Introduction to Game Theory

Administrative Things

- Assignment 8 due Friday. Questions?

- Another reference on game theory is W. Poundstone, *Prisoner's Dilemma* (1992).

  - Poundstone's book includes some of the history of game theory, particularly its role in the decision making that surrounded the deployment of nuclear weapons and the policy of mutually assured destruction (MAD).

Today's Topics

- Game Theory

  - Experience game playing

  - Introduction to strategies and terminology of game theory

  - Tit-for-tat, prisoner's dilemma, and the paradox of this situation

- What's this matieral mean to me and this course?

Review from Last Time

- Density pooling

  - More extensive set of notes for Lecture 25 on this methodology will be distributed in class and are on the class web page.

Introduction to Game Theory

- Before discussing prisoner's dilemma, we'll play several rounds of a so-called two-person game to get the feel for these artificial things.

➢ Two-person game, with the following "payoff matrix" for players labeled generically as "Row" and "Column"

	Column A	**Column B**
Row A	**3,3**	**0,5**
Row B	**5,0**	**1,1**

- Interpret this table of payoffs as in the following examples:

  If Row picks A and Column picks A, then Row earns 3 points and Column earns 3 points.

  If Row picks A and Column picks B, Row earns no points and column earns 5 points.

➢ Rules for playing our two-person game

- Find a partner, and decide who is "row" and who is "column". (Since the payoff matrix is symmetric, this choice is arbitrary and needed only for labeling the results.)

- Number a list with about 10 rows, with "Row" and "Column" as headings for two columns.

- Play game for about 10 rounds. I'll stop the game at some time

- For each round of the game

  (1) Each player secretly picks A or B.

  (2) Players simultaneously reveal choices

  (3) Record payoff in each round from the payoff matrix, and repeat.

- The play resembles the kids' game paper, scissors, rock in that you simultaneously reveal hidden choices.

- When game is complete, record total score for both players.

Analysis of our Game

- What strategies were used, and what was the outcome?
- Do any strategies emerge as successful from our small experiment?
  - Plans, scripts (as occasionally used in professional football)
  - Last round (backward induction)
  - Competition between pairs playing the game
  - Retribution, retaliation
  - Cooperation
- This *is* the prisoner's dilemma game
  - Two prisoners being interviewed for a crime. Will either give up the other for a reduced punishment, or count on the other staying silent as well.
  - "Payoffs" resemble those from above

    Both cooperating is the highest net gain, but if either prisoner "defects" and gives up the accomplise, the defector wins a lot (gets let off) whereas the other gets a severe punishment ("sucker payoff").
- Other situations that resemble a prisoner's dilemma
  - Allowing a safe distance between cars when driving on crowded highway
  - Bicycle racing
  - Real estate transactions
  - From Axelrod

    - WWI trench warfare "truces" among front-line troops

    - Biological examples
  - What other situations have you observed?

Axelrod's Tournaments

- Computer-based competitions or tournaments

  - Players are computer programs written by a variety of sponsors, from academics to programmers.

  - Payoff matrix is the one just used in our experiment

  - Two types of tournaments, both with repeated encounters

    (1) Round-robin tournament

    Every program plays every other program, for 200 rounds.

    (2) Biological tournament

    Programs randomly paired, but against a population whose membership changes in response to previous successes. Programs that score twice as much as another will have twice the representation in the next round.

  - Computer programs submitted by various experts in the field.

    The programs implement whatever algorithm the author chose. The idea was to write a program that would win the competition. Some were quite elaborate and complex. Others were quite simple.

    The results of the first tournament were known at the time of the second tournament.

  - Winning algorithm is "tit-for-tat"

    Winner based on highest total winnings (highest average score).

    Tit-for-tat, namely a program that attempts to "cooperate" by choosing the upper left cell of the payoff matrix, unless the other player "defects". Then tit-for-tat defects on the next round as punishment.

    Tit-for-tat never initiates defection, but retaliates one-for-one if the other defects. It's one of the "nice" programs.

    Basically, a computerized version of the "golden rule."

    It was well-known at the time of contests (1980), but initially submitted by only one participant.

- Some important properties of tit-for-tat that help it win these sort of tournaments

  - Very simple and predictable; you know what it will do if you take a given action.

  - No advantage to secrecy in this context

  - Easy to follow strategy that avoids role for emotions (beat this guy) and potential harmful escalation.

  - Wins in tournament since it does very well when it finds a cooperative opponent, and is not a “patsy” to be taken advantage of by aggressive strategies.

