Valuing Investments
A Statistical Perspective

Bob Stine
Department of Statistics
Wharton, University of Pennsylvania

Overview

- Principles
  - Focus on returns, not cumulative value
  - Remove market performance (CAPM)
  - Watch for unseen volatility (peso problem)
  - Adjust for multiplicity
- How to evaluate...
  - Investments **as if** they behave like familiar random processes.
  - Plethora of choices offered by financial advisors
  - Specific investments using data

Data

Financial Data

- Examples
  - Indices
  - Portfolios
  - Mutual Funds
  - Hedge Funds
  - Commodities
  - Eurodollars
  - ...
- Time series without signal!

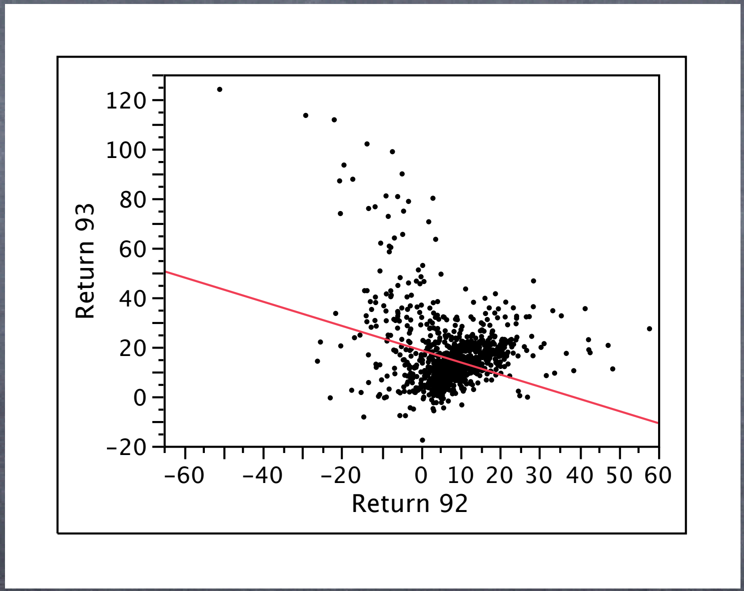
Case study in selection bias.

Multivariate co-integrated time series.

Chua, Foster, Ramaswamy, Stine (2007)
"Dynamic model for forward curve," Rev. Fin. Studies.

Mutual Funds

- Regress growth in current year on prior growth
  - Annual results for 1500 mutual funds
- "Statistically significant"



But the sign changes!

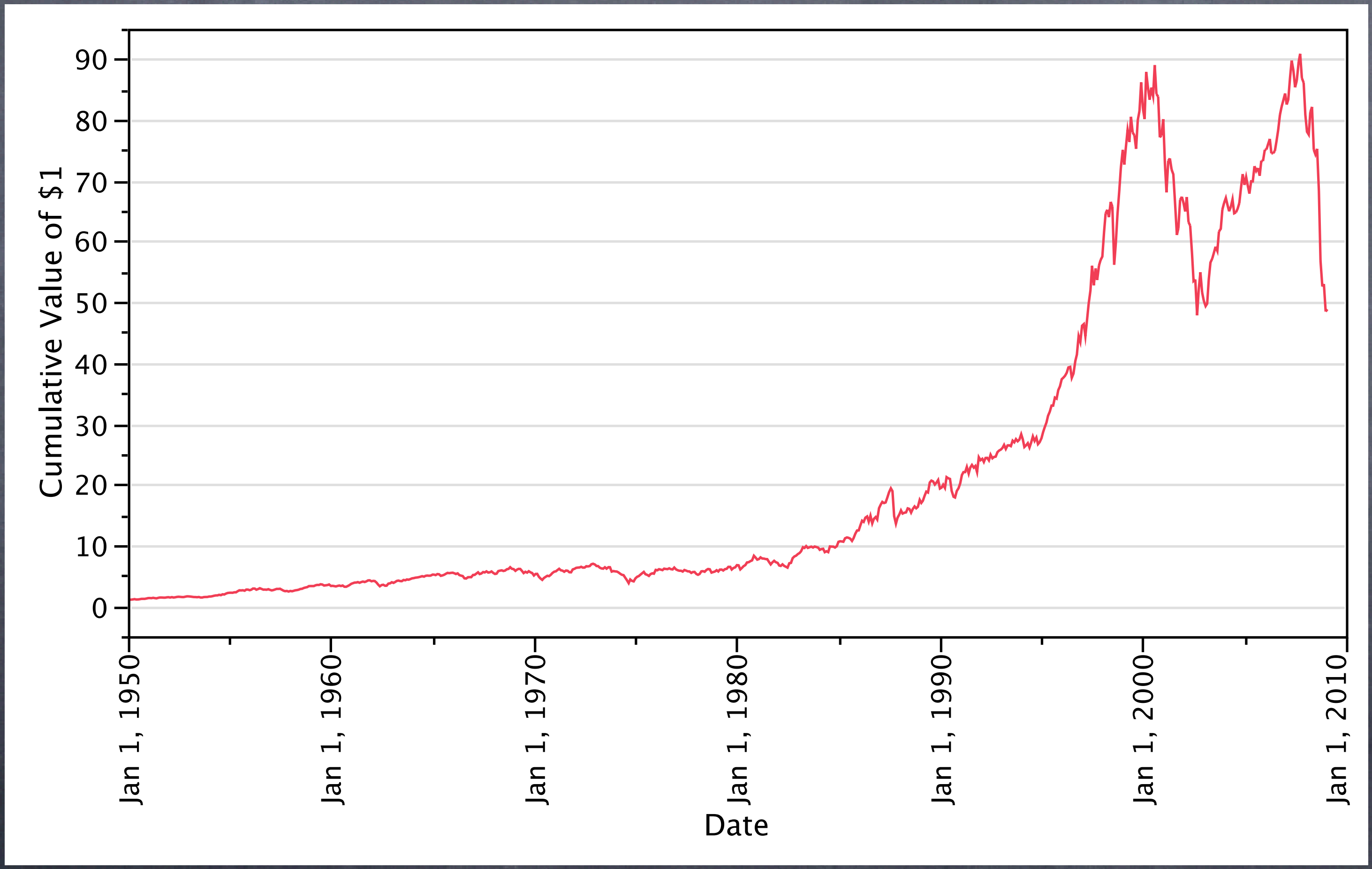
Term	Estimate	Std Error	t Ratio
Intercept	18.634313	0.467789	39.83
Return 92	−0.49079	0.04243	−11.57

