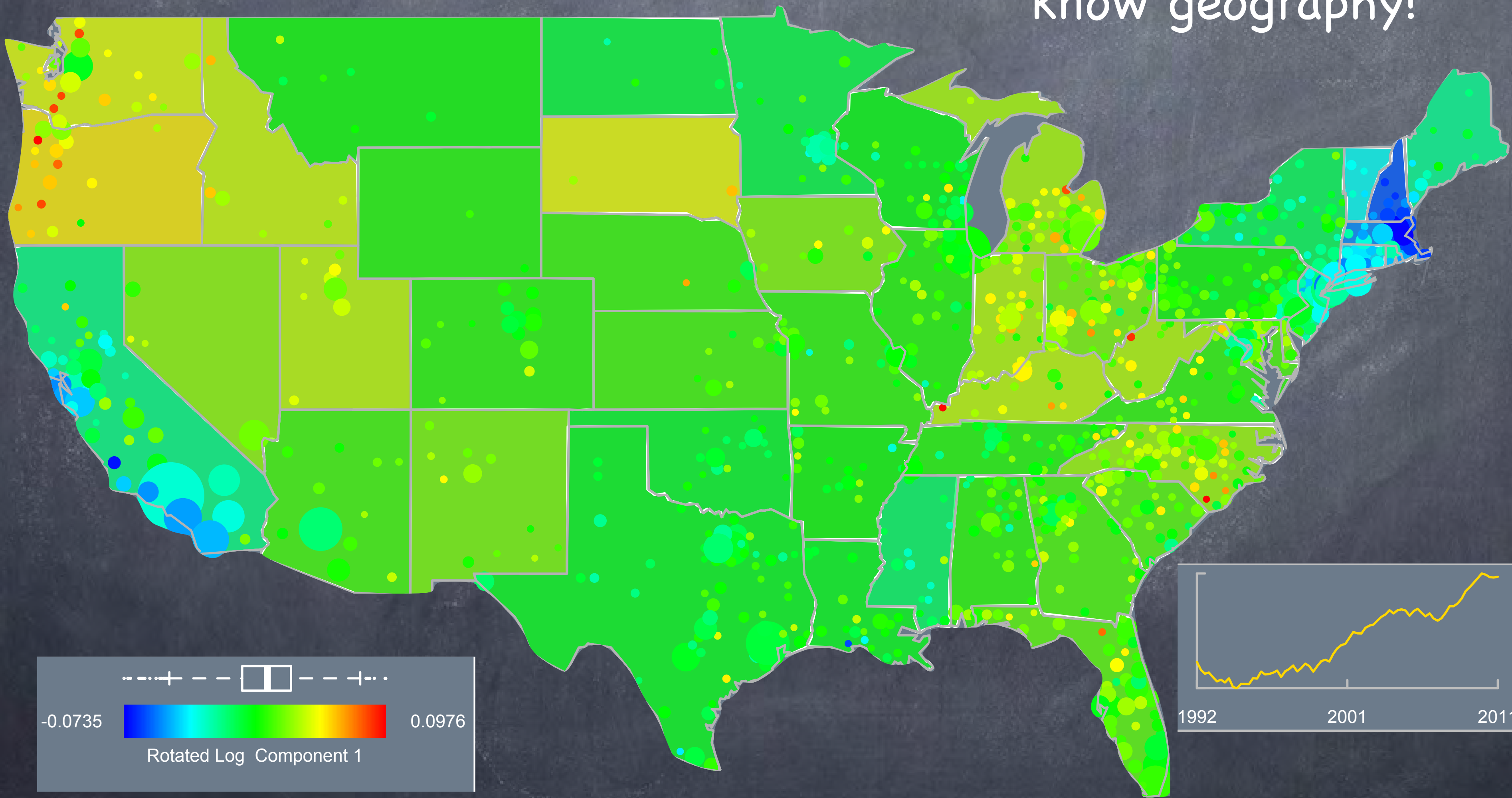



- Rotated components
  - Sacrifice orthogonality to improve interpretation
  - Each rotated component has ≈⅓ of variance

# First Component

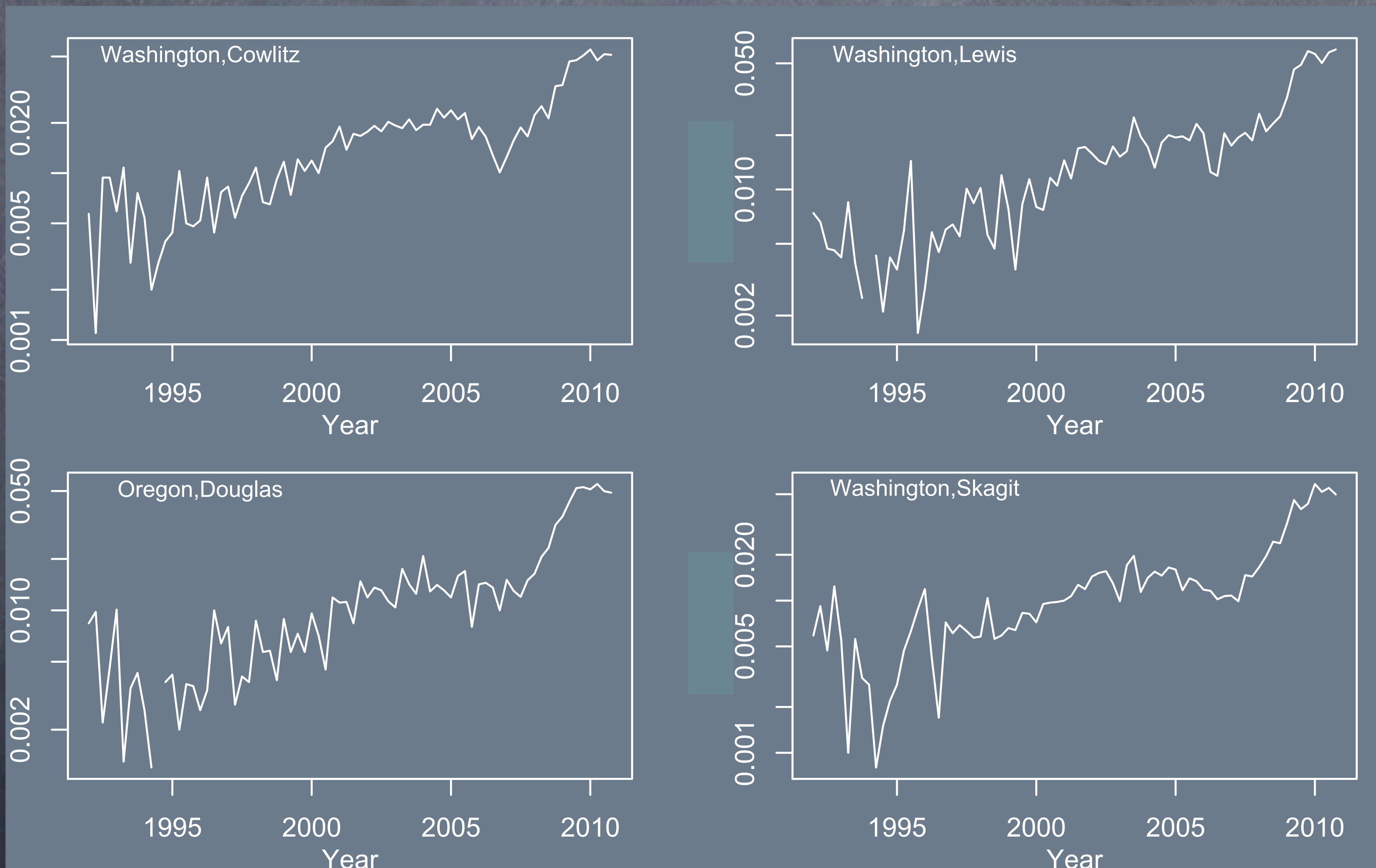
- Long term problems...

SVD does not know geography!



