STAT 430/510: Lecture 1

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Quick Overview of Class

- Approximately 20 lectures, with the remaining classes reserved for catching up and midterm/final reviews.
- Syllabus to come, but plan is to cover *at least* first 7 chapters of books (last topic in Chapter 7 is *Conditional Expectations*).
- Meet on MTWTh for 10:45 - 12:15.
- Class located at F50 JMHH.
- **REQUIRED**: Ross, S. *A First Course in Probability, 8th edition.*
About Me

- 3rd year PhD candidate in Statistics.
- My office number is 433 here in Huntsman (read: 4th floor in the Stat Dept).
- Hours are (tentatively) scheduled on MW 2:00 - 3:00.
- If none of that works, available by appointment.
- Email is jpiette(at)wharton.upenn.edu.
Grading Schema and Class Inner Workings

- Everything will be on a page off of my website (not up right now).

- Grade breakdown:
  - Final = 40%
  - Midterm = 30%
  - Homework = 20%; 4 homeworks.
  - Participation = 10%

- You’ll have one week after the homework has been graded to contest said grade (same goes with midterm).

- However, none of this is set in stone; I am willing to be flexible, both in terms of grading schema and syllabus. If you have any suggestions, cause of concerns, etc., let me know.
**Origin and History**

- **Probability** is the study of randomness and uncertainty.
- From the Latin word *probabilis*, which was often applied to an opinion to mean plausible or generally approved.
- It developed as a way of dealing with uncertainties of evidence in court, then ideas of risk and gambling.
- But the favorite in the beginning came from games, such as dice.
First "Probabilist"

In fact, in 1526, Girolamo Cardano wrote a book about games of chance, which became his main source of income and is known as the first systematic treatment of probability. One of the first things he looked at was dice.
Example 1

- Craps is a game where two dice are thrown and if the sum comes up as 7 or 11, everyone wins.

- **Question:** There is one roll of the die. What is the probability that everyone wins?
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- **Solution:** \(\frac{8}{36} = 0.22\). Why?
Example 1

- Craps is a game where two dice are thrown and if the sum comes up as 7 or 11, everyone wins.
- **Question**: There is one roll of the die. What is the probability that everyone wins?
- **Solution**: \( \frac{8}{36} = 0.22 \). Why?
  - There are 8 possible die outcomes that lead to everyone winning: (16), (25), (34), (43), (52), (61), (65), (56).
  - There are a total of 36 possible die outcomes \((6 \cdot 6)\).
Basic Principle of Counting

Suppose that two experiments are to be performed. If experiment 1 can result in anyone of $m$ possible outcomes and if, for each outcome of experiment 1, there are $n$ possible outcomes of experiment 2, then together there are $mn$ possible outcomes of the two experiments.
Example 2

**Question**: How many different 7-place license plates are possible if the first 3 places have to be occupied by letters and the last 4 places have to be numbers?
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**Question:** How many different 7-place license plates are possible if the first 3 places have to be occupied by letters and the last 4 places have to be numbers?

**Solution:**

\[
26 \cdot 26 \cdot 26 \cdot 10 \cdot 10 \cdot 10 \cdot 10 = 175760000
\]
Generalized Basic Principle of Counting

If $r$ experiments that are to be performed are such that the first one may result in any of $n_1$ possible outcomes; and if, for each of these $n_1$ possible outcomes, there are $n_2$ possible outcomes of the second experiment; and if, for each of the possible outcomes of the first two experiments, there are $n_3$ possible outcomes of the third experiment; and if . . . , then there is a total of $n_1, n_2, \ldots, n_r$ possible outcomes of the $r$ experiments.
Question: Suppose that any one letter or number can only appear at most once in the license plate. If that is the case, then how many different 7-place license plates given the same placement constraints as before?
Example 2 (cont.)

- **Question**: Suppose that any one letter or number can only appear at most once in the license plate. If that is the case, then how many different 7-place license plates given the same placement constraints as before?

- **Solution**:

\[ 26 \cdot 25 \cdot 24 \cdot 10 \cdot 9 \cdot 8 \cdot 7 = 78624000 \]
**Def:** A *permutation* of size $k$ from $n$ elements is the ordered arrangement of $k$ distinct objects taken from a set of $n$ objects.

The total number of *permutations* of size $k$ from $n$ elements is denoted by $P_{k,n}$ and can be calculated by:

$$P_{k,n} = n(n-1)(n-2)\ldots(n-k+1) = \frac{n!}{(n-k)!}$$

The special case, when $k = n$, we have $P_{n,n} = n!$. 
Example 3

- Suppose we are talking about pool balls. There are 16 total in a rack.

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- Suppose we are talking about pool balls. There are 16 total in a rack.

- **Question**: How many different ways can we rack the 16 pool balls?

- **Solution**: 
  
  \[ P_{16,16} = 16! = 20922789888000 \]
Example 3 (cont.)

- Suppose we don’t have the 16-ball rack, but only the 9-ball rack.

  **Question:** How many different ways can we rack our 16 pool balls in the 9-ball rack?
Example 3 (cont.)

- Suppose we don’t have the 16-ball rack, but only the 9-ball rack.

**Question:** How many different ways can we rack our 16 pool balls in the 9-ball rack?

**Solution:**

\[ P_{9,16} = \frac{16!}{(16 - 9)!} = 4151347200 \]
Suppose that we have $n$ objects. $n_1$ of the objects are alike, $n_2$ of the rest of the objects are alike, . . . , and the last $n_k$ are alike.

**Question**: How many different permutations of the $n$ objects are there, given you have all of these that are similar?

**Solution**:

$$\frac{n!}{n_1! n_2! \ldots n_r!}$$
To Do

- Read Ch. 1 (less section 1.6).
- Start gearing up for 4 classes a week!