Hypothesis Testing

$Hypothesis$	$H_0 : \mu = \mu_0$ $Ha : \mu \neq \mu_0$	$H_0 : \mu \leq \mu_0$ $Ha : \mu > \mu_0$	$H_0 : \mu \geq \mu_0$ $Ha : \mu < \mu_0$
Acceptance Region	$\mu_0 \pm z^* \frac{\sigma}{\sqrt{n}}$	Up to $\mu_0 + z_1^* \frac{\sigma}{\sqrt{n}}$	From $\mu_0 - z_1^* \frac{\sigma}{\sqrt{n}}$
$Z - score$	$z = \frac{\overline{x}-\mu_0}{\sigma/\sqrt{n}}$ $If\ \lvert z\rvert \geq z^* reject$	$z = \frac{\overline{x}-\mu_0}{\sigma/\sqrt{n}}$ $If\ z \geq z_1^* reject$	$z = \frac{\overline{x}-\mu_0}{\sigma/\sqrt{n}}$ $If\ z \leq -z_1^* reject$
$P - value$	$2P\left(Z \geq \lvert z\rvert\right)$	$P\left(Z \geq z\right)$	$P\left(Z \leq z\right)$

Notes:

1. z^* is the z-value associated with the probability of $1 - \frac{\alpha}{2}$
2. z_1^* is the z-value associated with the probability of $1 - \alpha$
3. For proprotions: μ_0 is replaced by p_0
4. σ is replaed by $\sqrt{p_0(1 - p_0)}$
5. In the case of unkown variance:

- σ is replaced by s
- $z^*$is replaced by t^* where $t^*$is in the $\frac{\alpha}{2}$ column and n-1 row of the t-table
- $z_1^*$is replaced by t_1^* where $t_1^*$is in the α column and n-1 row of the t-table

Other Calculations

1. Type II errors: You need to be given a value of $\mu_a$that belongs to the alternative hypothesis. Compute the probability of being in the acceptance region using the fact that $\overline{X}$ is normal with mean equal to μ_a and standard deviation equal to $\frac{\sigma}{\sqrt{n}}$.
2. Find the sample size to produce a Type II error of a given β at a given α. $n = \frac{(z_\alpha + z_\beta)^2 \sigma^2}{(\mu_a - \mu_0)^2}$

Review Problem

A car manufacturer redesigned the plant. They want to see if the average time of assembly of a car is less than 3 hours. A sample of 100 cars is taken to yield: $\overline{x} = 2.8\, hours$ and $s = 1\, hour$. In addition, the number of items that take at least 3 hours was 40 out of the 100 items.

a) Test whether the population mean time is less than 3 hours, using $\alpha = .01$

b) If the true value of μ was 2.7 hours, what would be the probability of an error using the test in a)?

c) What sample size is required so that the probability of a type II error when $\mu = 2.7$ is .05. Assume $\alpha = .01$

Solution

$H_0 : \mu \geq 3$ versus $H_a : \mu < 3$; n=100; $\alpha = .01$; Assume $\sigma = 1$.

a)

1. Acceptance Region: $\overline{x}$ must be at least $3 - 2.33\frac{1}{\sqrt{100}} = 2.767$. $\overline{x} = 2.8\, hours$ implies retain H_0
2. $z = \frac{\overline{x}-\mu_0}{\sigma/\sqrt{n}} = \frac{2.8-3}{1/\sqrt{100}} = -2$ Since -2>-2.33 retain H_0
3. $P - value = P(\overline{X} \leq 2.8|\mu = 3) = P(Z \leq \frac{2.8-3}{1/\sqrt{100}} = -2) = .0228$.Since the P-value exceeds α retain H_0

b) Since $\mu_a = 2.7$ that implies that $\overline{X}$ is normal with mean = 2.7 and standard deviation = .10

$$P(\overline{X} > 2.767) = P(Z > \frac{2.767 - 2.7}{.1} = .67) = 1 - .7486 = .2514$$

c) $n = \frac{(z_\alpha + z_\beta)^2\sigma^2}{(\mu_a - \mu_0)^2} = \frac{(2.33+1.645)^2\, 1}{(2.7-3)^2} = 175.5625\ or\ 176$