Variance and the Volatility of Investments

Overview

Variance, often called volatility when speaking of returns on financial investments, is an important characteristic of investments like stocks. When we consider investing in something like a stock, it's not enough to know – or more likely guess – the average rate of return. The variance of the investment will also impact how much we can expect to profit from owning the stock over the long haul.

To get a sense for how variability in the returns on an investment eats away at your gains, consider the following example. Suppose upon leaving Wharton that you start with a $100,000 salary (just to keep the math easy!). Things go well, and you get a 10% raise in your first year after Wharton, so your salary is now $110,000. But then, having gone the dot-com route, things do not go so well in the second year and you are hit with a 10% cut. Are you better off or have things gotten worse because of the "volatility" in your income? At the end of the second year, your salary is now $99,000. Even though your average rate of growth is zero (up 10%, down 10%), the volatility means that you now make only 99% of the initial salary.

Volatility hurts by eating away at the average rate of return in a way that can be made fairly precise. Most people have an intuitive sense that investments that promise potentially high returns are often risky – they might lose value as well. Intuition aside, though, there is a systematic way to balance risk and return.

For this homework exercise, a **team** of you and several classmates will simulate a small financial market that has three different investments. To capture some of the uncertainty of real investments, we are going to use dice to simulate the returns on the three investments. Your team will work best if you have **three** people. One person will play the role of nature (or the market forces) and roll the dice. Another will need to keep track of where the dice land and read off their values, and a third will record the outcomes and compute the returns on the investments.

The three investments in this simulation have different characteristics, summarized below. One investment is quite risky whereas another resembles an old-fashioned savings account whose interest has been adjusted for the effects of inflation. A third lies between these two. Long-run experience with these investments yields the following summary characteristics. The labeling of the investments with colors as shown here matches the colors of the dice that we will use momentarily.

Color Die	Expected Annual Return	SD of Annual Return
Red	71%	132%
White	0%	6%
Green	7.5%	20%

The measure of variability shown here is the standard deviation (SD) of the past, long-run returns of each investment. We will discuss the origin of these choices for the average return and variability at the end of class after we finish performing a small simulation of this market.

Question #1
Before starting the simulation, which of the three investments looks most appealing to you? Does the team agree on which is the "best" investment or is it not so clear?

Why might different investors prefer different choices of these three alternatives?

The Simulation

Now it's time to start the simulation. Each roll of all three dice will represent a "year" in the simulated market. For each "year" of the simulation, you will roll all three dice, and use the outcomes to determine what has happened to the value of each investment. Each of the three investments starts off with an initial value of $1000.

The following table shows how the rolls of the dice affect the values of the three investments. The number in each cell of the table is the value of each dollar invested at the end of the year, as determined by the outcome when you roll the three dice. (The values in the table are thus 1 plus the return on the investment during the year.)

Roll	Red	White	Green
1	0.06	0.9	0.8
2	0.2	1	0.9
3	1	1	1.05
4	3	1	1.1
5	3	1	1.2
6	3	1.1	1.4

For example, suppose that on the first roll of all three dice, you obtain

(Red 5) (White 3) (Green 2)

Then the values of the investments after the first year are

Red: $1000 □ 3 = $3000
White: $1000 □ 1 = $1000
Green: $1000 □ 0.9 = $900

For the next roll, the values are compounded from these. Suppose that the second roll gives

(Red 2) (White 6) (Green 4)

The values of the three investments after two years are

Green: $900 □ 1.1 = $990
Red: $3000 □ 0.2 = $600
White: $1000 □ 1.1 = $1100

Question #2
Carry out this simulation for 30 "years". Which investment turns out to be the best at the end?

Is it the same one that you picked initially in Question #1? Think about why this investment won – was it chance, or something that would hold up over the long term?

A New Investment

The final part of this exercise considers the performance of a hybrid investment. This fourth investment mixes the outcomes of *Red* and *White*. We'll call it *Pink*. *Pink* represents a continuously re-balanced portfolio with half of its value invested in *Red*, and the other half in *White*. To compute the value of this new investment, use the *previously recorded rolls of both* the red and white dice for each round. It's easiest to describe what to do with an example. (Don't do the calculations, though, until *after* you consider Question #3. The example given here clarifies how the new investment performs.)

For the first round, using the same dice rolls as above (*Red*=5, for a value of 3 and *White*=3, for a value of 1), the value of the "Pink" investment is

Pink: $$\$1,000 \times \frac{3 + 1}{2} = \$2,000$$

Compounded in the second round (which had values *Red*=2 for a value 0.2 and *White*=6 with value 1.1), the result is

Pink: $$\$2,000 \times \frac{0.2 + 1.1}{2} = \$1,300$$

Before you compute your outcomes for the hybrid, consider this next question:

> **Question #3**
> What does the group think about this hybrid? How do you anticipate it to rank among the other three?

Now, compute the returns and final value for "Pink" and answer the following question.

> **Question #4**
> Carry out the calculations for the investment *Pink* for 30 years, using the data from your previous experiment. How does *Pink* compare to the others? Did it perform as you expected, or do better or worse?

We will discuss these results as well as more general concepts related to portfolios like *Pink* in class. As part of your preparations, see if you can explain the performance of *Pink*. To help you along with this, see the next two questions.

The remaining questions suggest some answers to the problems posed as part of the dice simulation. The questions make use of a notion called the "volatility adjusted growth rate." This measure of the value of an investment is

Volatility Adjusted Growth Rate = E(R) – Var(R)/2,

where R is the annual return. The idea is that if you have a 10% gain one year followed by a 10% drop the next year, you end up at 0.99 time where you started. So, the standard deviation of 10% in this illustration lowers your return by 1% for every two years – which happens to be the variance divided by 2 (0.10/2 = 0.05 per year).

Question #5
Find the volatility adjusted growth rate for the first three investments. Does this adjustment capture what happened in your simulation?

Question #6
Find the mean and variance for the Pink investment, and use these values to find the volatility adjusted growth rate for Pink. Does this help explain what happened in your dice simulation?

From the textbook, exercises

Chapter 6, problems 6.2, 6.7 (page 290)
Chapter 7, problems 7.4 (page 380), t7.19 (page 392)

