1.9

This situation is equivalent to calculating the number of possible arrangements when you have \( n \) objects, but \( n_1 \) objects are alike, \( n_2 \) objects are alike, \( n_3 \) objects are alike and \( n_4 \) objects are alike. Using that formula and applying it to the different colors blocks, we get

\[
\frac{n!}{n_1!n_2!n_3!n_4!} = \frac{12!}{6!4!1!1!} = 27,720
\]

1.10

(a)

This is just the total number of possible arrangements, or permutations of size 8 from a set with 8 elements. Thus, the answer is \( 8! = 40,320 \).

(b)

First, we can sit down the pair of people as \( A \) then \( B \). There are 7 locations to do that in a line of 8 spots. Second, we fill out the rest of the people: \( 6! \). Lastly, we repeat the first and second spots switching the pair to \( B \) then \( A \). Thus, the answer is \( 2 \cdot (7 \cdot 6!) = 10,800 \).

(c)

There are 8 people to choose from for the "first spot". After that, whatever gender was chosen first is taken from the possibilities in the "second spot", etc. Thus, we get \( 8 \cdot 4 \cdot 3 \cdot 3 \cdot 2 \cdot 2 \cdot 1 \cdot 1 = 1152 \).

(d)

We can put the 5 straight males in the line in four ways: \( MMMMFFFF \), \( FMMMMMF \), \( FFMFFFF \), \( FFFMMM \). Then, it is the usual \( 5! \) and \( 3! \) ways of picking the ordered arrangements of males and females respectively. So, the answer is \( 4 \cdot 5! \cdot 3! \).

(e)

There are \( 4! \) ways to place the couples (think \( P_{4,4} \) with couples being \( ABCD \) and finding orderings of them like \( ABCD, ACBD \), etc.). Then, each couple has 2 permutations. Thus: \( 4! \cdot 2^4 = 384 \).
1.15

There are \( \binom{10}{5} \binom{12}{5} \) possible choices of the 5 men and 5 women. They can then be paired up in 5! ways, since if we arbitrarily order the men then the first man can be paired with any of the 5 women, the next with any of the remaining 4, and so on. Hence there are \( 5! \cdot \binom{10}{5} \binom{12}{5} = 23950080 \) possible results.

1.19

(a)

There are \( \binom{8}{3} \binom{4}{3} \) committees that do not contain either of the 2 men and there are \( \binom{2}{1} \left( \binom{8}{3} \binom{4}{2} \right) \) committees that contain exactly 1 of them. Thus, there are \( \binom{8}{3} \binom{4}{3} + \binom{2}{1} \left( \binom{8}{3} \binom{4}{2} \right) \) possible committees.

(b)

Same logic in the previous part can be used in this case, which produces the answer of \( \binom{6}{3} \binom{6}{3} + \binom{2}{1} \left( \binom{6}{2} \binom{6}{3} \right) = 1000 \) possible committees.

(c)

There are \( \binom{7}{3} \binom{5}{3} \) committees in which neither feuding party serves, \( \binom{7}{2} \binom{5}{3} \) committees in which the feuding woman serves and \( \binom{7}{3} \binom{5}{2} \) committees in which the feuding man serves. So, we get \( \binom{7}{3} \binom{5}{3} + \binom{7}{2} \binom{5}{3} + \binom{7}{3} \binom{5}{2} = 910 \) possible committees.

1.24

We can employ the multinomial coefficient here, since \( 13 + 13 + 13 + 13 = 52 \) is fits in the framework of \( n_1 + n_2 + n_3 + n_4 = n \). Thus, there are \( \binom{52}{13, 13, 13, 13} \) bridge deals.

1.31

In the first part, we want the number of nonnegative integer solutions of \( x_1 + x_2 + x_3 + x_4 = 8 \), so using Proposition 6.1, we get \( \binom{11}{3} = 165 \) divisions. In the second part, we are instead interested in the number of positive integer solutions of \( x_1 + x_2 + x_3 + x_4 = 8 \). By Proposition 6.2, we get that there are \( \binom{7}{3} = 35 \) divisions.

2.3

\( E \cap F = \{(1, 2), (1, 4), (1, 6), (2, 1), (4, 1), (6, 1)\} \).

\( E \cup F \) occurs if the sum is odd or if at least one of the dice lands on 1.

\( F \cap G = \{(1, 4), (4, 1)\} \).

\( E \cap F^C = E \setminus F = \) is the event that neither of die lands on 1 and the sum is
odd, such as \( \{(3, 2), (3, 4), (3, 6), (5, 2), \ldots \} \).

\[ E \cap F \cap G = F \cap G. \]

### 2.4

**a**

1 means that a head landed and 0 means that the outcome was a tail. We also know that for any \( i \in \mathbb{N} \) and vector \( x = \{x_1, x_2, \ldots \} \), \( x_{3i} = \) person \( A \)'s \( i \)th toss, \( x_{3i+1} = \) person \( B \)'s \( i \)th toss and \( x_{3i+2} = \) person \( C \)'s \( i \)th toss.