Explanation: 1500 dependent observations...

Overall Market Performance

- Cumulative value of a $1 investment in the S&P 500 on January 1, 1950.

Log Scale

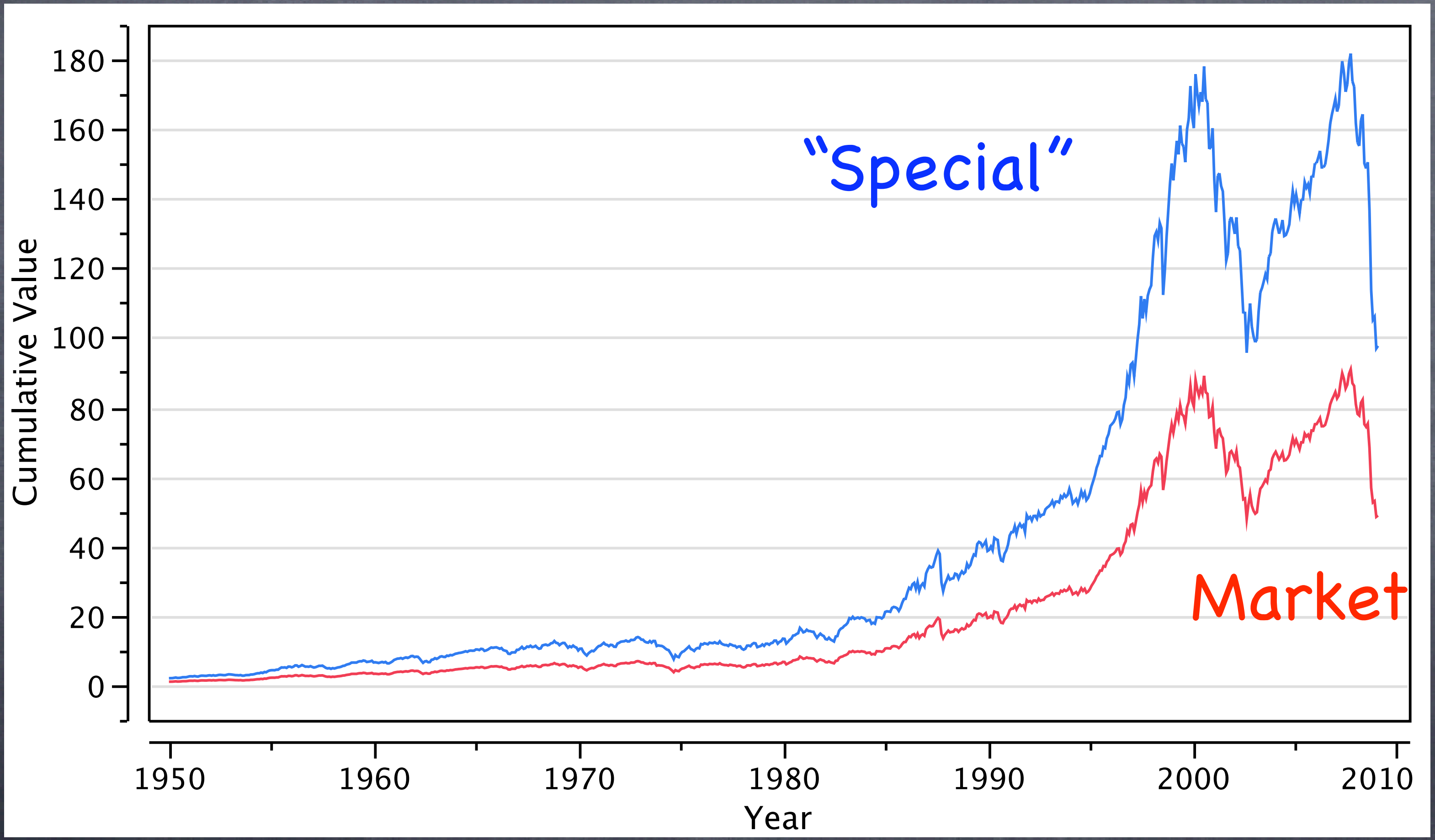


7% Annual growth

Data: Yahoo finance, Jan 1950 - Feb 2009, 710 months

Cumulative Returns?

- Too easy to be deceived...



Moral: Stick to returns...

Wharton