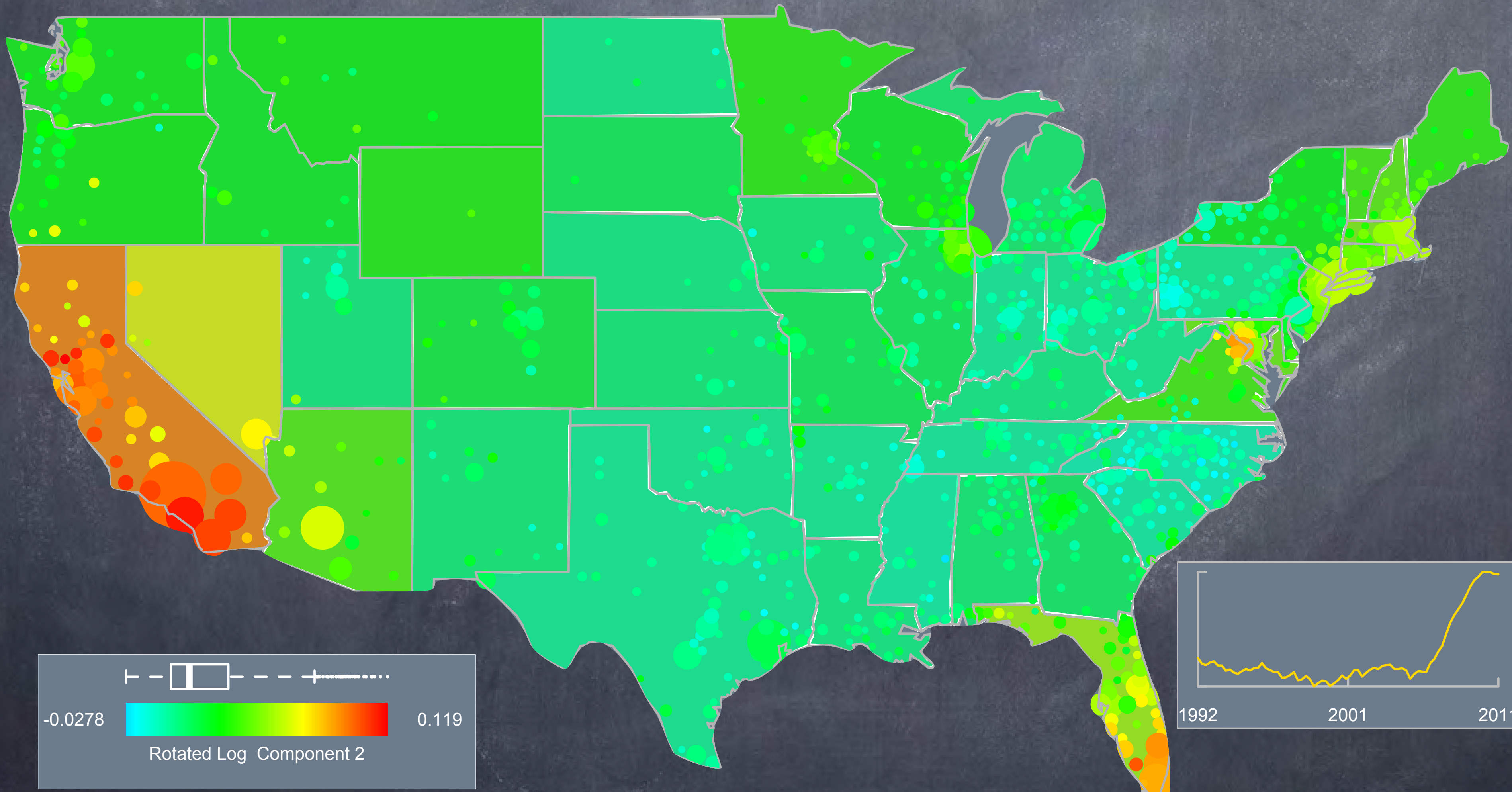
# Eg: Long-term Problems

- Defaults in the Northwest US



# Second Component

- Recent surge in defaults

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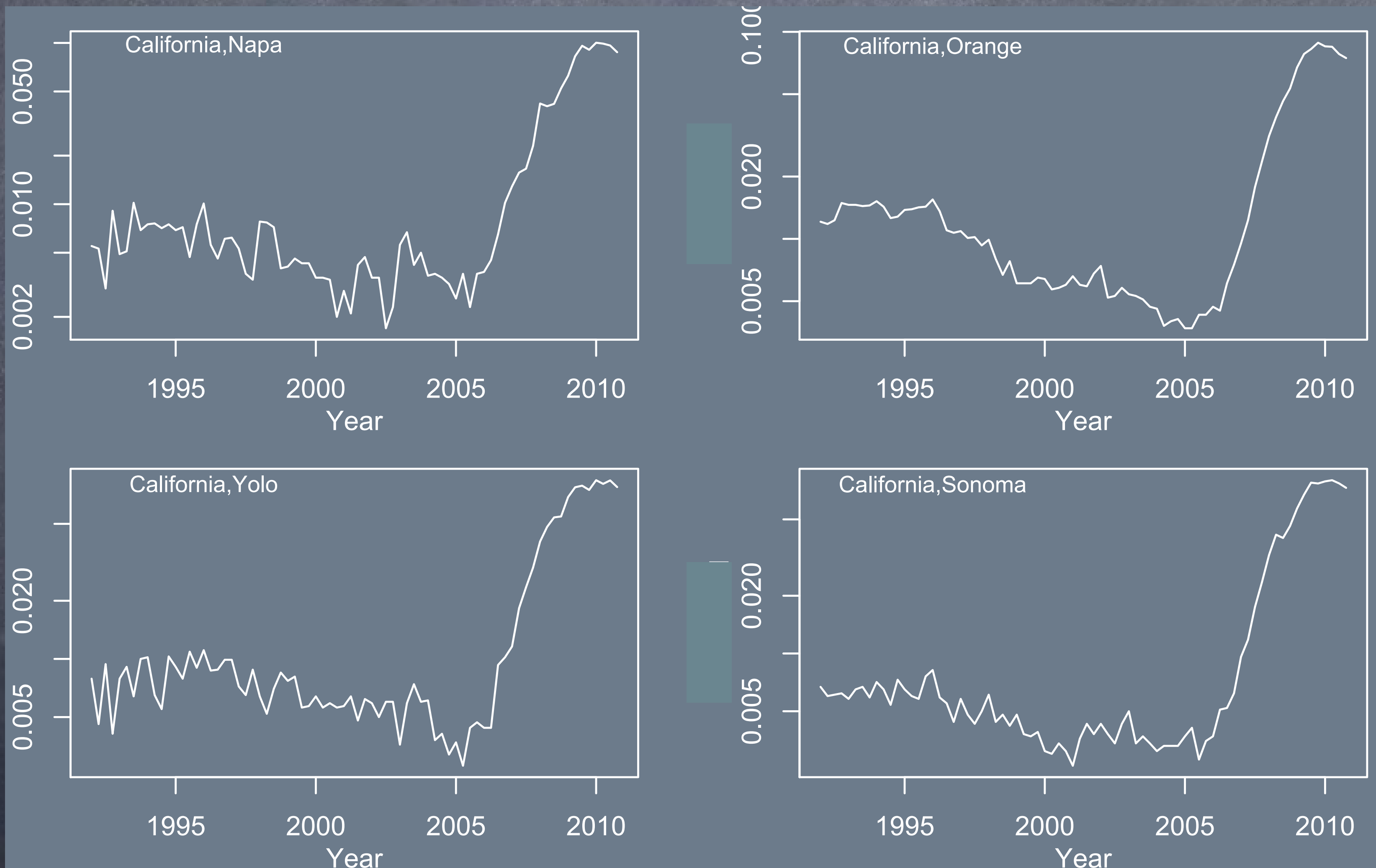
# Eg: Recent Surge

- Coastal problems: California, southern Florida

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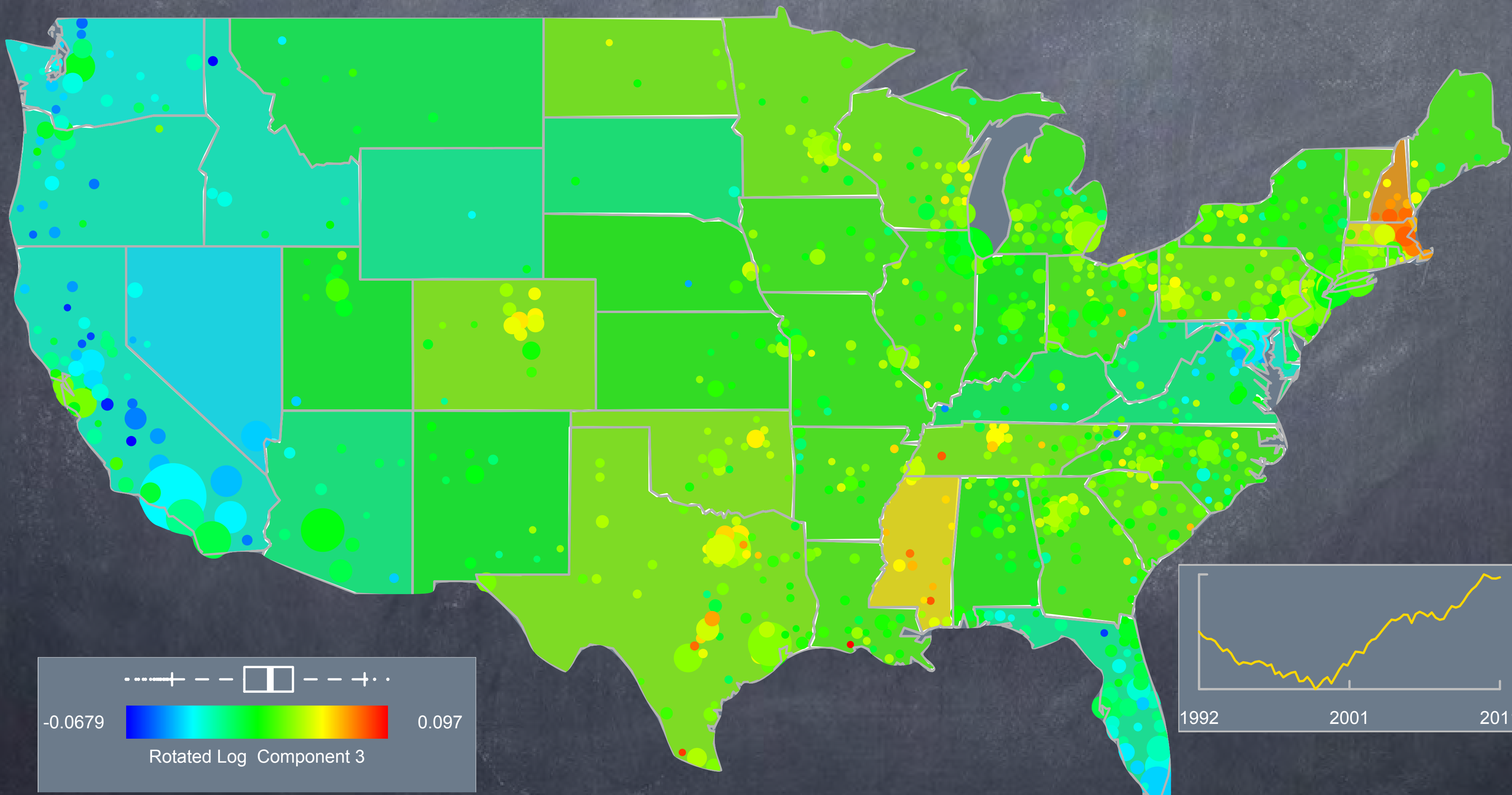
# Third Component