- Why are these results a paradox?

  - Study the payoff matrix from the point of view of a “rational” player who is attempting to maximize his own utility, and expects the other player to do the same.

    From point of view of “Row” player, which of A and B would he want to choose if he thinks that

    “Column” will pick A? → B, since the payoff of 5 is highest.

    “Column” will pick B? → B, since the payoff of 1 is highest.

    Optimal rational strategy for “Row” in a *one-play game* is to pick B.

  - However, if both “Row” and “Column” together “cooperate” and choose A, they both gain more that by this “optimal” choice (which guarantees that each gets 1 point rather than 3).

- Further points from Axelrod

  - Fact that program win by cooperating shows that human characteristics are not needed for this strategy to emerge as a winner.

  - Even happens among combatants and animals (symbiotic).

Further Discussion of Game Strategies

- Cooperation emerges as the winning strategy. Does this always happen?

  - Not all games have the features (payoffs) that lead to a prisoner's dilemma. This is a pretty special type of payoff matrix.

  - Repeated play allows cooperation and tit-for-tat to emerge as a useful strategy.

    Optimal for one-time encounter is to defect, and hence the paradox.

    Discounting of future results also weakens the importance of repeated play. With sufficient discounting, all games start to resemble single-round games with little or no need for cooperation.

  - Finite number of encounters leads to similar paradox (backward induction) since you want to defect on the "last" round, then the one before, etc.

- Some things that get in the way of cooperation

  - Retaliation

    How much to retaliate when the other defects? One-for-one, two-for-one?

  - How to avoid the incumbent escalation?

  - Envy and competitive instincts

    Do you win by getting more than your opponent or by doing well overall?

- Cooperation nonetheless emerges in real life examples

  - Trench warfare of WWI, cooperation among people.

  - Symbiotic relationships, among animals.

  - In simulated tournaments among computer programs that don't require the notion of "understanding" or "guilt" to find that cooperation can be a good strategy.

Distinction from Other Types of "Games"

- Prisoner's dilemma is not a zero-sum game.
  - In a zero-sum game, the payoffs in each cell add to zero. Thus, what one player wins, the other must lose.
  - In prisoner's dilemma type games, the winnings of one player do not come directly from the losses of the other player.
  - There's no opportunity for cooperation in a zero-sum game.
  - Strategies are not so interesting in zero-sum games since von Neumann proved that there is always an optimal rational strategy to follow in such games.
- Not like playing chess either
  - Moves are not sequential. You have to make your choice without knowing what your adversary is doing.
  - With chess, you get to see the moves of the other before you have to choose. Like a complex version of tick-tack-toe, there is a "game tree" that will lead to the best strategy given the current position.
  - Resembles some of the issues in planning R&D for new products in business.

    You cannot wait for competitors to release new products without having started to develop your own.

Relationship of Game Theory to Topics in This Course

- Limited, but quite important
  - Remind you of the importance of always thinking about what your competition is doing, and how your actions will affect those that your competition will choose.
  - Cooperation as a strategy is often a novel thought.
- Predictability
  - We discussed various methods of forecasting and illustrated the difficulties that you encounter in getting an accurate prediction.

- Sometimes, you want to be predictable. If your strategy is vague, you can invite "defection" because it is not clear that you will retaliate.

- Example: Patent cases for 3-M. Should they always pursue someone infringing a patent, and incur the associated legal costs?

➢ Coverage of subjective confidence intervals

- Why do subjective intervals only have 50% coverage?

- Consider the interactions of a consulting firm with a client company.

- Consultant's choices (keeping it simple)

  Report accurate interval that might be quite wide, or report a very narrow interval to look very knowledgeable.

- Client company's choices

  Retain consulting firm, or fire consultant

- Payoffs resemble a prisoner's dilemma?

Consult\Firm	**Retain**	**Fire**
Honest	**3,3**	**0,5**
Narrow	**5,0**	**1,1**

- Consulting firm is interesting in retaining client, and client benefits from having a reliable source of information.

- To produce an accurate, honest assessment will cost the consulting firm a lot to learn about the client. A narrow, "off-the-cuff" interval will look impressive and be cheap to do.

- One-time encounter versus many times

- Priorities/utility for length and coverage

Next Time

- ➢ Quick discussion of "Shed Load" as an opportunity to talk about the role of trees to organize information

    - Trees also provide another way to look at a game, though one that encourages a sequential rather than simultaneous perspective.

- ➢ Auctions