Department of Statistics

Monthly Returns

- Much simpler structure, almost iid...

$$\frac{P_t - P_{t-1}}{P_{t-1}}$$

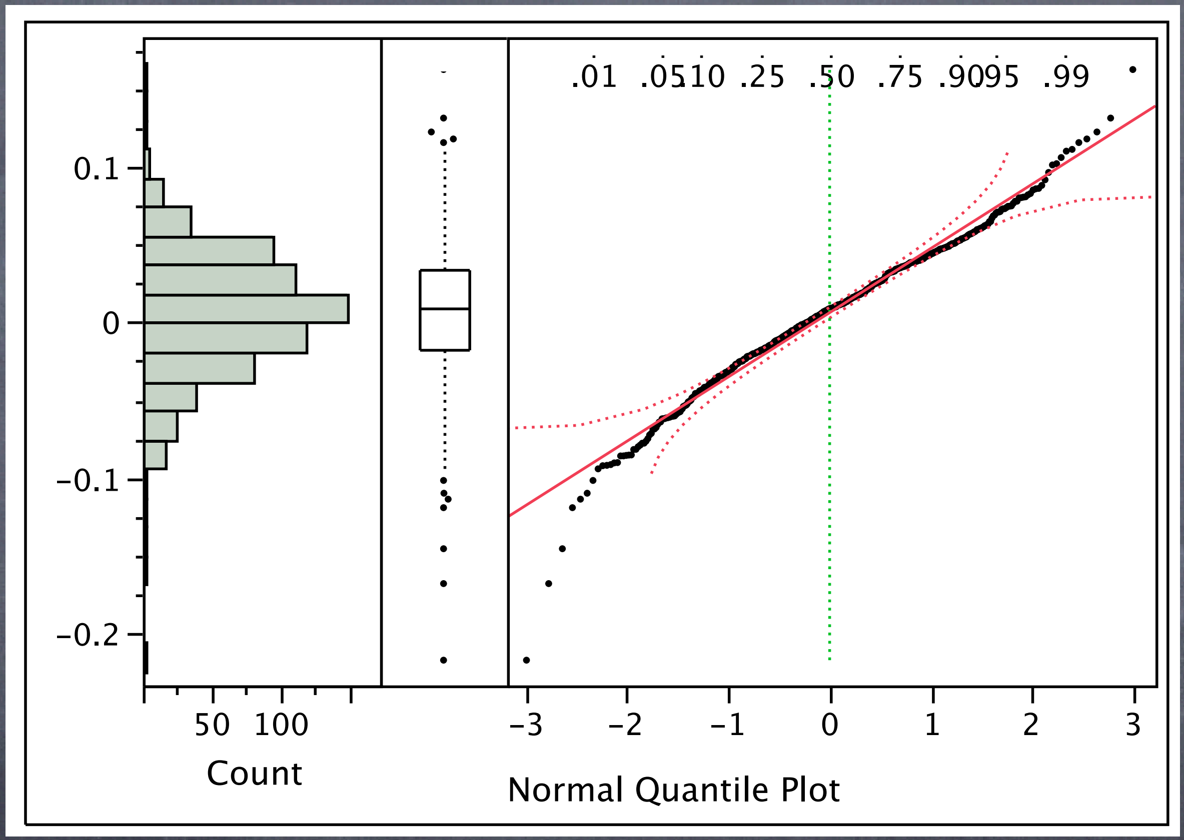
October 1987
"Black Monday"

August 1998
Long Term Capital

October 2008
Banking Crisis

Distribution of Returns

- mean = 0.0064, s = 0.0415, s^2 = 0.0017



Fat tails more apparent in daily data.

