Automobile Design

Car89.jmp

A team charged with designing a new automobile is concerned about the gasoline mileage that can be achieved. The team is worried that the car's mileage will result in violations of Corporate Average Fuel Economy (CAFE) regulations for vehicle efficiency, generating bad publicity and fines. Because of the anticipated weight of the car, the mileage attained in city driving is of particular concern.

The design team has a good idea of the characteristics of the car, right down to the type of leather to be used for the seats. However, the team does not know how these characteristics will affect the mileage.

The goal of this analysis is twofold. First, we need to learn which characteristics of the design are likely to affect mileage. The engineers want an equation. Second, given the current design, we need to predict the associated mileage.

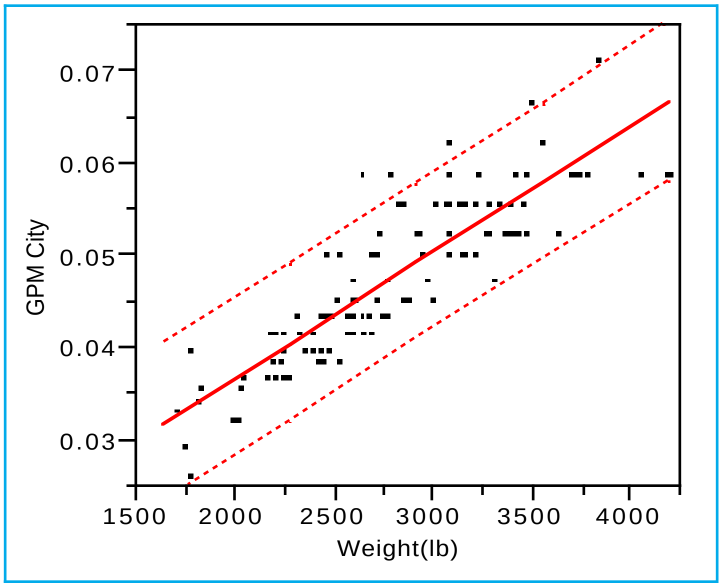
The new car is planned to have the following characteristics:

Cargo	18 cu. ft.	
Cylinders	6	
Displacement	250 cu. in.	(61 cu. in. ≈ 1 liter)
Headroom	40 in.	
Horsepower	200	
Length	200 in.	
Leg room	43 in.	
Price	$38,000	
Seating	5 adults	
Turning diameter	39 ft.	
Weight	4000 lb.	
Width	69 in.	

An observation with these characteristics forms the last row of the data set. The mileage values for this observation are missing. One model for this relationship is described in the solutions for that assignment.

One model for the relationship between weight and mileage uses mileage expressed in gallons per mile rather than miles per gallon. The plot below reproduces a summary of that fit on the transformed scale of gallons per mile.

GPM City by Weight(lb)



Linear Fit

GPM City = 0.00943 + 0.00001 Weight(lb)

Summary of Fit

RSquare	0.765
RSquare Adj	0.763
Root Mean Square Error	0.004
Mean of Response	0.048
Observations (or Sum Wgts)	112.000

Analysis of Variance

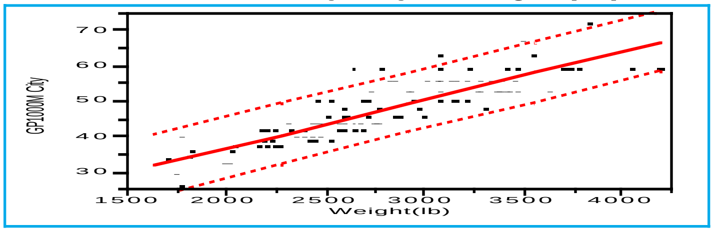
Source	DF	Sum of Squares	Mean Square	F Ratio
Model	1	0.006	0.006426	358.62
Error	110	0.002	0.000018	**Prob>F**
C Total	111	0.008		<.0001

Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	0.0094323	0.002055	4.59	<.0001
Weight(lb)	0.0000136	7.19e-7	18.94	<.0001

The units of gallons per mile produce a very small slope estimate since each added pound of weight causes only a very small increase in fuel consumption per mile. We can obtain a "friendlier" set of results by rescaling the response as gallons per 1000 miles. The results follow. Little has changed, but the slope and intercept are 1000 times larger. The goodness-of-fit measure R^2 is the same.

GP1000M City by Weight(lb)



Linear Fit

GP1000M City = 9.43234 + 0.01362 Weight(lb)

Summary of Fit

RSquare	0.765
Root Mean Square Error	4.233
Mean of Response	47.595
Observations (or Sum Wgts)	112