- Counties that had been doing well.



# Eg: Were going well

- Some in the South, some in New England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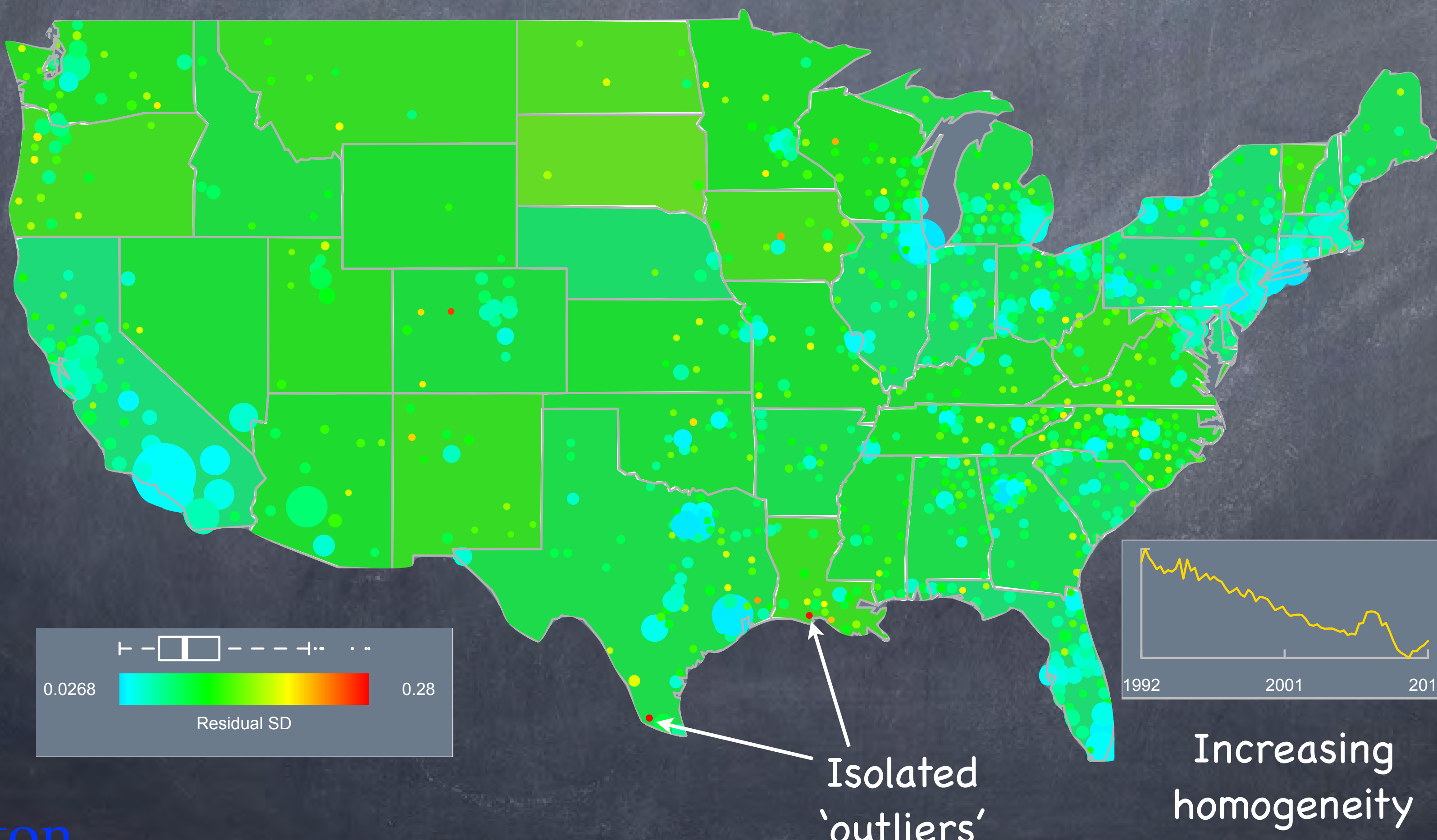
# Residual Analysis

- Subtract retained components from data
  - Map shows SD for locations
  - Trend line shows SD over time




Increasing homogeneity

# Covariates

- Covariate effects depend on region
- Regional unemployment
  - Variability changes over time
  - Association with mortgage default changes

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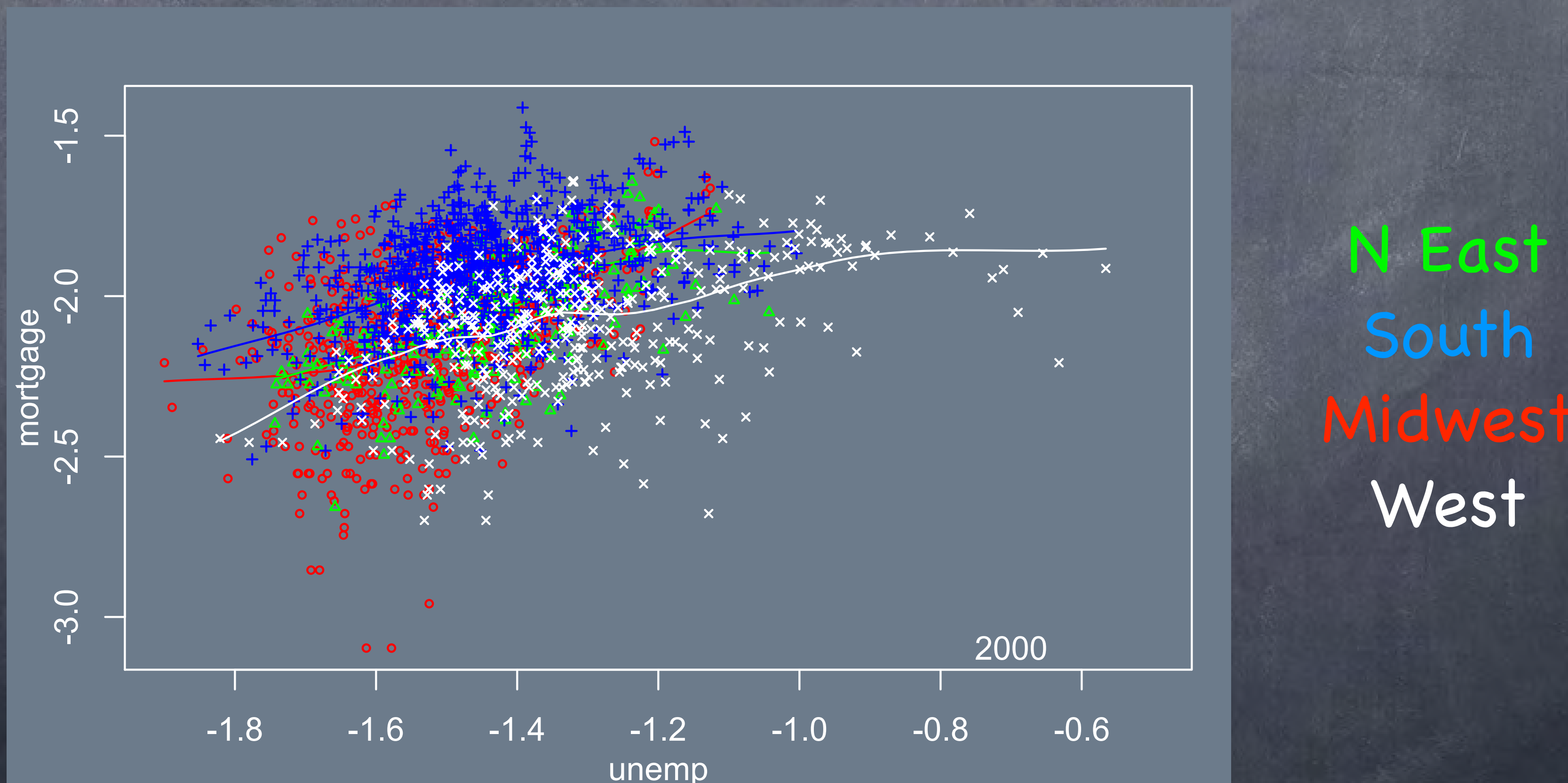