Wharton
Department of Statistics

The Dice Game

What makes a good investment?

- Consider 3 investments...

Investment	Average Annual Return	SD Annual Return
Green	7.5%	20%
Red	71%	132%
White	0%	6%

- Questions
  - Which of these do you like, if any?
  - How do you decide: risk versus return?

Wharton
Department of Statistics

Hands-on Simulation

- 3 dice determine outcomes:

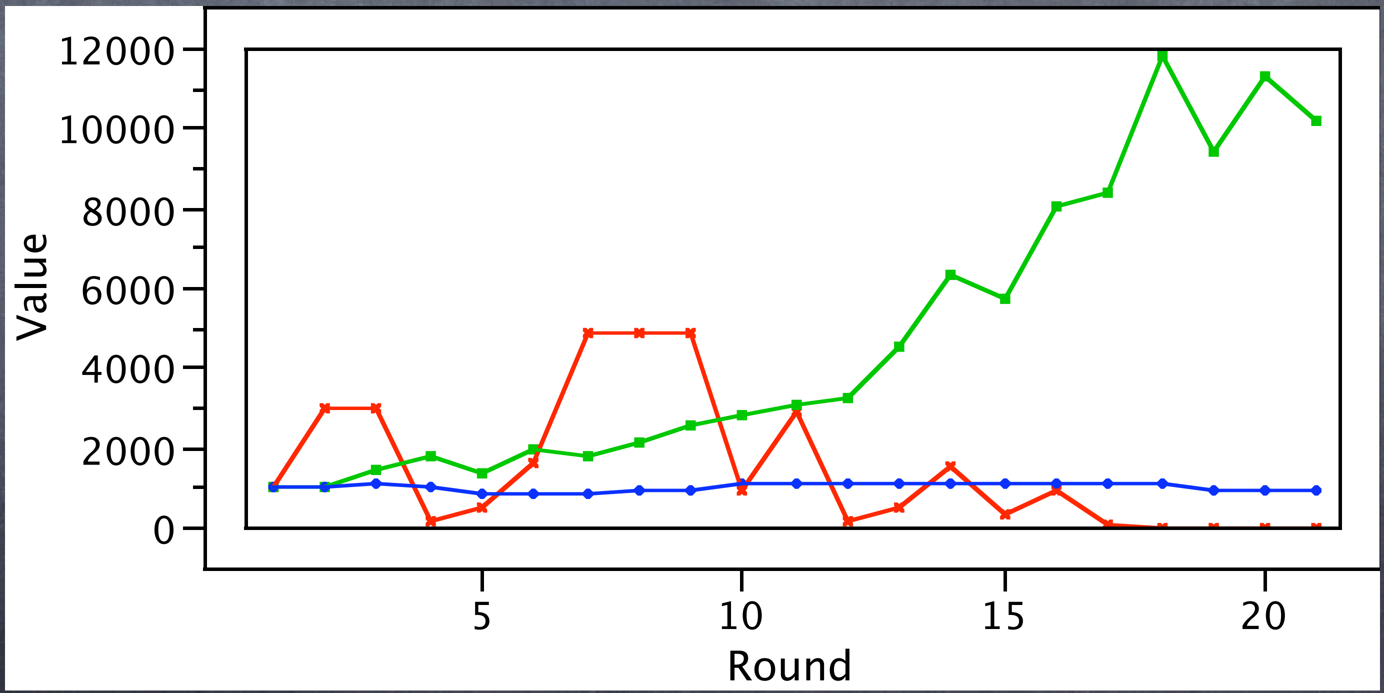
$$W_t = (\text{Table Result})\ W_{t-1}$$

Outcome	Green	Red	White
1	0.8	0.06	0.9
2	0.9	0.2	1
3	1.05	1	1
4	1.1	3	1
5	1.2	3	1
6	1.4	3	1.1

"Being Warren Buffett", Amer Statistician, 2006

Typical Results

- Red is "exciting" but generally loses value.
- Green offers steady growth.
- White goes nowhere.