Analysis of Variance

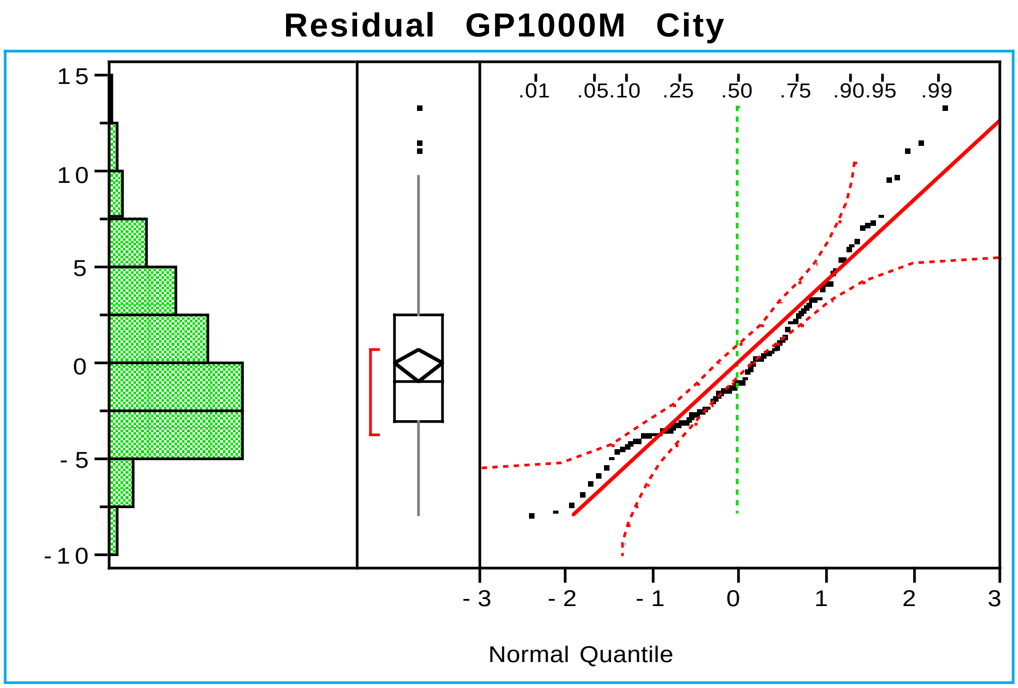
Source	DF	Sum of Squares	Mean Square	F Ratio
Model	1	6426.4	6426.44	358.6195
Error	110	1971.2	17.92	**Prob>F**
C Total	111	8397.6		<.0001

Parameter Estimates

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	9.4323	2.0545	4.59	<.0001
Weight(lb)	0.0136	0.0007	18.94	<.0001

Before we turn to the task of prediction, we need to check the usual diagnostics. The residuals in the previous plot do not appear symmetrically distributed about the line. Notice that several high-performance vehicles have much higher than predicted fuel consumption.

Saving the residuals lets us view them more carefully. The skewness stands out in the following normal quantile plot. How does this apparent skewness affect predictions and prediction intervals from the model?



From the summary of the regression of gallons per 1000 miles on weight, we can obtain a prediction of the mileage of the new car being designed. From the output shown, the equation for the fitted line is

Fitted GP1000M = 9.43 + 0.0136 *Weight*.

If we substitute the design weight of 4000 lb., we obtain the prediction

Predicted GP1000M for new design = 9.43 + 0.0136 (4000) = 63.8 *GP1000M*

This prediction agrees with fitted line shown two pages back.

We also need to determine the associated prediction interval. We can either estimate the interval endpoints from the plot or, more accurately, use the *Fit Model* platform to compute both the prediction and interval for us. The *Fit Model* platform lets us save the predictions and the prediction intervals. (Use the $ button to save the predictions and prediction intervals, labeled *Save Indiv Confidence*.) From the last row of the spreadsheet, we find that using weight to model the *GP1000M* of the vehicle leads to the prediction and interval:

	Predicted *GP1000M*	95% Prediction Interval *GP1000M*
Weight = 4000	63.9	[55.3 – 72.5] ,

which implies an interval of [13.8, 18.1] miles per gallon. (Note: The predicted *GP1000M* found by JMP differs slightly from the value obtained above due to rounding in our earlier calculation.) Since confidence intervals transform in the obvious way, we can also find intervals for related quantities like the gasoline operating cost per 1000 miles. At $1.20 per gallon, the cost interval is

$$[55.3 - 72.5] \times 1.2 = [66.36 - 87.00] \text{ \$/1000M.}$$

The search for other factors that are able to improve this prediction (make it more accurate with a shorter prediction interval) begins by returning to the problem. *Weight* aside, what other factors ought to be relevant to mileage? Right away, the power of the engine (horsepower) comes to mind. Some other factors might be the size of the engine (the engine displacement is measured in cubic inches; 61 $in^3 \approx$ 1 liter) or the amount of space in the vehicle, such as the passenger capacity or the cargo space available. Correlations show that the latter two have a slight relationship with the response. Horsepower, like weight, has a substantial correlation. Horsepower and displacement are highly correlated with the response, with each other, and with the weight of the car.