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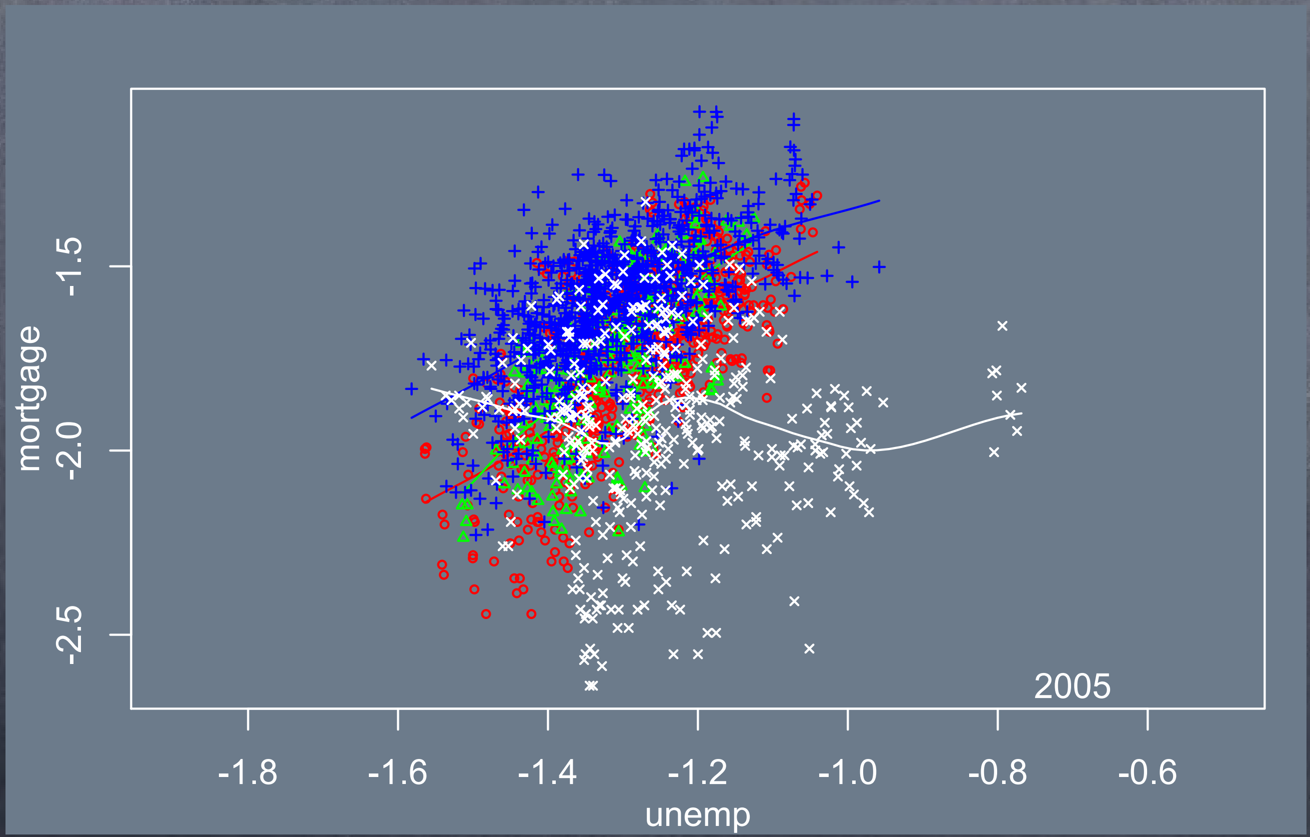


N East
South
Midwest
West

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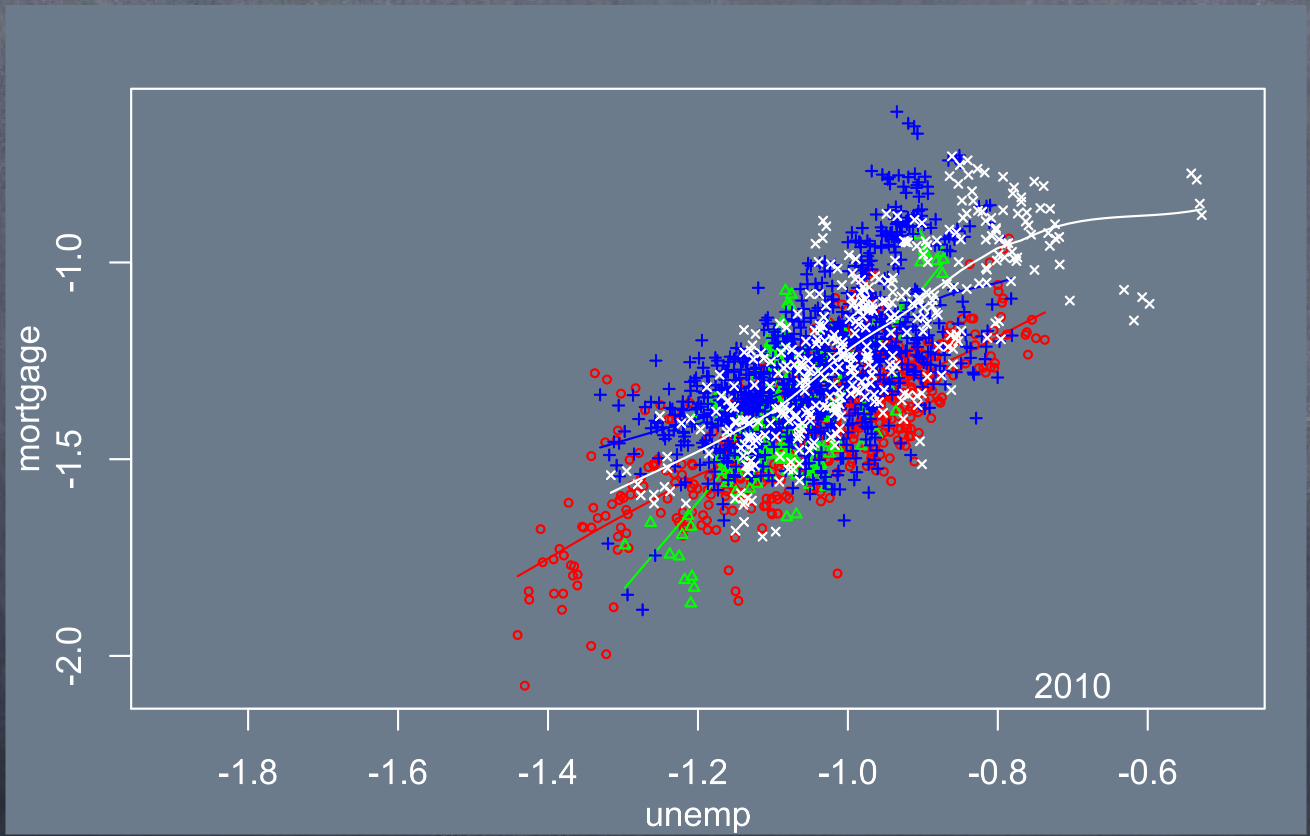


N East
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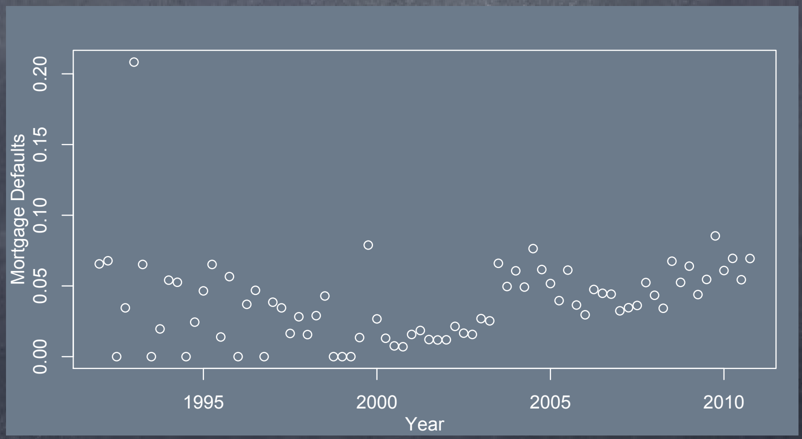
# Discussion: Spatial Patterns

- General trends
  - Rising defaults
  - Increased spatial concentration
- Timing of mortgage defaults
  - Some have struggled for a long time.
  - Bubble exploded in California, Florida.
  - Less discussed...
    Surge around 2000 in less talked-about locations: Deep South, New England
- Aside
  - SVDs are great for finding outliers!