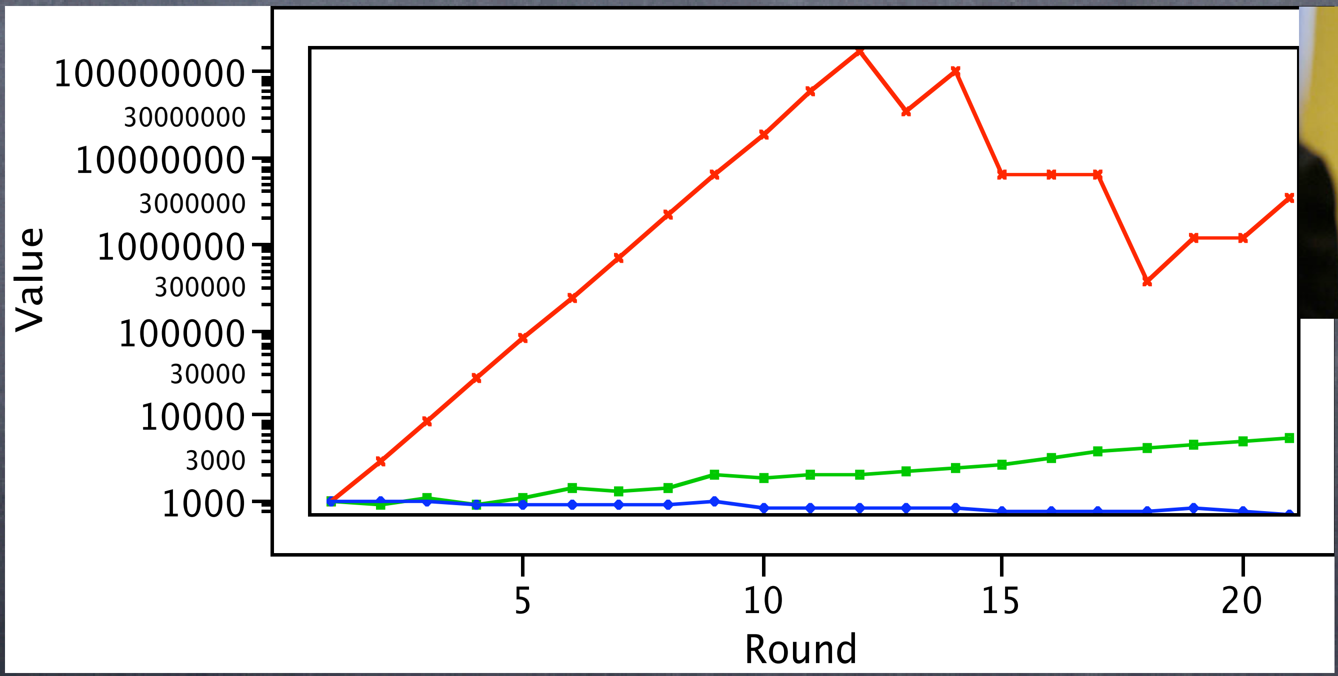
Green is calibrated to match annual excess returns on US stock market.

White is calibrated to match returns on Treasury Bills.

We made up Red!

Occasional Results

- Red soars...
  - In 20 rounds, the expected value of Red is
    $1.71^{20} = 45{,}700$
    times initial value



Digesting the Results

- Something to ponder
  - Most simulations with the dice result in Red having lost most of its value.
  - A few simulations end with Red being fabulously wealthy, the "Warren Buffetts" of the class
- In the long run, Red will lose (w.p. 1)
  - How can I recognize that Red will lose without waiting for it to happen?
  - Even so, how can I take advantage of Red?

A Special Opportunity!

- While you are thinking about those dice, here's a special opportunity...

The Bob Fund

- Guarantees 2% excess annual returns above any benchmark you want. Guaranteed.
- Rest assured, it's not a Ponzi/Madoff scheme.
- Contact me after the talk...

Investment Objective

- Long-run wealth

  $W_t = W_{t-1}(1+r_t)$

  $= W_0 (1+r_1)(1+r_2) \ldots (1+r_t)$

- If the r_t are independent over time, then

  $W_t \approx W_0 (1 + E(r_t) - Var(r_t)/2)^t$

Volatility Drag

	$E(r_t)$	$Var(r_t)$	$E(r_t)-Var(r_t)/2$
Green	0.075	$(0.20)^2 = 0.04$	$.075-.04/2 = .055$
Red	0.71	$(1.32)^2 = 1.74$	-0.16
White	0	$(0.06)^2 = 0.003$	-0.002

Can buy this one

Diversifying is good.

- Mix investments rather than leaving everything in one.
- Pink is a 50/50 mixture of Red & White.

E(Pink) = E(0.5 Red + 0.5 White)
= E(Red)/2 = 0.355

Var(Pink) = Var(0.5 Red + 0.5 White)
= Var(Red)/4 = 0.435

Sacrifice **half** of the return to reduce the variance by 4.

- Long-run value of Pink is positive:

E(Pink) - Var(Pink)/2 = 0.14

even though neither Red not White perform well taken separately.

Lessons from Dice Game

- Long-run value determined by
  E(return) - (1/2) Var(return)
- Over short horizons, a poor long-term investment might appear very attractive.
- Portfolios succeed by trading expected returns for reductions in variance

Cautions

- Real investments lack some properties of the investments in the dice simulation
- Independence
  - The dice fluctuate independently of one another. The returns of Red are not affected by what happens to Green.
- Stability
  - The properties of the dice stay the same throughout the simulation. The chance for a good return on Red does not change.
- Parameters known
  - We know the properties of the random processes in the dice game.

