STAT 430
prof shine

Solutions, Assignment # 5

4.32 Let N = # tests required for the 10 people.

$$EN = 1\ P(N=1) + 11\ P(N=11)$$
$$= 1\ P(\text{all ok}) + 11\ P(\text{any sick})$$
$$= 1 \cdot (.9)^{10} + 11\,(1-.9^{10})$$
$$= 11 - 10\cdot(.9)^{10} \qquad (.9)^{10} = .35$$

So, it will be much cheaper on avg to do the test this way rather than one at a time. (Saves tests if $p > \frac{1}{10^{.1}} \doteq .79$)

4.33 Given that demand $D \sim Bi(10, 1/3)$, let B = # bought.
then the profit of the newsboy is

$$R = \begin{cases} .15D - .10B, & D < B \\ .05B & D \geq B \end{cases}$$

If the newspaper boy buys 3 papers (just below expected demand) then his profit is (or if buys 4, just above ED)

d	P(D=d)	buys 3 R	buys 4
0	.0173	-.30	-.40
1	.087	.15-.30 = -.15	.15-.40 = -.25
2	.195	.30-.30 = 0	.30-.40 = -.10
3	.260	.45-.30 = .15	.45-.40 = .05
4	.228	.15	.60-.40 = .20
5	.137	.15	.20
⋮	⋮	⋮	⋮
10		.15	.20
	Expected profit =	.087	= .053

⟹ Buy 3

4.32 RTFB. See example 6f, page 143

£4.9 $EY = \frac{1}{\sigma}E[Y-\mu] = 0$

$\text{Var}\,Y = \frac{1}{\sigma^2}\text{Var}[Y-\mu] = \frac{1}{\sigma^2}\text{Var}[Y] = 1$

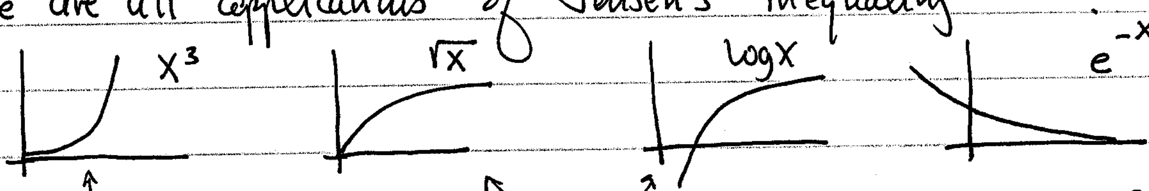
$E[a+bY] = a + bEY$
$V[a+bY] = b^2V(Y)$

t 4.25 Verbal rationale is to return to the underlying "coin tosses" and note these are independent. Analytically, the <u>memoryless</u> property follows from $(k>0)$

$$P(X=n+k \mid X>n) = \frac{P(X=n+k,\, X>n)}{P(X>n)} = \frac{P(X=n+k)}{P(X>n)}$$

$$= \frac{(1-p)^{n+k-1}\,p}{(1-p)^n} = (1-p)^{k-1}p = P(X=k)$$

8.19 These are all applications of Jensen's inequality:



Convex (like X^2), so

concave, so $E[f(X)] \le f(EX)$

convex

$EX^3 \ge (EX)^3$

8.20 Start at the left inequality. $(EX)^2 \le EX^2$ by Jensen, or just from the calculation of the variance. Now for the second inequality, start with $E[X^2]^{3/2} \overset{?}{\le} E[X^3]$ and write this as $(Y=X^2)$, or

$$E[Y]^{3/2} \le E[Y^{3/2}]$$

and now Jensen is evident. Keep going in same way.

8.22 a $\quad P(X \ge 26) \le \dfrac{EX}{26} = 20/26$

b $\quad P(X \ge 26) \le \dfrac{Var(X)}{Var(X)+26} = \dfrac{20}{46}$ $\Leftarrow$ quite abit smaller

t 8.6 a $\quad EX = \sum_x x\,P(X=x) = \sum_{x \le k} x\,P(X=x) + \sum_{x>k} x\,P(X=x)$

$$\ge \sum_{x \le k} x\,P(X=x) \ge P(X=k)\sum_{x=1}^{k} x = P(X=k)\,\frac{n(n+1)}{2}$$

$$\ge n^2/2 \; P(X=k)$$