2.25

Probability that a 5 occurs on the $n$th roll and no 5 or 7 occurs on the first $n-1$ rolls (i.e. $E_n$) is

$$P(E_n) = \left(\frac{26}{36}\right)^{n-1} \cdot \frac{6}{36}$$

The probability that a 5 occurs first is the sum of the probabilities of each sequence where the $n$th roll is a 5 and every roll before it has no 5 or 7 occurring:

$$\sum_{i=1}^{\infty} P(E_i) = \frac{6}{36} \sum_{i=1}^{\infty} \left(\frac{26}{36}\right)^{n-1}$$

$$= \frac{6}{36} \cdot \frac{1}{10/36}$$

$$= \frac{2}{5}$$

2.45

(a)

Let $A_k$ be the event that the $k$th try results in the door opening. We know that:

$$P(A_k) = P(A_k A_{k-1} A_{k-2} \ldots A_1)$$

That is, the $A_k$ is the supset of a failure to open the door on try $i$, where $1 \leq i \leq k - 1$. So, by using the multiplication rule, we get

$$P(A_k) = P(A_1) P(A_2|A_1) \ldots P(A_k|A_{k-1})$$

$$= \frac{n-1}{n} \cdot \frac{n-2}{n-1} \cdot \ldots \cdot \frac{1}{n-(k-1)}$$

$$= \frac{1}{n}$$
Using that same framework from above, we can just look at the probabilities with replacement:

\[
P(A_k) = P(A_1^c)P(A_2^c|A_1^c) \cdots P(A_k|A_{k-1}^c)
\]

\[
= \frac{n-1}{n} \cdot \frac{n-1}{n} \cdots \frac{1}{n}
\]

\[
= \frac{(n-1)^{k-1}}{n^k}
\]

3.5

\[
P(\text{first 2 white, last 2 black}) = \frac{6 \cdot 5 \cdot 9 \cdot 8}{15 \cdot 14 \cdot 13 \cdot 12} = 0.066
\]

3.19

(a)

Let \(W\) be the event that the person is a woman and \(A\) be the event that the person attends the party. Then

\[
P(W|A) = \frac{P(A|W)P(W)}{P(A|W)P(W) + P(A|W^c)P(W^c)}
\]

\[
= \frac{0.48 \cdot 0.38}{0.48 \cdot 0.38 + 0.37 \cdot 0.62}
\]

\[
\approx 0.443
\]

(b)

\[
P(A) = 0.48 \cdot 0.38 + 0.37 \cdot 0.62 = 0.412
\]

3.26

Let \(M\) be the event that the person is male and \(C\) be the event that he/she is color blind. Also, let \(p\) denote the proportion of the population that is male. Then

\[
P(M|C) = \frac{P(C|M)P(M)}{P(C|M)P(M) + P(C|M^c)P(M^c)}
\]

\[
= \frac{(0.05)p}{(0.05)p + (0.0025)(1-p)}
\]

3.38

\[
P(\text{tails}|w) = \frac{\frac{1}{15}}{\frac{1}{15} + \frac{3}{12}}
\]

\[
= \frac{36}{36 + 75} = \frac{36}{111}
\]
\[ P(\text{two-headed} | \text{heads}) = \frac{\frac{1}{3} \cdot 1}{\frac{1}{3} \cdot 1 + \frac{1}{2} \cdot \frac{1}{2} + \frac{1}{3} \cdot \frac{1}{3}} = \frac{4}{9} \]

\[ P(5\text{th coin} | \text{heads}) = \frac{\sum_i P(\text{heads} | i\text{th coin})P(i\text{th coin})}{P(5\text{th coin})} = \frac{\frac{5}{10} \frac{1}{10} 110}{\sum_{i=1}^{10} \frac{1}{10} \frac{1}{10} 110} = \frac{1}{11} \]

\[ P(\text{silver in other} | \text{silver found}) = \frac{P(\text{silver in other, silver found})}{P(\text{silver found})} = \frac{1}{2} \frac{P(\text{silver found} | \text{cabinet } A) \cdot \frac{1}{2} + P(\text{silver found} | \text{cabinet } B) \cdot \frac{1}{2}}{1 + \frac{1}{2}} = \frac{2}{3} \]

\[ P(X = 1) = \frac{1}{2} \]
\[ P(X = 2) = \frac{5 \cdot 5}{10 \cdot 9} = \frac{5}{18} \]
\[ P(X = 3) = \frac{5 \cdot 4 \cdot 5}{10 \cdot 9 \cdot 8} = \frac{5}{36} \]
\[ P(X = 4) = \frac{5 \cdot 4 \cdot 3 \cdot 5}{10 \cdot 9 \cdot 8 \cdot 7} = \frac{10}{168} \]
\[ P(X = 5) = \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 5}{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6} = \frac{5}{252} \]
\[ P(X = 6) = \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6} = \frac{1}{252} \]
4.8

(a)

\[ p(6) = 1 - (5/6)^2 \]
\[ p(5) = \frac{21}{64}/6 + (1/6)^2 \]
\[ p(4) = \frac{21}{63}/6 + (1/6)^2 \]
\[ p(3) = \frac{21}{62}/6 + (1/6)^2 \]
\[ p(2) = \frac{21}{61}/6 + (1/6)^2 \]
\[ p(1) = 1/36 \]

(b)

\[ p(5) = 1/36 \]
\[ p(4) = 2/36 \]
\[ p(3) = 3/36 \]
\[ p(2) = 4/36 \]
\[ p(1) = 5/36 \]
\[ p(0) = 6/36 \]
\[ p(-j) = p(j), j > 0 \]