# Exploratory Models

# Transition

- Switch type of debt
  from mortgage to cards
- Revolving default rates
  - Data cover most of the US
  - Less political upheaval
- But similar problems remain:
  - Substantial flight to quality in later years
  - Demographic shifts remain relevant
  - Heterogenity in size and characteristics

# Local Models

- Consider a reduced-form, economic model
  - Response $Y_{cq}$ = log(default rate)
  - Lags of default rate
  - Economics (unemployment, income)
  - Credit data (utilization, other debt)
- Issues
  - What variables to use in the models?
  - How to obtain an honest standard error?
  - Where's the independence?
- Fit within "slice" of time or space
  - Time: 3,000 counties during a specific quarter
  - Space: Subset of counties over many years

# Slices in Time

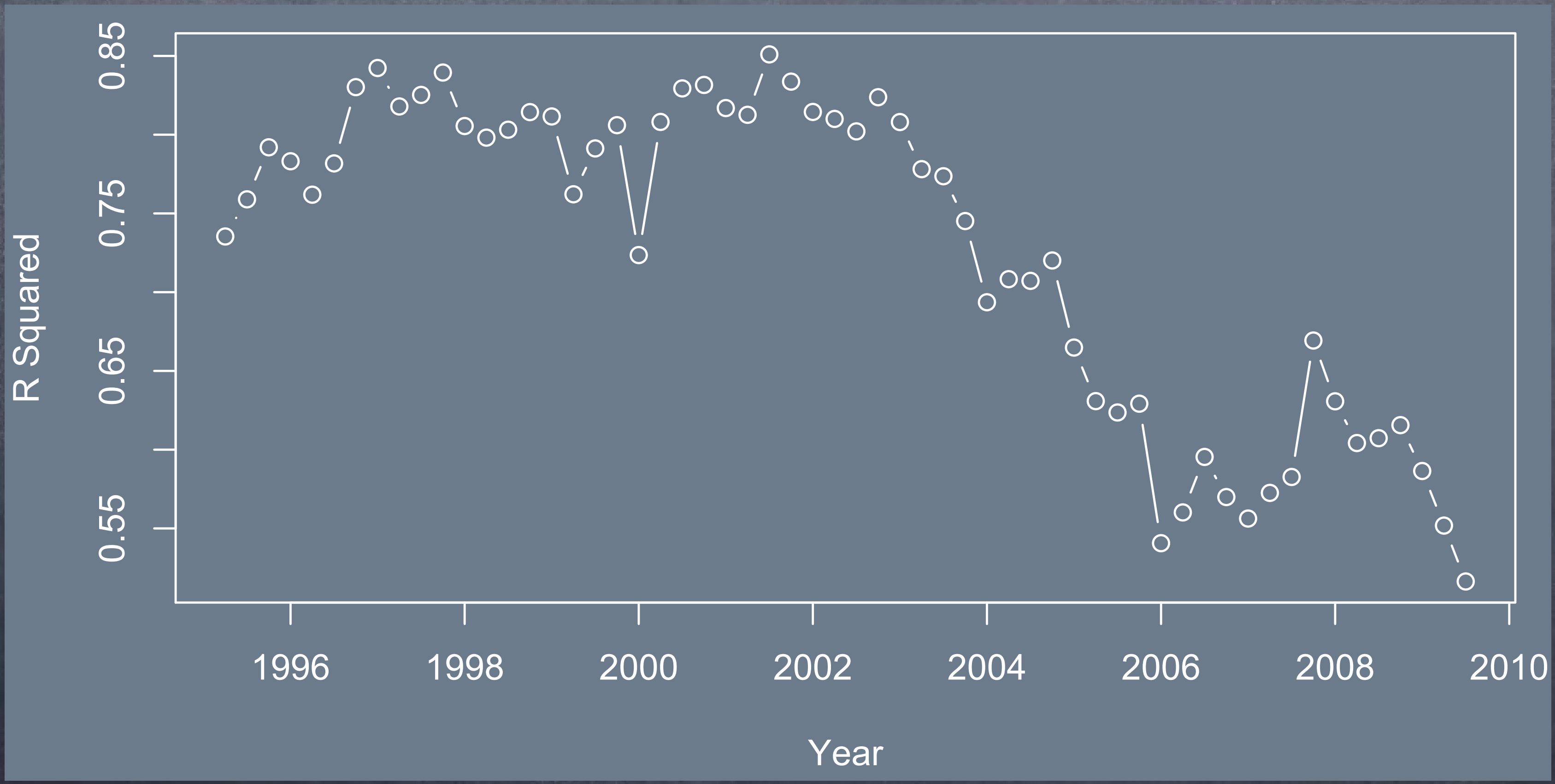
$Y_{cq}$ = log(default rate) in county c, quarter q

| | | | | |
|---|---|---|---|---|
| $Y_{11}$ | | $Y_{1q}$ | | $Y_{1T}$ |
| | | … | | |
| $Y_{c1}$ | | $Y_{cq}$ | | $Y_{cT}$ |
| | | … | | |
| $Y_{n1}$ | | $Y_{nq}$ | | $Y_{nT}$ |

Time

# Local-time Models

- Procedure
  - Fit over counties within a given quarter
  - Plot over time, "population drift"
- Goodness-of-fit deteriorates in later years

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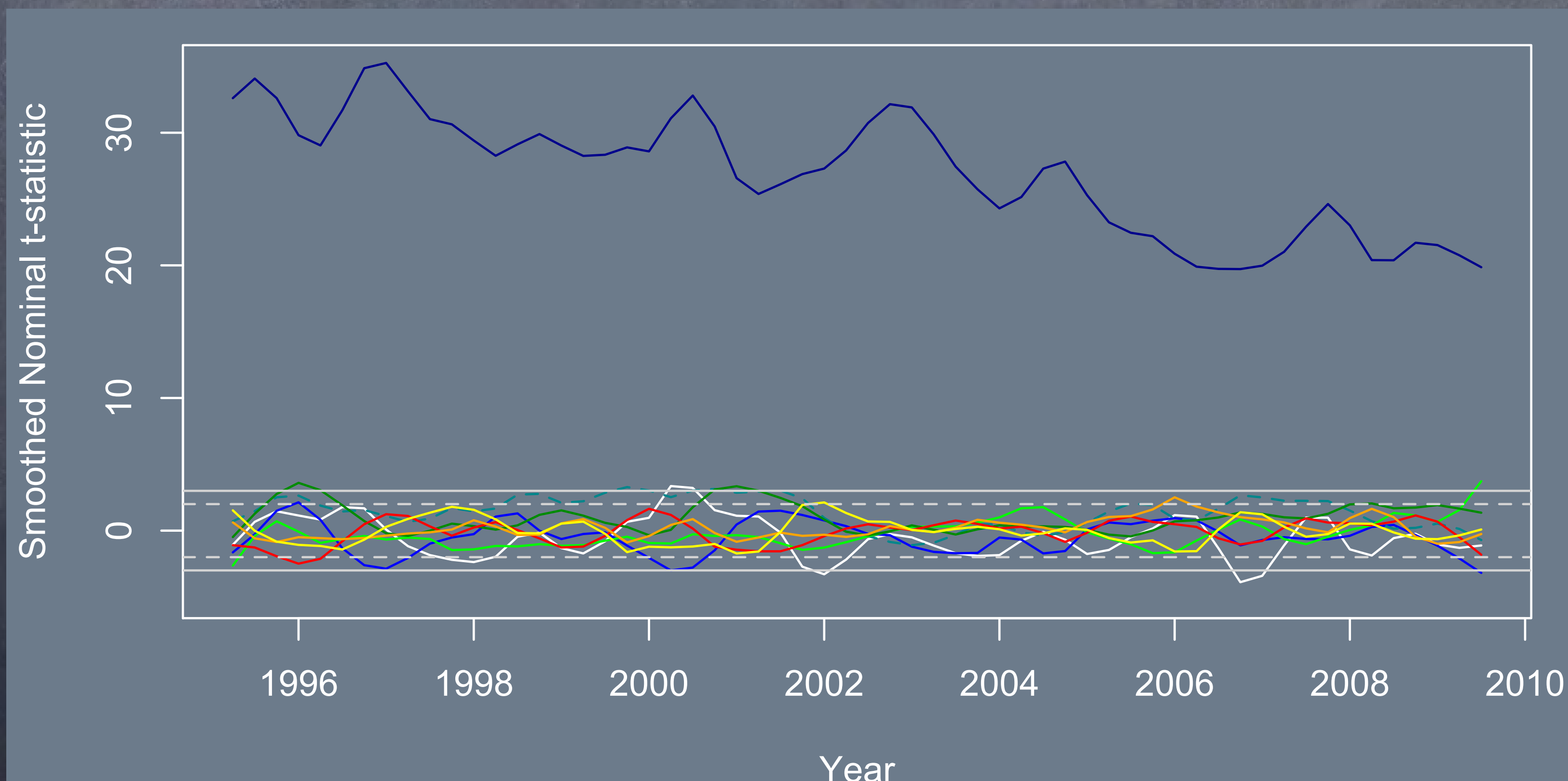
# Local-time Models