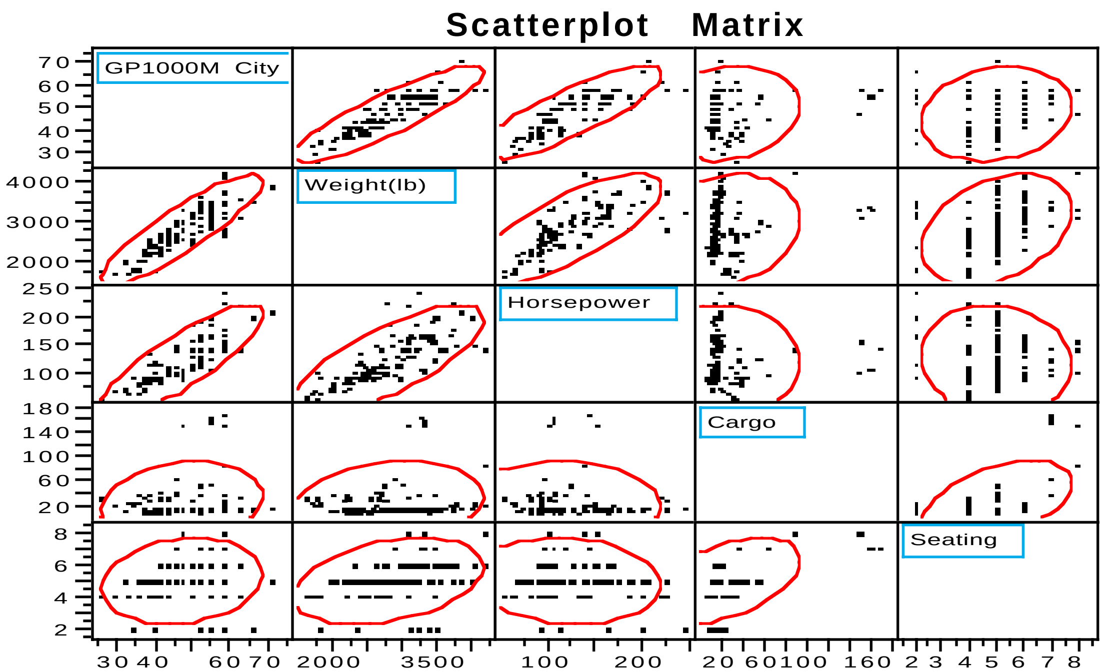
Correlations

Variable	GP1000M City	Weight(lb)	Horsepower	Cargo	Seating
GP1000M City	1.00	0.88	0.83	0.17	0.16
Weight(lb)	0.88	1.00	0.75	0.18	0.35
Horsepower	0.83	0.75	1.00	-0.05	-0.09
Cargo	0.17	0.18	-0.05	1.00	0.49
Seating	0.16	0.35	-0.09	0.49	1.00

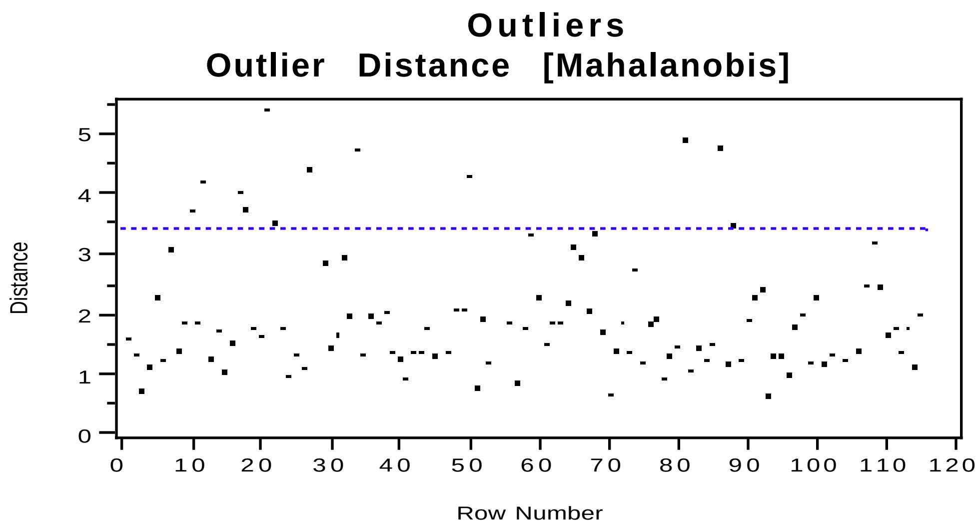
7 rows not used due to missing values.

Previous analyses have shown that the correlation can be a misleading summary. Plots tell a more complete story. The scatterplot matrix on the next page (with variables arranged as in the correlation matrix above) shows the data that go into each of the 10 distinct correlations in the previous table. The cargo variable essentially captures a few unusual vehicles. (Use point-labeling to identify these cars.)

The ellipses in the scatterplot matrix graphically convey the size of each correlation: the more narrow the ellipse is and tilted toward the 45˚ line, the higher the correlation. If an ellipse looks like a circle, the correlation between that pair is near zero.



Outliers are also important in multiple regression, only it becomes harder to spot them with so many more variables. The *Correlation* platform in JMP produces a useful summary plot which helps spot overall outliers, but does not suggest how they will affect the multiple regression equation. (Other plots will.) This plot shows a measure of how far each observation lies from the center of the selected variables, plotted on the row number from the associated spreadsheet.



The outliers in this view are the same as we can see in the scatterplot matrix: point-labeling identifies them as mostly vans and sports cars.

With the addition of horsepower to our model, the regression equation using both weight and horsepower is

$$\text{Fitted } GP1000M = 11.7 + 0.0089\ Weight + 0.088\ Horsepower.$$

The addition of horsepower improves the explanatory power of the initial regression (R^2 is higher, rising from 77% to 84%) by a significant amount (the *t*-statistic for the added variable is t = 7.21). The addition of horsepower captures about a third of the residual variation remaining from the regression using weight alone.

The coefficient for weight, however, is smaller than when considered initially in the bivariate regression (also with a smaller *t*-statistic in the multiple regression). The *t*-statistic for weight was 18.9 in the previous simple regression.

Response: GP1000M City

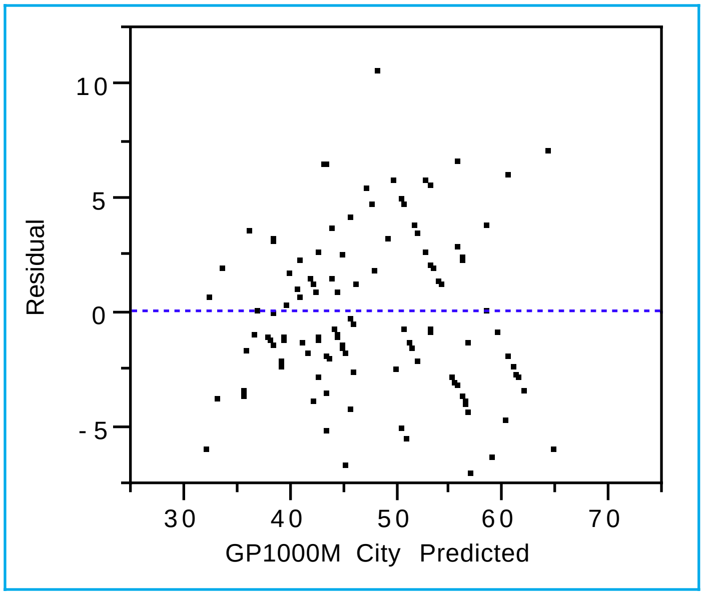
RSquare	0.841
RSquare Adj	0.838
Root Mean Square Error	3.50
Mean of Response	47.6
Observations	112

