Solutions : Assignment #4

STAT 430
Prof Stine

4.46 Jury convicts if 9 out of 12 say guilty. It reaches correct decision if convicts a guilty person ($Y = \#$ convict ≥ 9) and if it does not convict an innocent person ($Y < 9$).

$$P(\text{correct}) = P(Y \geq 9 \mid \text{Guilty})\, P(G) + P(Y < 9 \mid \text{Inno})\, P(\text{Inno})$$
$$= P(Bi(n=12, p=.8) \geq 9)(.65) + P(Bi(n=12, p=.1) < 9)(.35)$$
$$=$$

4.51 (a) Under usual Poisson assumptions, $Y = \#$errors is Poisson with $\lambda = .2$ errors/page.

$$P(Y=0) = e^{-\lambda}$$

(b) $P(Y \geq 2) = 1 - P(Y \leq 1) = 1 - e^{-\lambda} - \lambda e^{-\lambda}$

4.60 $P(\text{Beneficial}) = .75$ $\quad C = \#$colds $\quad$ Use Bayes

$$P(B \mid C=2) = \frac{P(C=2 \mid B)\, P(B)}{P(C=2 \mid B) P(B) + P(C=2 \mid B^c) P(B^c)}$$
$$= \frac{(3^2 e^{-3}/2)(.75)}{(3^2 e^{-3}/2)(.75) + (5^2 e^{-5}/2)(.25)}$$

4.61 $H = \#$ hands with full house in 1000 attempts

$H \sim Bi(n = 1000, p = .0014)$. Use poisson approximation,

$$P(H \geq 2) = 1 - P(H \leq 1) = 1 - (e^{-\lambda} + e^{-\lambda}\lambda)$$

with $\lambda = np = 1.4$.

4.67 Better team must win the last game

$$P(\text{Wins in } 4) = .6^4$$
$$P(\text{wins in } 5) = .4^1\, .6^4 \cdot \binom{4}{1}$$
$$P(\text{wins in } 6) = (.4)^2 (.6)^4 \cdot \binom{5}{2}$$

4.68 Check your work to see that adds to one!

$$P(X=j) = \underset{\substack{\uparrow \\ \text{either team} \\ \text{wins} \\ \text{series}}}{2} \underset{\substack{\uparrow \\ \text{winner wins} \\ \text{last game}}}{\binom{j-1}{j-4}} (1/2)^j \quad j=4,5,6,7 \quad = \begin{cases} 1/8 & j=4 \\ 1/4 & j=5 \\ 5/16 & j=6 \\ 5/16 & j=7 \end{cases}$$

4.70 The first 9 heads can be in any of the previous locations,

$$\underbrace{\underline{h}\ \_\ \_\ \_\ \_\ \underline{h}\ \underline{h}\ \_\ \_\ \_\ \_\ \cdots\ \underline{h}}_{\substack{9 \text{ heads among} \\ \text{prior } x-1 \text{ positions}}}\ \_\ \underset{10^{th} \text{ in}}{\underline{h}}$$

$x = \#\text{ tails} + 9$

$$P(T=t) = \binom{t+9}{t} (P_{head})^{10} (1-P_{head})^{t} \qquad t = 0,1,2,\dots$$

4.77 $P(4 \text{ ok}) = (.9)^4$, so prob of a problem found is $1-.9^4$

8.2 (a) Use Markov since one sided and positive r.v. $\quad P(S \geq 86) \leq \frac{75}{86} = .87$

(b) Use Chebyshev since want an interval

$$P(65 \leq S \leq 85)$$
$$= P\left(\frac{65-75}{5} \leq \frac{S-\mu}{\sigma} \leq \frac{85-75}{5}\right)$$
$$= P\left(-2 \leq \frac{S-\mu}{\sigma} \leq 2\right)$$
$$\geq \frac{3}{4}$$

(c) Skip!

8.4 (a) $E[X_1 + \cdots + X_{20}] = 20$ so by markov

$$P\left(\sum_1^{20} X_i > 15\right) = P\left(\sum X_i \geq 16\right) \leq \frac{16}{20} = \frac{4}{5}$$