- Procedure
  - Fit over counties within a quarter
  - Plot coefficients over time, "population drift"
- Nominal t-statistics identify only lag




t-stat of lag $Y_t$

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# Slices in Space

$Y_{cq}$ = log(default rate) in county c, quarter q

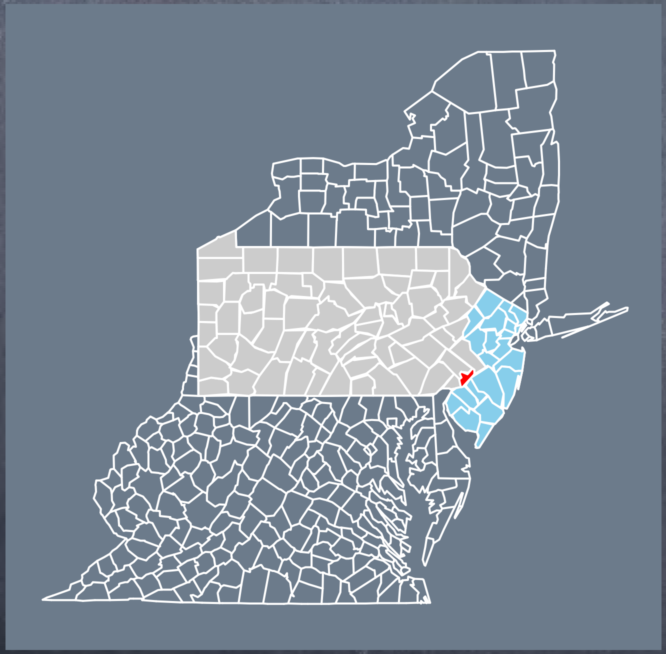
| | | | | |
|---|---|---|---|---|
| $Y_{11}$ | | $Y_{1q}$ | | $Y_{1T}$ |
| | | … | | |
| $Y_{c1}$ | | $Y_{cq}$ | | $Y_{cT}$ |
| | | … | | |
| $Y_{n1}$ | | $Y_{nq}$ | | $Y_{nT}$ |

Time

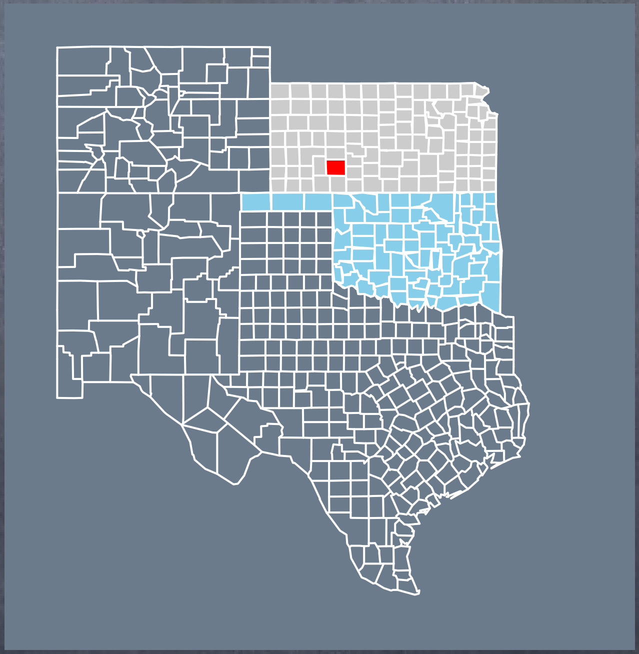
# Local-space Models

- Procedure
  - Fit regression model in cluster of counties
  - Measure residual dependence

Urban, densely populated



Philadelphia, PA



Ford County, KS

Rural, sparsely populated

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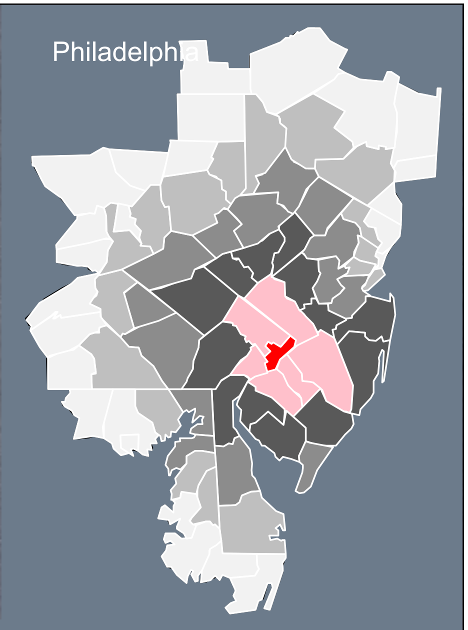
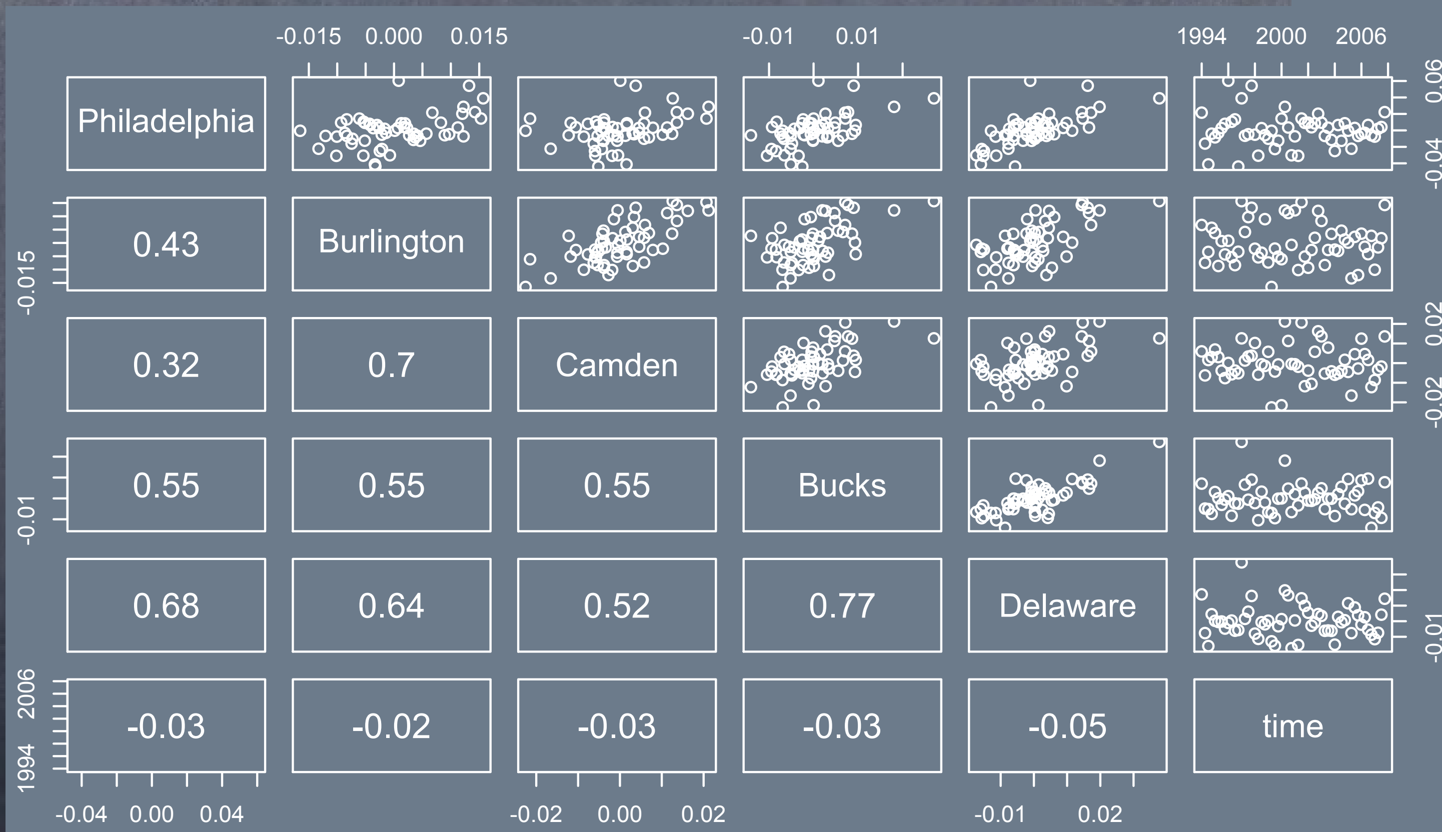
# Urban Models

- Models fit well, $R^2 \approx 80\%$ or more
- Spatial correlations depend on proximity, political boundaries
- No residual autocorrelation





