STAT 430

Jan 2003

Assignment 2 Solutions (Chapter 3)

(8) Assume for this question that $P(B) = P(G) = \frac{1}{2}$. then sample space is just (with births in order)

$$(\{B,G\}, \{B,B\}, \{G,B\}, \{G,G\})$$

$$P(\text{both girls if oldest is girl}) = P((G,G) \mid (G,B) \text{ or } (G,G))$$
$$= \frac{1}{2}$$

(11) Let E denote event of ectopic pregnancy (most often, egg attaches after fertilization in fallopian tube). Given that $P(E|S) = 2\,P(E|S^c)$ and that $P(S) = .32$, then

$$P(S|E) = \frac{P(E|S)\,P(S)}{P(E)}$$

$$= \frac{P(E|S)\,P(S)}{P(E|S)\,P(S) + P(E|S^c)\,P(S^c)}$$

$$= \frac{\cancel{P(E|S)}\,P(S)}{\cancel{P(E|S)}\,P(S) + (\frac{1}{2}\cancel{P(E|S)})\,P(S^c)}$$

$$= \frac{2\,P(S)}{1 + P(S)} = \frac{.64}{1.32} \simeq .45$$

note: We never had to explicitly find $P(E|S)$. We only required that it was twice $P(E|S^c)$.

(16) (a) $P(F|C) = \dfrac{P(FC)}{P(C)} = \dfrac{.02}{.05} = .4$

(b) $P(C|F) = \dfrac{P(FC)}{P(F)} = \dfrac{.02}{.52} \doteq .04$

note how different as conditioning is reversed.

S_i = event of success, passes exam i ; F_i denotes S_i^c (fails)

(18) (a) $P(S_1S_2S_3) = P(S_1)\cdot P(S_2|S_1)\cdot P(S_3|S_1S_2) = .9\cdot.8\cdot.7 = .504$

(b) $$P(S_1F_2 \mid F_1 \cup F_2 \cup F_3) = \frac{P(S_1F_2 \cap (F_1\cup F_2 \cup F_3))}{P(F_1 \cup F_2 \cup F_3)}$$ ⟸ disjoint, but just use part (a)

$$= \frac{P(S_1F_2)}{1-.504}$$

$$= \frac{.9\,(1-.8)}{.496} = .36$$

(19) See example 5h(b) on page 38 for the counting argument. It seems harder, to me anyway. Conditioning is "mechanical."

$$P(A_1A_2A_3A_4) = P(A_1)\cdot P(A_2|A_1)\cdot P(A_3|A_1A_2)\cdot P(A_4|A_1A_2A_3)$$

$$P(A_1) = \frac{\binom{4}{1}\binom{48}{12}}{\binom{52}{13}} \qquad P(A_2|A_1) = \frac{\binom{3}{1}\binom{36}{12}}{\binom{39}{13}}$$

$$P(A_3|A_1A_2) = \frac{\binom{2}{1}\binom{24}{12}}{\binom{26}{13}} \qquad P(A_4|A_1A_2A_3) = \frac{\binom{1}{1}\binom{12}{12}}{\binom{13}{13}} = 1$$

(26) Refer back to example 5j, page 40 for a similar problem (as discussed in class).

(a) Remove 19 cards that are not aces from deck. Let A_i denote event that in drawing from remaining cards that Ace of Spades appears in ith draw; then

$$P(\text{ace of spades follows first ace}) = P(A_2)$$

$$= P(A_2|A_1)P(A_1) + P(A_2|A_1^c)P(A_1^c)$$

$$= 0 + \frac{1}{32}\cdot\frac{3}{4}$$

↑ one of 32 remaining cards ↑ ace drawn was not ace of spades

(26) (b) Condition on event $F = \{2 \text{ clubs in First } 19\}$

$$P(2 \text{ clubs next}) = P(2 \text{ club next} \mid F)\, P(F) + P(2 \text{ club next} \mid F^c)\, P(F^c)$$
$$= 0 + \frac{1}{32} \cdot \frac{29}{48}$$

Think of the 48 spaces/positions that are not aces. 29 of these are after first ace.

(33) (a) Let F denote coin is fair, H is heads showing.

$$P(F \mid H) = \frac{P(H \mid F)\, P(F)}{P(H)} = \frac{P(H \mid F)\, P(F)}{P(H \mid F)\, P(F) + P(H \mid F^c)\, P(F^c)}$$
$$= \frac{1/2 \cdot 1/2}{1/4 + 1 \cdot 1/2} = \frac{1}{3}$$

(b) Let H_1 denote heads first time, H_2 is heads second time

$$P(F \mid H_1 H_2) = \frac{P(HH \mid F)\, P(F)}{P(HH \mid F)\, P(F) + P(HH \mid F^c)\, P(F^c)}$$
$$= \frac{1/4 \cdot 1/2}{1/4 \cdot 1/2 + 1/2} = \frac{1}{5}$$

(c) It cannot be the coin with HH, so the probability is 1.

(37)

choose interchanged card

$$P(A) = P(A \mid C)\, P(C) + P(A \mid C^c)\, P(C^c)$$

(good hint)

$$= 1 \cdot \frac{1}{27} + \frac{3}{51} \cdot \frac{26}{27} = \frac{1}{27}\left(1 + \frac{26}{17}\right)$$

(43) Condition on rolled number. Easy, but a little tedious ...

(a)
$$P(W) = P(W \mid R=1)\, P(R=1) + \cdots + P(W \mid R=6)\, P(R=6)$$
$$= \frac{1}{6}\left(\frac{1}{3} + \frac{5 \cdot 4}{15 \cdot 14} + \frac{5 \cdot 4 \cdot 3}{15 \cdot 14 \cdot 13} + \frac{5 \cdot 4 \cdot 3 \cdot 2}{15 \cdot 14 \cdot 13 \cdot 12} + \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{15 \cdot 14 \cdot 13 \cdot 12 \cdot 11} + 0\right)$$

(b)
$$P(R=3 \mid \text{all white}) = \frac{P(W \mid R=3)\, P(R=3)}{P(W)} = \frac{\frac{5 \cdot 4 \cdot 3}{15 \cdot 14 \cdot 13} \cdot \frac{1}{6}}{P(W)}$$

(49) $$P(C_1 \mid C_1 \cup C_2 \cup \cdots \cup C_n) = \frac{P(C_1 \cap (C_1 \cup C_2 \cup \cdots \cup C_n))}{1 - P(C_1^c \cap C_2^c \cap \cdots \cap C_n^c)}$$

$$= \frac{P(C_1)}{1-(1/2)^n}$$

(54) (a) This version uses pairs of possible tosses, so the sample space is HH, TT, HT, & TH. So

$$P(\text{return } T) = P(HT \mid HT \cup TH)$$

$$= \frac{P(HT)}{P(HT) + P(TH)} \quad \text{disjoint union}$$

$$= \frac{p(1-p)}{2p(1-p)} = \frac{1}{2}$$

(b) This version does not group tosses into pairs. So the sample space now looks like sequences of tosses. the only ways to deliver tails now are to observe

$$P(\text{del. } T) = P(HT) + P(HHT) + P(HHHT) + \cdots$$

$$= p(1-p)\left[1 + p + p^2 + \cdots\right]$$

$$= p(1-p) \sum_{j=1}^{\infty} p^j$$

$$= p\cancel{(1-p)} \frac{1}{\cancel{1-p}} = p$$

Heuristically, we only return T if the prior toss was heads.