Back to the Real World

Questions

- Two fundamental questions

- How much?
  - How much of my wealth should I invest to meet my financial goals?

- Which assets?
  - Start with the whole-market index
  - Which other investments in addition to index?

How much to invest?

- If we accept the objective to maximize long-run wealth, then the proportion of our wealth p to put in an investment is

$$p = \frac{\mu - r_f}{\sigma^2}$$

r_f is the risk-free rate of interest

- Example suggests we're more risk averse...
  - μ and σ for the history of the market gives

    $p = 0.075/0.040 = 1.75$

    times wealth.

- Nonetheless, we ought to invest **some** fraction of our wealth in any asset for which we know $\mu \neq 0$ (short it if $\mu < 0$).

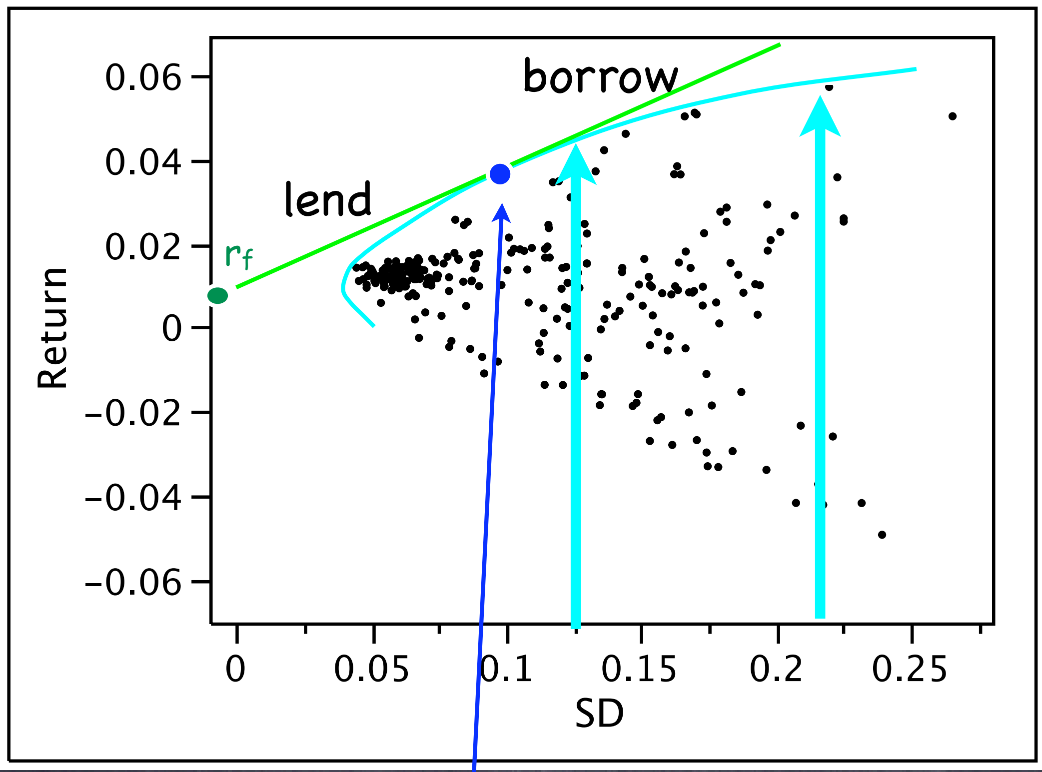
Problem: So many choices?

- The simple analysis of how much to invest considers one asset, in isolation.
- Role of dependence
  - Need to consider the correlation among the returns when investing in several
  - Messy problem of portfolio analysis is to anticipate correlations going forward.
- Theory from finance
  - Invest first in the market as a whole
  - Then consider other assets.

Efficient Frontier

- Plot average return on SD of return for a collection of randomly formed portfolios

Leverage



Efficient Frontier

Mixing the "tangent" portfolio with cash obtains better performance

The tangent portfolio is the market portfolio.

Capital Asset Pricing Model

- Linear equation
  - Excess returns on an asset are related to those on whole market by a linear equation