Parameter Estimates

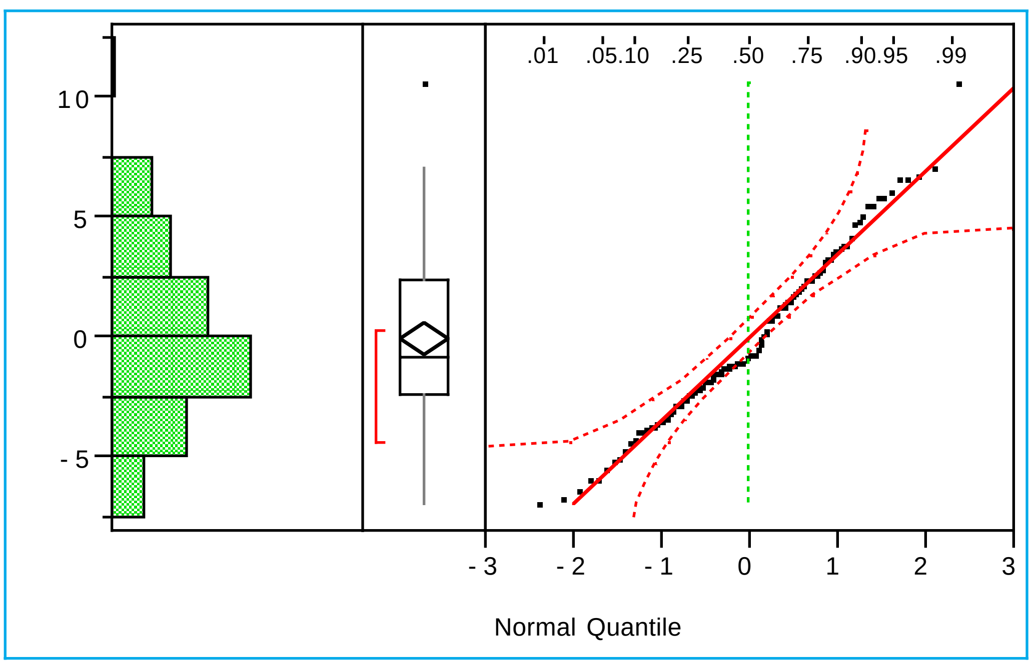
Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	11.6843	1.7270	6.77	<.0001
Weight(lb)	0.0089	0.0009	10.11	<.0001
Horsepower	0.0884	0.0123	7.21	<.0001

Focusing on the difference between marginal and partial slopes, consider this question. For a typical car in this data, how much more gas will it use to carry an additional 200 pound passenger for 1000 miles? Using the marginal slope suggests an increase of $0.0136 \times 200 = 2.72$ gallons. By comparison, the partial slope suggests that the fuel consumption will rise by only $0.0089 \times 200 = 1.78$ gallons. Which is right? Well, did adding the weight change the horsepower? No. The horsepower of the car is the same, with or without the added 200 pounds, and the partial slope gives the better estimate (1.78 gallons).

Some "stripes" are visible in the residual plot. Looking back at the original scatterplots, we can see that these are caused by discreteness in the response.



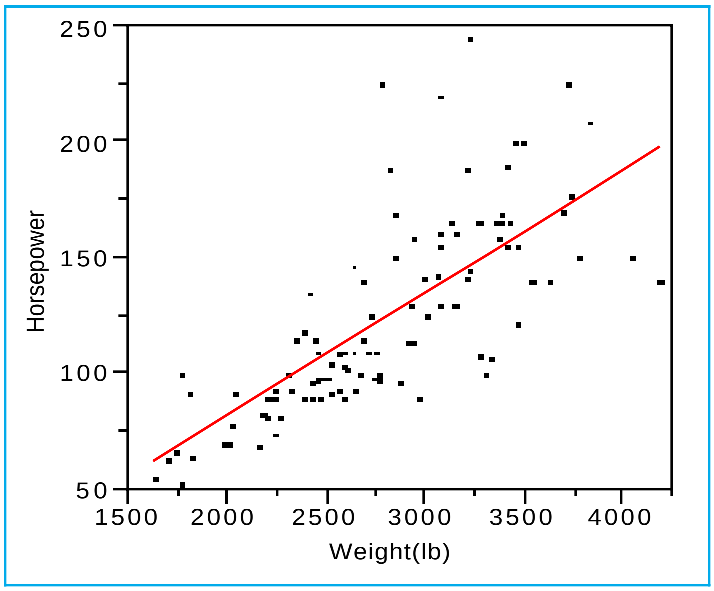
The residual plot displayed below shows that the addition of horsepower to the equation has reduced the amount of skewness in the residuals. The large outlier is the Mazda RX-7, the only car with a rotary engine.



The addition of horsepower to the model has produced a better fit with narrower prediction limits. However, the increase in the standard error for the coefficient of weight is notable: the overall fit is better, but the estimate is different and its SE is larger. Why has the estimate changed?

From the original correlation matrix or scatterplot matrix, notice that the two predictors used here, weight and horsepower, are highly related. The correlation is 0.75 between these two factors. A plot of the two appears below.

Horsepower by Weight(lb)



When used together in a regression model, these two factors interact with each other as both describe similar features of cars – both are related to the size of the car.