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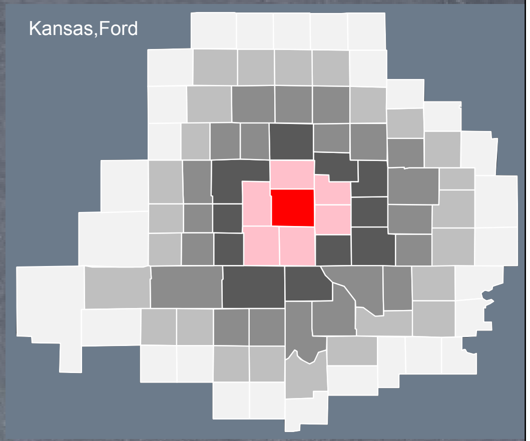
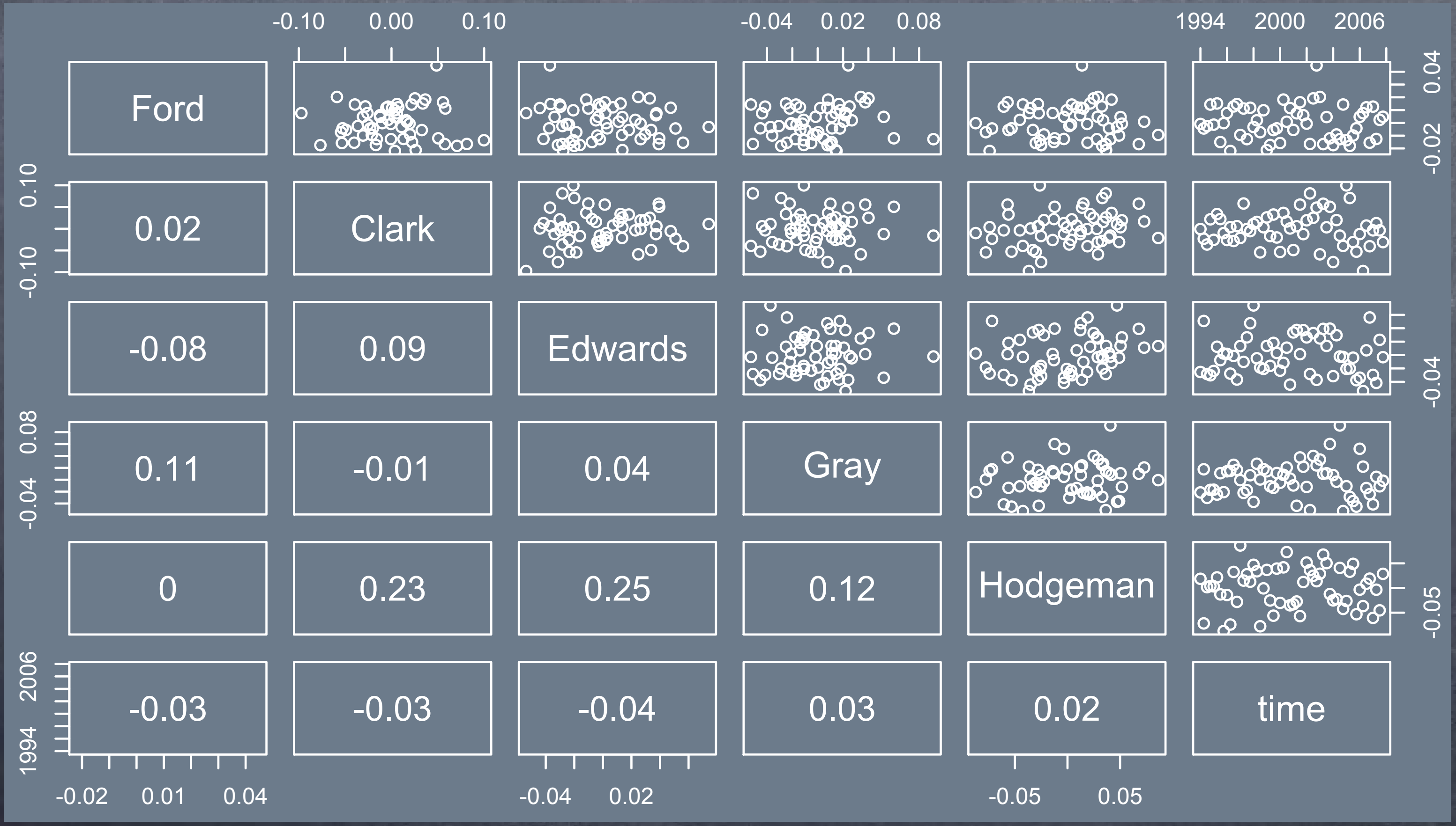
# Rural Models

- Models fit weakly
- Small spatial correlation
- No residual autocorrelation

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# Lessons from Exploration

- Over time…
  - Evolving, simple models describe much of the variation in default rates, leaving…
  - Errors that appear uncorrelated over time
- Over space…
  - Complex spatial dependence
- Explanatory variables
  - Subtle contribution from local explanatory variables such as income
  - Adjustments for spatial dependence needed to avoid over-fitting

# Confirmatory Models

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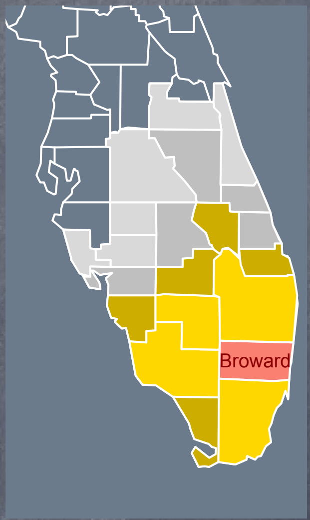
# Markov Random Fields

- Idea
  Describe spatial distribution by the collection of conditional distributions

- Conditional independence
  Default rate $Y_k$ in location k depends only on its neighbors N(k),
  $$\{ Y_k \mid Y_m, m \neq k \} = \{ Y_k \mid Y_{N(k)} \}$$

- Gaussian MRF… CAR model
  Covariates model broad structure, with spatial correlations for errors
  $$\{Y_k \mid Y_{N(k)}\} = N(\mu_k + \Sigma\ W_{km} (Y_m - \mu_m), \sigma_k^2)$$



# Conditions for MRF

- Not every set of conditional distributions specifies a valid joint distribution.
- Gaussian MRF (Besag 1974)

  $\{Y_k \mid Y_{N(k)}\} = N(\mu_k + \Sigma\ W_{km}(Y_m - \mu_m),\ \sigma_k^2)$

  implies that joint distribution is

  $\{ Y \} = N(\mu, (I-W)^{-1}S^2)$

  for $S = \mathrm{diag}(\sigma_k)$.

- Implications
  - $(I-W)$ must be positive definite
  - $(I-W)^{-1}S^2$ must be symmetric
- Spatial pattern matrix (Cressie et al 2005)
  - Obtain spatial correlation parameter $\gamma$ by

# Is CAR right?

- What is the residual structure?
  - Likelihood ratio test between nested models
  - Equal correlation model is a CAR model with lots of neighbors, N(k) = all indices but k.
- Use all 3,000 counties
  - Logit response p/(1-p)
  - Var(logit) ≈ 1/(np(1-p)) determines $\sigma_k$
  - Covariates include local unemployment, poverty...
- Compare error specifications
  - CAR with single layer neighborhood
  - Equal-correlation model

# Testing Procedure

- Covariance structure
  - $Cov(Y_t) = (I-W)^{-1}S^2 = (I- \gamma H)^{-1}S^2$
- Different CAR models specify different neighborhood structures in H
  - Local spatial neighborhood
  - Global equal correlation
- Models are "overlapping" if $\gamma = 0$
- Two-part testing process (Vuong 1989)
  - Test first whether $\gamma = 0$
  - If reject, then test models using expected

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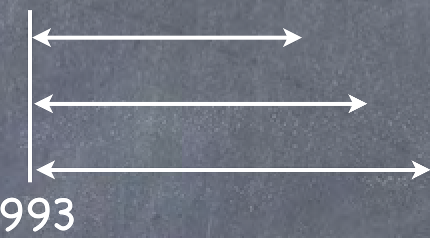
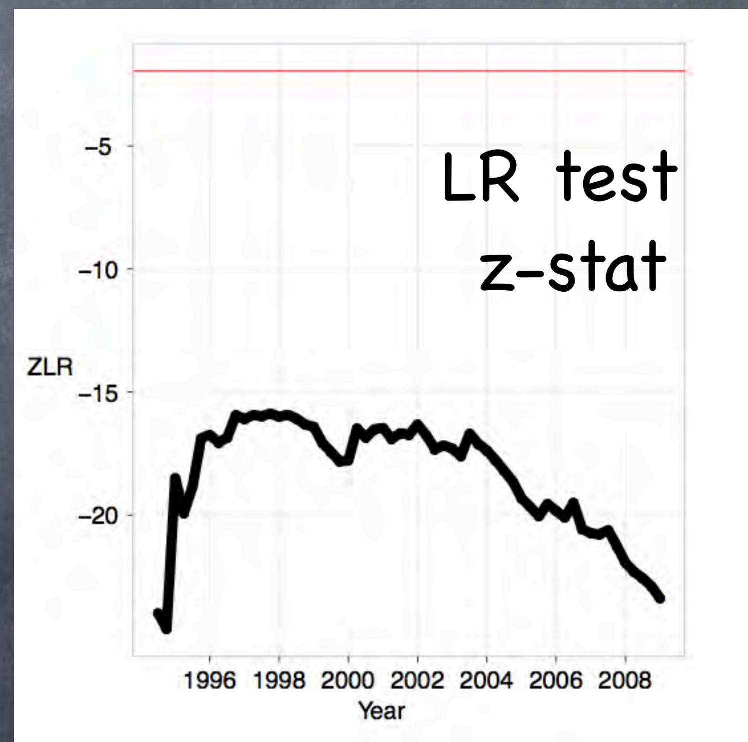
# Results of CAR Test