CAPM $$r_t - r_f = \alpha + \beta\,(M_t - r_f) + \varepsilon_t$$

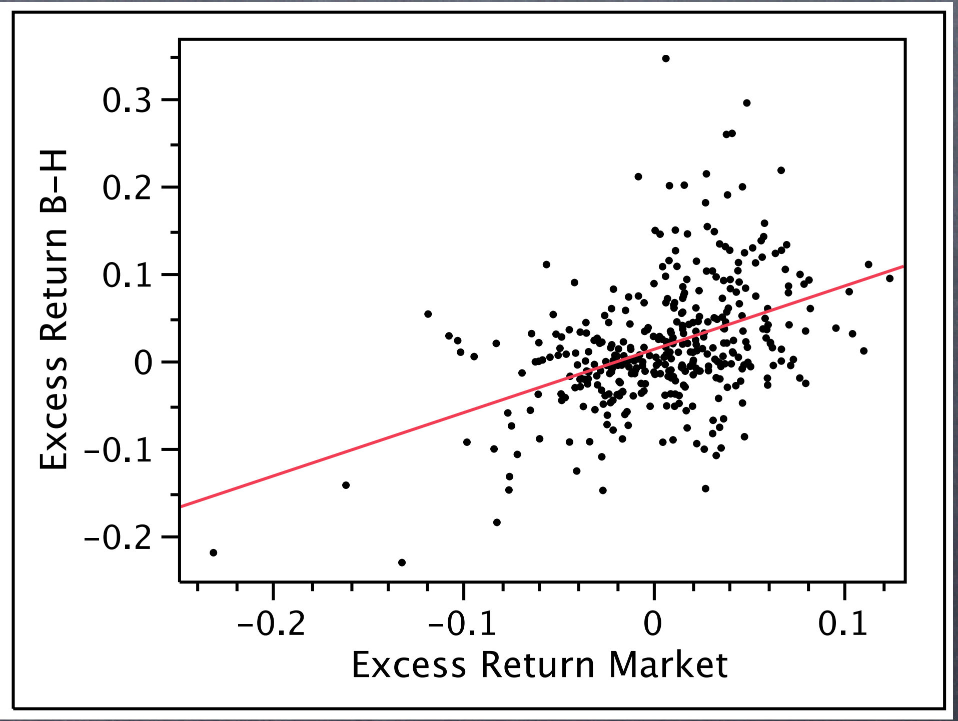
  - r_f is the risk-free rate
  - $\beta = Cov(r_t - r_f, M_t - r_f)/Var(M_t - r_f)$
  - $\alpha = 0$
- Orthogonal
  - Intrinsic returns uncorrelated with market

$$(r_t - r_f) - \beta\,(M_t - r_f) = \alpha + \varepsilon_t$$

- If $\alpha \neq 0$?
  - Intrinsic variation in asset has non-zero mean
  - Buy (or sell) some amount of it.

Testing Alpha

- Example: Berkshire-Hathaway
- Regress out the market, obtaining estimates for α and β.
  - beta = 0.722
  - alpha = 0.014
- Test H_0: $\alpha = 0$
  - Standard procedure relies on t-distribution to obtain p-value



Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	0.013962	0.003397	4.11	<.0001*
Excess Return Market	0.7223495	0.077763	9.29	<.0001*

Testing Alpha

- Procedure
  - Regress out the market, obtaining estimates a for α and b for β.
  - Test H_0: $\alpha = 0$ using regression estimates
- Doubts?
  - What's the distribution of the t-statistic? Some investments produce returns that are far from Gaussian. Cannot rely on t-distribution.
  - How to handle the issue of multiplicity? It is unlikely that we only consider only one other asset aside from the market as a whole. Methods (FDR, Bonferroni,...) require p-value.

Alternative Test for Alpha

- Returns after removing market

  $R_t = 1 + (r_t - r_f) - \beta\,(M_t - r_f)$

- Null hypothesis H_0

  The investment has α=0, so $E(R_t) = 1$. The alternative does not "beat the market"

- Compound these returns

  $C_t = R_1\, R_2\, \dots\, R_t \geq 0, \quad t = 1,2,\dots,n$

- Test p-value

  $P(C_1,\dots,C_n|H_0) \leq 1/\max(C_t)$

- Easy to use

  To reject H_0 at 0.05 level, compound returns have to exceed 20 during observed period

Wharton
Department of Statistics

Foster, Stine, Young (2008) "A martingale test for alpha," WFIC working paper.

Martingale Test

- Martingale
  Stochastic process $\{X_t\}$ for which
  $E(X_{t+1}|X_t, X_{t-1}, X_{t-2}) = X_t$
- Classic examples
  - Sum of coin tosses
  - Random walk
  - Martingale does not require independence
- Test for alpha treats compound returns C_t as a non-negative martingale with conditional expected value 1.
- Doob's martingale inequality
  $P(\max(X_1, X_2, \ldots X_n) \geq \lambda) \leq E\, X_n / \lambda$