**b**

\[ A = \{1, 0001, 0000001, \ldots \}. \]
\[ B = \{01, 00001, 00000001, \ldots \}. \]
\[ (A \cup B)^c = \{001, 0000001, \ldots \}. \]

### 2.6

**a**

\[ S = \{(1, g), (0, g), (1, f), (0, f), (1, s), (0, s)\}. \]

**b**

\[ A = \{(1, s), (0, s)\}. \]

**c**

\[ B = \{(0, g), (0, f), (0, s)\}. \]

**d**

\[ \{(1, s), (0, s), (1, g), (1, f)\}. \]

### 2.8

**a**

Since the two events \( A \) and \( B \) are mutually exclusive, we can simply add their probabilities by **Axiom 3**: \( 0.3 + 0.5 = 0.8 \).

**b**

This is the same as writing \( A \cap B^c \) or \( A \setminus B \). However, \( A \cap B = \emptyset \), so there is no part of \( A \) that \( B \) can subtract from. Thus: \( P(A \setminus B) = P(A) = 0.3 \).
Like it was mentioned before, \( A \cap B = \emptyset \), so the probability of both happening must be 0.

### 2.12

(a) 

\[
P(S \cup F \cup G) = \frac{28 + 26 + 16 - 12 - 4 - 6 + 2}{100} = \frac{1}{2}
\]

However, we are interested in the complement of \( S \cup F \cup G \), so we need to do

\[
1 - \frac{1}{2} = \frac{1}{2}.
\]

(b) 

In these scenarios, a Venn diagram can prove very useful:

Thus, the answer we want is \( \frac{14 + 10 + 8}{100} = 0.32 \).

(c) 

First, we note that the complement event to at least one of the two students is taking a language course is neither of the two students is taking a language course. There are 50 students not taking a single language class, so the probability that both of them are not taking a language course is

\[
\frac{\binom{50}{2}}{\binom{100}{2}} = \frac{49}{198}
\]

Now, since that is the probability of the complement of the event of interest, we simply:

\[
1 - P(\text{Neither student takes language}) = 1 - \frac{49}{198} = \frac{149}{198}
\]
2.15

(a)

Need to choose one of the suits, \( \binom{4}{1} \), and choose five cards from that suit, \( \binom{13}{5} \). OPTIONAL: You’ll notice if we finish there, we are counting straight flushes in our calculation of flushes, so, technically, we would want to remove those possibilities. Those possibilities happen in 10 different ways for any suit (straight to the 5, 6, 7, . . . , Q, K, A). This would give us the following for the final probability:

\[
\frac{\binom{4}{1} \binom{13}{5} - \binom{4}{1} \binom{10}{1}}{\binom{52}{5}} = 0.002
\]

(b)

Need to choose the value for the pair, \( \binom{13}{1} \), and then choose two of the cards of that value, \( \binom{4}{2} \). Now, we need to make sure not to make another pair (which would make the hand “two pair”) or get another card of that same value (which would make the hand “three of a kind”), so we need to choose three distinct values out of the ones left, \( \binom{12}{3} \), and select one card from each of those values, \( \binom{4}{1}^3 \). Thus:

\[
\frac{\binom{13}{1} \binom{4}{2} \binom{4}{2} \binom{4}{4} \binom{12}{3}}{\binom{52}{5}} = 0.390
\]

(c)

Seen in lecture, but the answer is:

\[
\frac{\binom{13}{2} \binom{4}{2} \binom{4}{2} \binom{4}{4}}{\binom{52}{5}} = 0.048
\]

(d)

Similar to the one pair case. First, we choose the value of the three of a kind, \( \binom{13}{1} \), and choose three of the cards of that value, \( \binom{4}{3} \). Then, we need to choose two distinct valued cards, \( \binom{12}{2} \), and pick one of each value, \( \binom{4}{1}^2 \).

\[
\frac{\binom{13}{1} \binom{4}{3} \binom{12}{2} \binom{4}{1} \binom{4}{1}}{\binom{52}{5}} = 0.021
\]

(e)

Choose the value of the four of a kind, \( \binom{13}{1} \), and choose all four of the cards of that value, \( \binom{4}{4} \). Then, choose any of the remaining cards to fill out the spot, \( \binom{48}{1} \).

\[
\frac{\binom{13}{1} \binom{4}{4} \binom{48}{1}}{\binom{52}{5}} = 0.0002
\]