- Recursive estimation
  - Use history from 1993 forward
  - Evaluate model at 'current' time




- Test CAR vs equal corr
  - Equal correlation with smaller, broader corr dominates
  - Hints at national latent variable

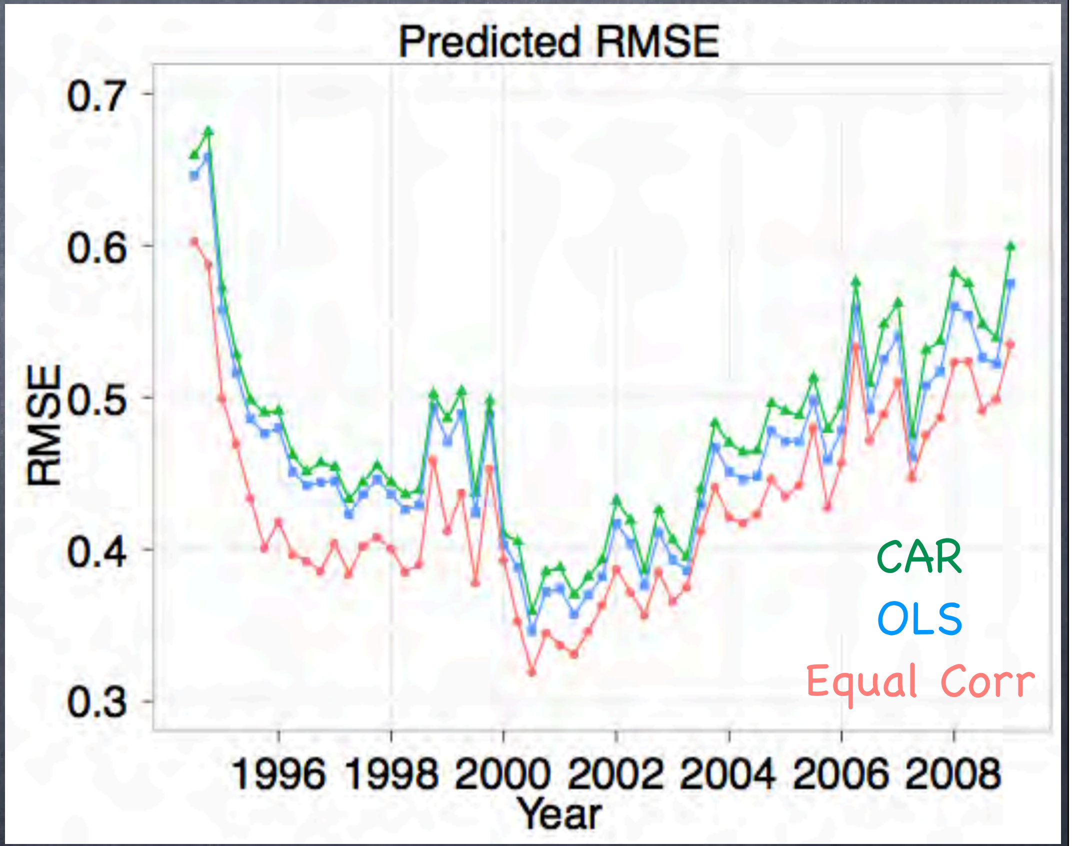



Ending Year

# Prediction Results

- Comparison of Prediction MSE
  - OLS
  - CAR (local neighborhood)
  - Equal correlation (global neighborhood)

- Results
  - Only lags of default as predictors
  - Equal correlation has smallest MSE
  - Model performance worse as time accumulates

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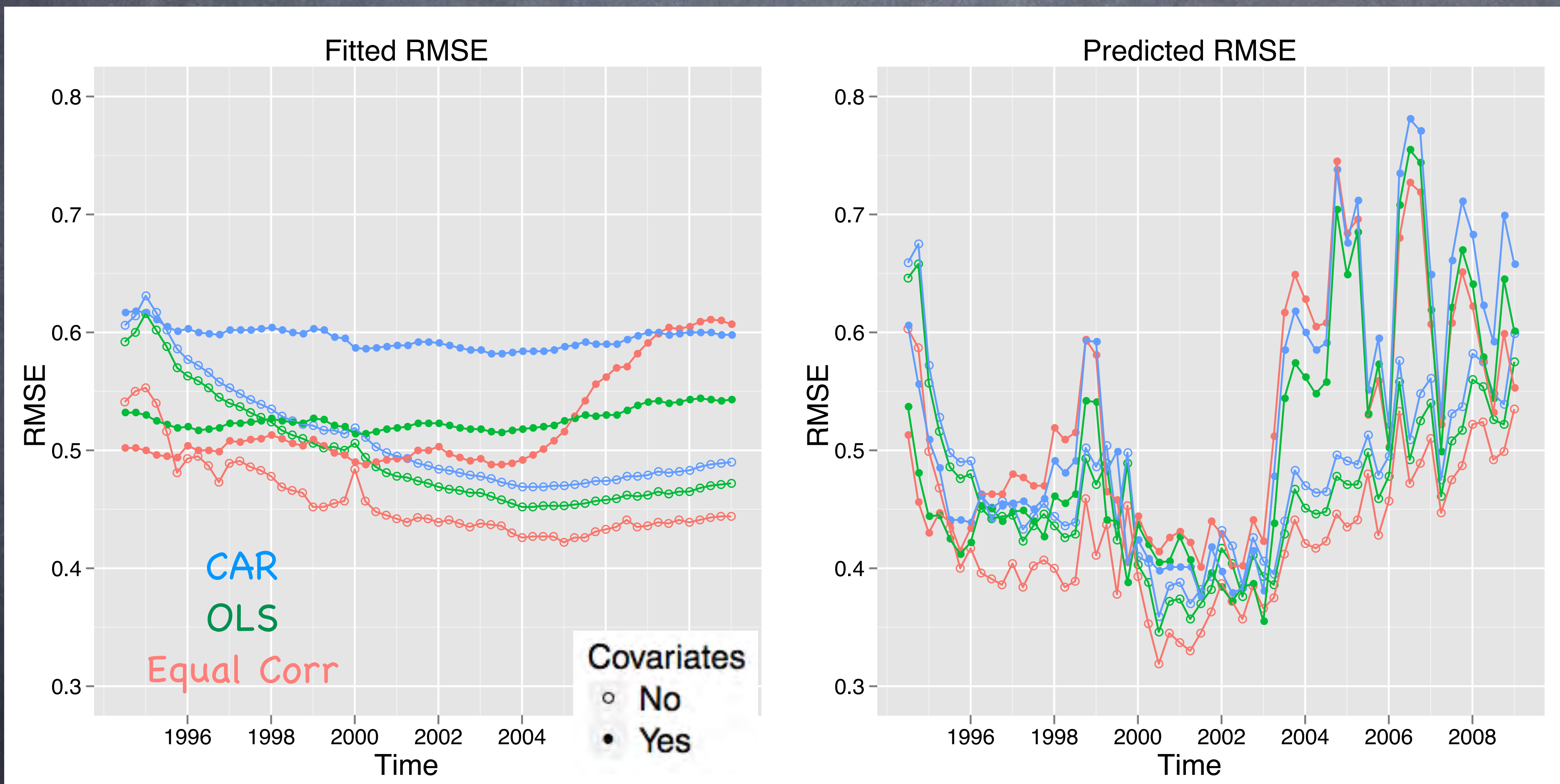
# Covariates

- Less accurate with explanatory variables

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# Summary & Discussion

# Key Points

- Substantial spatial correlations
  - Don't have 3,000 independent observations
    Cannot claim 3,000 x 80 = 240,000 d.f. in models
  - Over-stated claims of significant inference
- Time-specific, location-specific patterns
  - Population drift over sub-models
  - Complex models most likely overfit
- Possible remedies?
  - Better economic modeling at consumer level
  - Portfolio view of individual consumer debt
  - Expensive to develop and maintain

# Directions in Modeling

- Adaptive, data-driven strategies
- Hierarchical Bayesian models
  - Dirichlet process priors via Markov chain MC
  - Scalable? Have not been able to scale to US.
- Large scale data mining using regression
  - Fast selection from 100,000's of variables
  - Predictive, but not "explanatory"
- Latent process models
  - High dimension hidden Markov models
  - SVD of massive matrices (50,000,000 cases)
  - Currently requires stable training set

# Comments

- Epidemic models
- Surface diffusion model
- Multi-mode factor analysis (covariates)
- Voxel correlation analysis

Thanks for coming...

Papers will eventually appear at
stat.wharton.upenn.edu/~stine