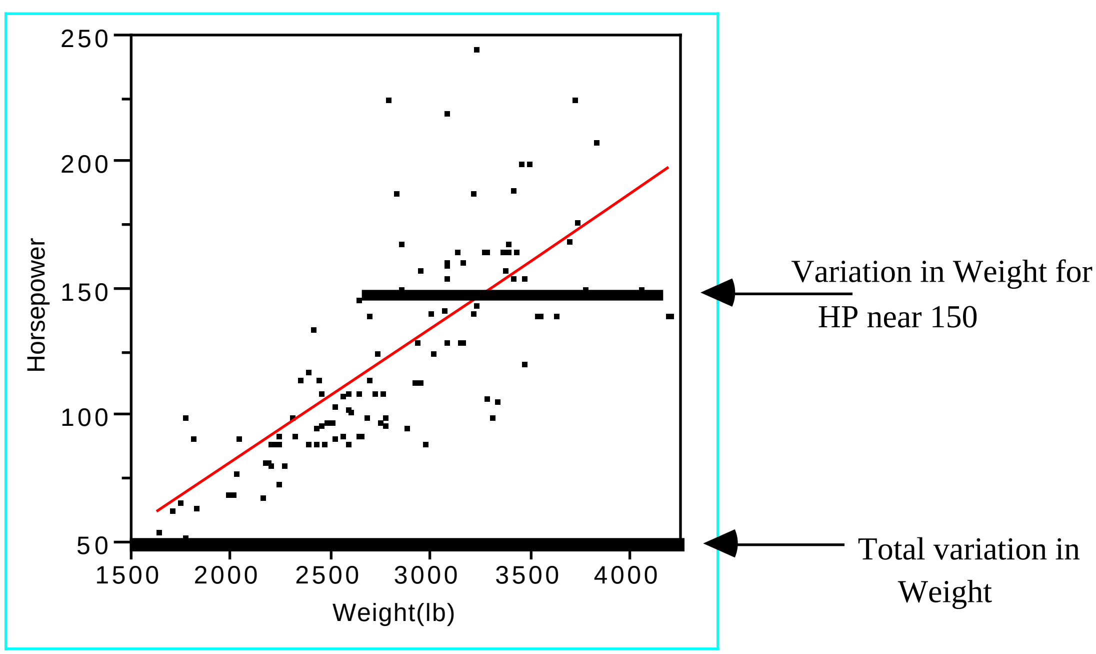
Recall that the SE of a slope estimate in a simple regression is determined by three factors:

(1) error variation around the fitted line (residual variation),
(2) number of observations, and
(3) variation in the predictor.

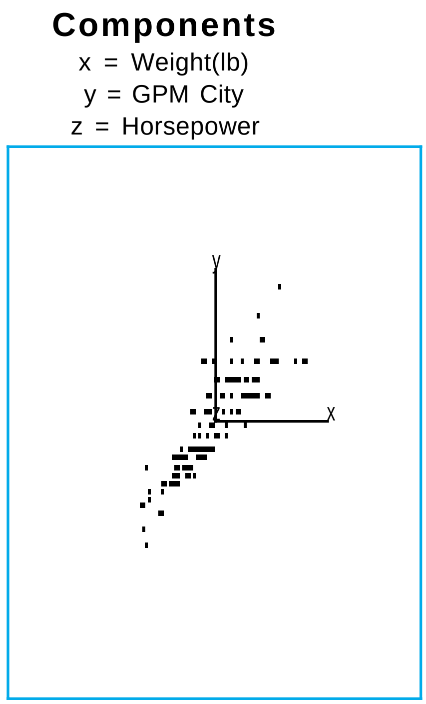
These same three factors apply in multiple regression, with one important exception. The third factor is actually

(3) "unique" variation in the predictor.

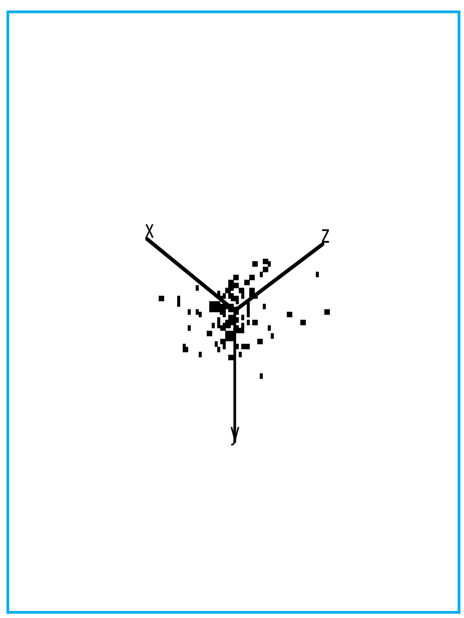
The effect of the correlation between the two predictors is to reduce the effective range of weight, as suggested in the plot below. Without *Horsepower* in the equation, the full variation of *Weight* is available for estimating the coefficient for *Weight.* Restricted to a specific horsepower rating, much less variation is available. As a result, even though the model fits better, the SE for *Weight* has increased.



A spinning 3-D plot of *GP1000M* , *Weight*, and *Horsepower* helps to visualize the problem. The initial view shows the clear association between *GP1000M* and *Weight*



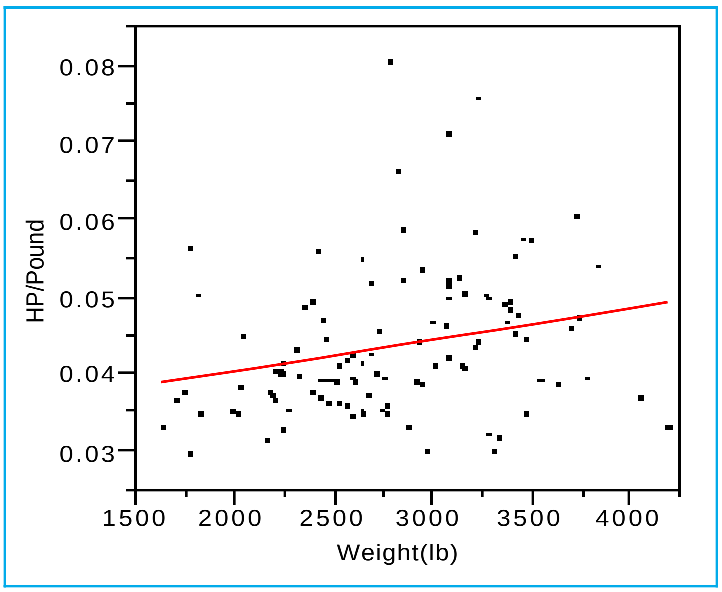
Rotating the plot shows, however, that most of the data fall into a cylindrical region. *Weight* and *Horsepower* are by and large redundant, with only a few points to identify the best fitting multiple regression surface.