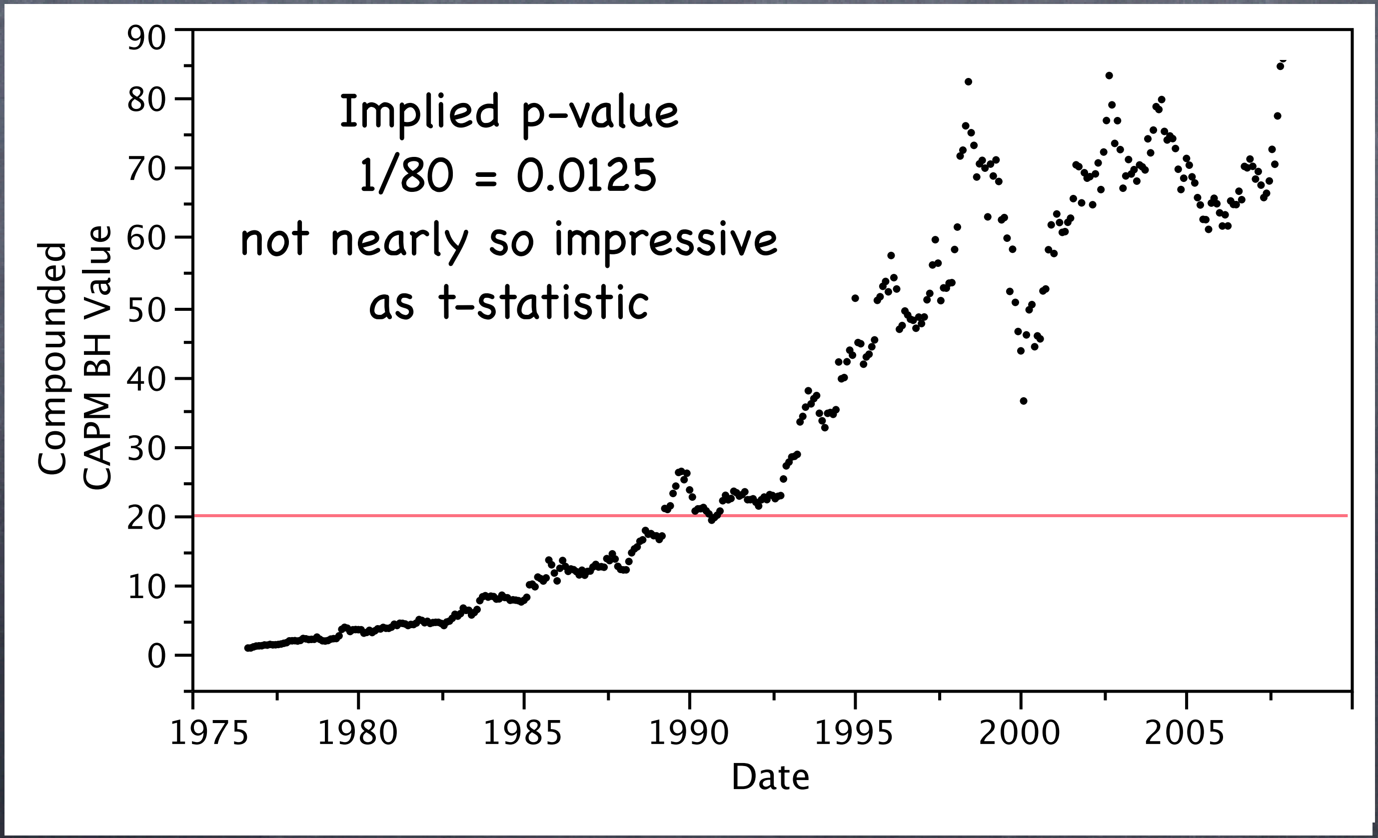
Wharton
Department of Statistics

Example

- "Residual" returns for Berkshire-Hathaway,

  $(r_t - r_f) - b\,(M_t - r_f)$

  - Note: since the martingale test does not care about n, we can use finely spaced data that essentially reveal β (if you believe its fixed!)



Discussion

- Multiplicity
  A p-value of 1/80 does not overcome even slight adjustments for multiplicity.
- Bonferroni p-value
  Multiply the p-value from martingale test by number of assets considered.
  - I bet that you have considered more than 4.
- Power
  The test is "tight" in the sense that there are processes you would not want to consider for which it gets the right answer.

Bob Fund

- How do you guarantee those 2% above benchmark returns?
- Unobserved volatility
  - $r_t = 1/k$ w.p. $k/(k+1)$
  - $r_t = -1$ w.p. $1/(k+1)$ busted
  - $E(R_t) = 1 + E(r_t) = 1$
- Example
  - k = 19, so returns a bit more than 2% growth
  - Smaller k give more exciting performance
- For any choice of k

  $P(C_t \text{ of Bob Fund} > 20) = 1/20$
- Martingale test protects against the "until it happens" unobserved volatility

Wharton
Department of Statistics

Summary

- Principles
  - Focus on returns, not cumulative value
  - Remove market performance
    Regress out market from returns
  - Watch for unseen volatility
    Martingale test
  - Adjust for multiplicity
    Bonferroni does fine, particularly since it's so hard to "count" the considered alternatives
- No free lunches or dinners!

Thanks!

www-stat.wharton.upenn.edu/~stine

Wharton
Department of Statistics