Regression models are more easily interpreted when the predictors are uncorrelated with each other. We can easily reduce the correlation in this example without losing our ability to interpret the factors. In particular, *based on knowing something about cars*, consider using the power-to-weight ratio, *HP/Pound*, rather than *Horsepower* itself.

This new predictor is not so highly correlated with *Weight*, as shown in the plot that follows. (The correlation is 0.26.) Typically, whenever companies make a heavier car, they also increase its horsepower. However, that does not imply that they also increase the power-to-weight ratio, and so the correlation is smaller.

HP/Pound by Weight(lb)



The 3-D spinning plot (not shown) also shows that the data are less concentrated in a cylinder and have a more planar shape when *HP/Pound* replaces *Horsepower*.

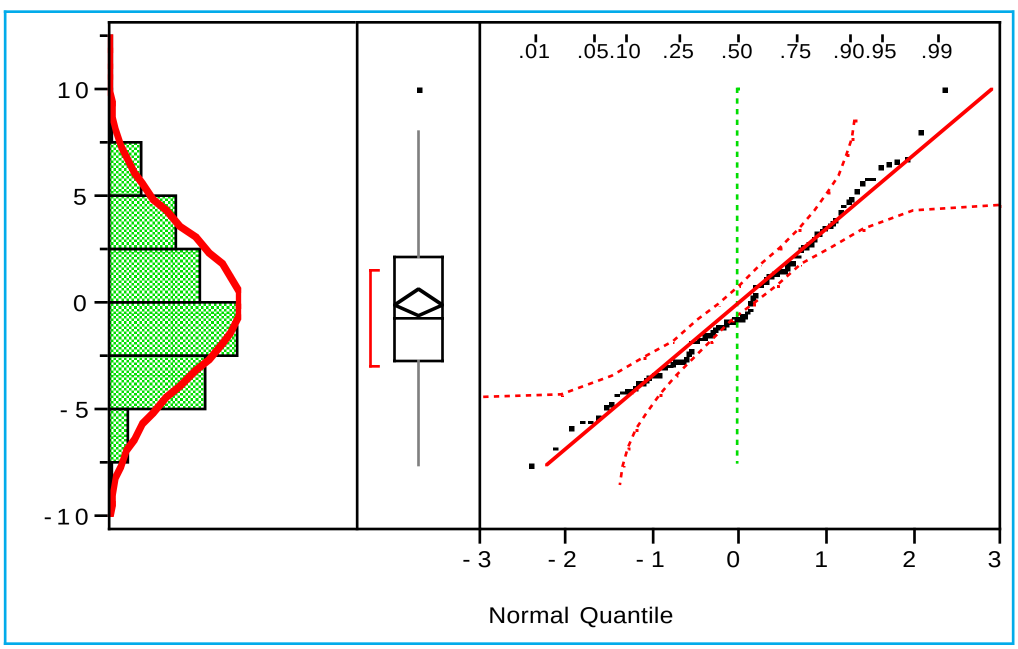
Using the power-to-weight ratio in place of horsepower alone yields the following multiple regression. The goodness-of-fit is comparable to what we obtained using *Horsepower*, and the *t*-statistic for the coefficient of *Weight* is much higher.

Response: GP1000M City

RSquare	0.845
RSquare Adj	0.842
Root Mean Square Error	3.458
Mean of Response	47.595
Observations (or Sum Wgts)	112.000

Term	**Estimate**	**Std Error**	**t Ratio**	**Prob>|t|**
Intercept	0.6703	2.0472	0.33	0.7440
Weight(lb)	0.0125	0.0006	20.71	<.0001
HP/Pound	270.7381	36.2257	7.47	<.0001

The residuals are similar to those from the prior model, though a bit more normal as you can check from the residual plot:normality.



The next type of residual plot, called a *leverage plot* in JMP, focuses on a single multiple regression coefficient. There is one leverage plot for each predictor in the model. A leverage plot shows the contribution of each variable to the overall multiple regression, exclusive of the variation explained by others.

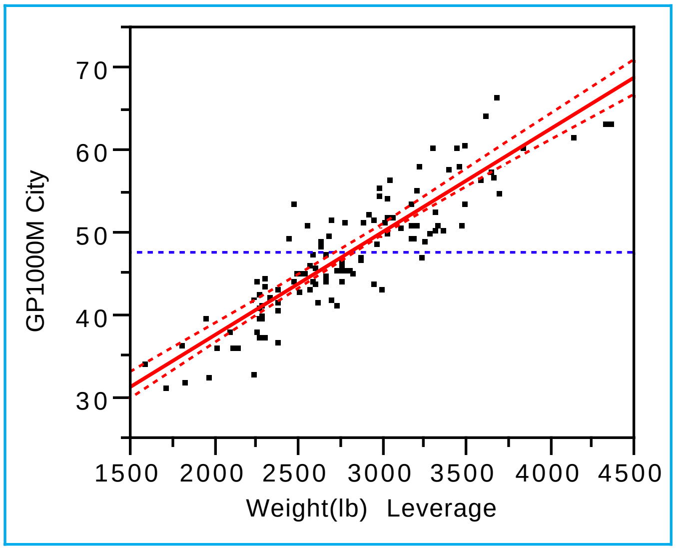
• The *slope* of the line shown in the leverage plot is equal to the coefficient for that variable in the multiple regression. In this sense, a leverage plot resembles the familiar scatterplot that makes regression with a single predictor so easy to understand.

• The *distances* from the points to the line in the leverage plot are the multiple regression residuals. The distance of a point to the horizontal line is the residual that would occur if this factor were not in the model. Thus, the data shown in the leverage plot are not the original variables in the model, but rather the data adjusted to show how the multiple regression is affected by each factor.

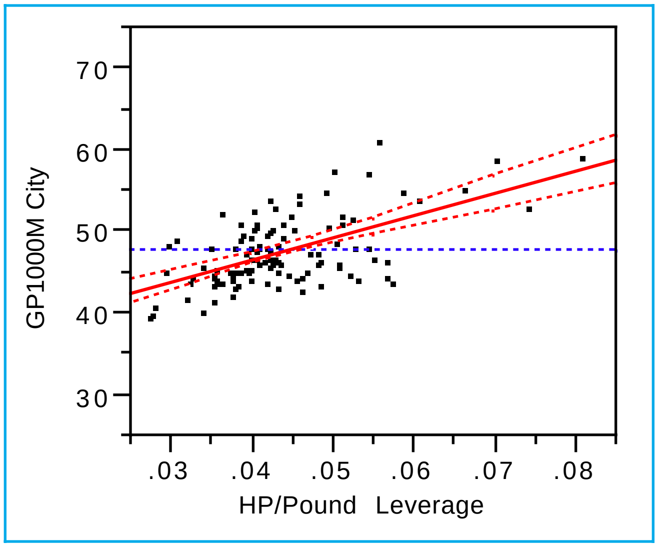
As in simple regression, the slope for a variable is significant if the horizontal line ever lies outside the indicated confidence bands.

Leverage plots for *Weight* and *HP/Pound* suggest the high precision of the slope estimates in our revised model. Performance cars stand out on the right side in the second plot.

Weight(lb)



HP/Pound



Returning to the problem of predicting the mileage of the proposed car, this multiple regression equation provides narrower prediction intervals than a model using weight alone. The prediction interval using both variables shifts upward (lower mileage) relative to the interval from simple regression. The higher than typical power-to-weight ratio of the anticipated car leads to a slightly higher estimate of gasoline consumption. With more variation explained, the prediction interval is also more narrow than that from the model with weight alone.

Model	Predicted *GP1000M*	95% Prediction Interval
Weight alone	63.9	[55.3 – 72.5]
Weight & *HP/Weight*	64.3	[57.3 – 71.3]

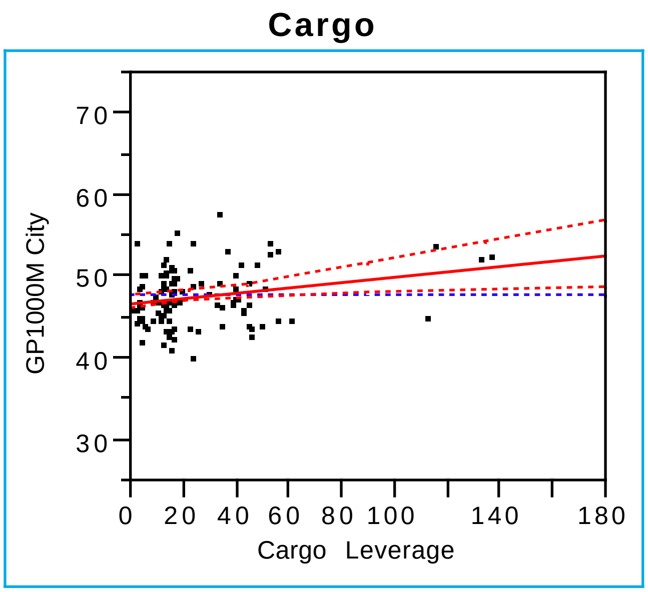
Other factors might also be useful in explaining the cars' mileage, but the analysis at this point is guided less by reasonable theory and becomes more exploratory. For example, adding both cargo and seating indicates that the cargo space affects mileage, even controlling for weight and horsepower. Seating has little effect (small *t*-statistic, *p*-value much larger than 0.05).

Response: GP1000M City

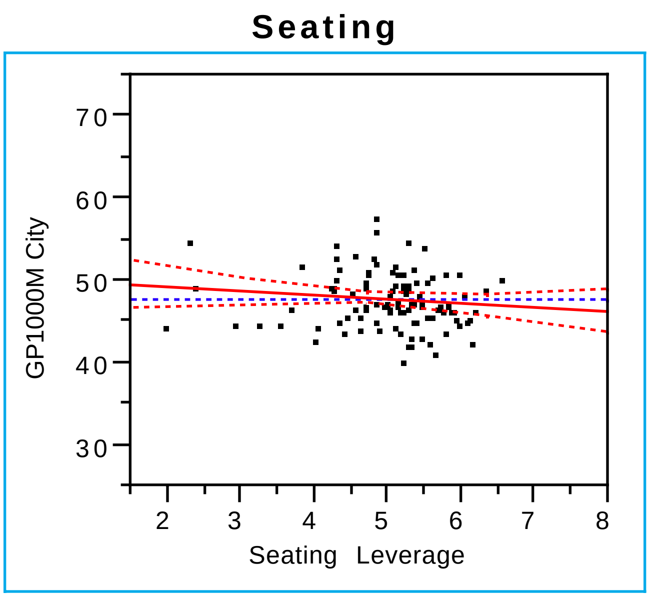
RSquare	0.854
RSquare Adj	0.849
Root Mean Square Error	3.39
Mean of Response	47.7
Observations	109

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	2.4895	2.8155	0.88	0.3786
Weight(lb)	0.0126	0.0007	17.55	<.0001
HP/Pound	262.0547	44.8848	5.84	<.0001
Cargo	0.0329	0.0132	2.49	0.0142
Seating	-0.4875	0.4066	-1.20	0.2333

Leverage plots make it clear, though, that the only reason that *Cargo* is significant is the presence of several vans in the data (here marked with x's).



Seating capacity is simply not relevant, even though the leverage plot below is dominated by the two-seaters shown at the left (marked with o's).



Further exploration suggests that price is a significant predictor that improves the regression fit. But why should it be included in the model?

Response: GP1000M City

RSquare	0.866
RSquare Adj	0.860
Root Mean Square Error	3.26
Mean of Response	47.8
Observations	107

Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	4.1616	2.8146	1.48	0.1424
Weight(lb)	0.0110	0.0010	11.12	<.0001
HP/Pound	255.8825	44.5396	5.75	<.0001
Cargo	0.0334	0.0127	2.64	0.0097
Seating	-0.2001	0.4209	-0.48	0.6355
Price	0.0001	0.0001	2.15	0.0339

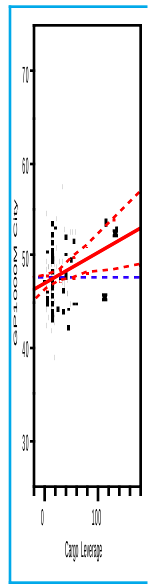
Has the addition of these three new predictors significantly improved the fit of our model. To answer this question, we need to go outside the realm of what JMP provides automatically and compute the partial F statistic. The idea is to see how much of the residual remaining after *Weight* and *HP/Pound* has been explained by the other three.

$$F = \frac{\text{Change in } R^2 \text{ per added term}}{\text{Remaining variation per d.f.}} = \frac{(0.866-0.845)/3}{(1-0.866)/(107-6)} = 5.28\ .$$

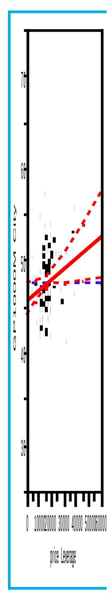
Each added coefficient explains about five times the variation remaining in each residual "degree of freedom". This is significant, as you can check from JMP's calculator.

But why should price be significant? Perhaps more expensive cars are more well engineered and have a more efficient design. The leverage plot for price suggests that several cars dominate this coefficient. Perhaps this regression model reaches too far, building on special features of this data set rather than those that might apply to our new car.

Cargo



Price



From looking at the leverage plot, it seems that just a small subset of the vehicles affects these new coefficients. Indeed, if we set aside the four vans and the three expensive cars on the right of the leverage plot for *Price* (BMW-735i, Cadillac Allante, Mercedes S), the regression coefficients for both *Cargo* and *Price* are no longer significant. The size of these changes to the fit suggests that the significant effects for these two factors are perhaps overstated, relying too much on a small subset of the available data.

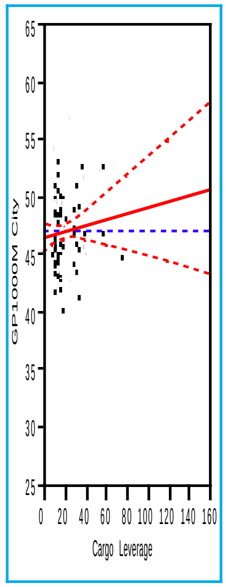
Response: GP1000M City

RSquare	0.856
RSquare Adj	0.848
Root Mean Square Error	3.205
Mean of Response	47.1
Observations	100

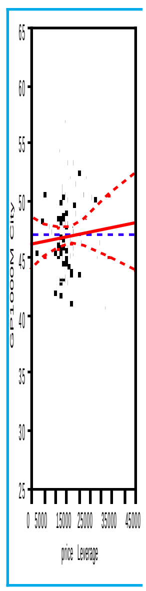
Term	Estimate	Std Error	t Ratio	Prob>|t|
Intercept	3.7488	2.9737	1.26	0.2105
Weight(lb)	0.0115	0.0010	11.16	<.0001
HP/Pound	254.2858	46.7636	5.44	<.0001
Cargo	0.0265	0.0262	1.01	0.3147
Seating	-0.1904	0.4381	-0.43	0.6649
Price	0.0000	0.0001	0.62	0.5393

Here are the leverage plots for *Cargo* and *Price*, with the seven outlying or leveraged points excluded. Neither slope estimate differs significantly from zero based on the reduced data set.

Cargo



Price



The design team has a good idea of the characteristics of the car, right down to the type of leather to be used for the seats. However, the team does not know how these characteristics will affect the mileage.

The goal of this analysis is twofold. First, we need to learn which characteristics of the design are likely to affect mileage. The engineers want an equation. Second, given the current design, we need to predict the associated mileage.

The analysis is done on a scale of gallons per 1000 miles rather than miles per gallon.

(1) As expected, both weight and horsepower are important factors that affect vehicle mileage. Adding the power-to-weight ratio to a simple regression equation leads to a better fit and more accurate (as well as shifted) predictions.

The inclusion of other factors that are less interpretable, however, ought to be treated with some skepticism and examined carefully. Often such factors appear significant in a regression because of narrow features of the data being used to fit the model; such features are not likely to generalize to new data and their use in prediction is to be avoided.

(2) Using the equation with just weight and the power-to-weight ratio, we predict the mileage of the car to be in the range

$$[57.3 - 71.3]\ \textit{GP1000M} \quad \Rightarrow \quad [14.0 - 17.5]\ \text{MPG}$$

Notice how prediction intervals (like confidence intervals) easily handle transformation — just transform the endpoints.