Review Problem on Continuous Random Variables

The bid that a competitor makes on a real estate property is estimated to be somewhere between 0 and 3 million dollars. Specifically, the bit X is viewed to be a continuous random variable with density function:

$$\begin{aligned} f(x) &= c\left(9-x^2\right) \text{ for } 0<x<3 \\ &= 0\ otherwise \end{aligned}$$

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You make a bid (without knowing the competitor's bid). The higher of the two bids win.

Questions

1. Find the value of c that makes f(x) a legitimate density function?
2. Find the cumulative distribution function, F(x). Use the cumulative distribution to determine the probability that you lose the bid if you make a bid of 2 million? 1 million?
3. Find the expected value and standard deviation for the competitor's bid. What is the probability that the competitor's bid is within one standard deviation of the mean?
4. How much should you bid so that you have a 90% chance of winning?

Answers

1. $c\int_0^3\left(9-x^2\right)dx=1$. This implies $c\left[9x-\frac{x^3}{3}\right]_0^3=1$. Hence $c\left[27-9\right]=1$. So $c=\frac{1}{18}$

2. $F(x)=\int\limits_{-\infty}^{x} f(x)dx=\int_0^x\frac{\left(9-x^2\right)}{18}dx=\frac{x}{2}-\frac{x^3}{54}$ for $0<x<3$. $F(x)=0\ for\ x<0\ and\ 1\ for\ x>3$.

   Hence $P(X>2)=\ 1-F(2)=1-\left(\frac{2}{2}-\frac{8}{54}\right)=\frac{8}{54}$

   $P(X>1)=\ 1-F(1)=1-\left(\frac{1}{2}-\frac{1}{54}\right)=\frac{28}{54}$

3. $E(X)=\int\limits_{-\infty}^{\infty} x\,f(x)dx=\int\limits_{0}^{3} x\,\frac{\left(9-x^2\right)}{18}dx=\left[\frac{x^2}{4}-\frac{x^4}{72}\right]_0^3=\frac{9}{8}$

   $E(X^2)=\int\limits_{-\infty}^{\infty} x^2 f(x)dx=\int\limits_{0}^{3} x^2\frac{\left(9-x^2\right)}{18}dx=\left[\frac{x^3}{6}-\frac{x^5}{90}\right]_0^3=4.5-2.7=1.8$

$V(X) = 1.8 - \left(\frac{9}{8}\right)^2 = .534375$.So the standard deviation is $\sqrt{.534375} = .731$

Finally, one standard deviation of the mean is $1.125 - .731 = .394$ to $1.125 + .731 = 1.856$.

$F(1.856) - F(.394) = \left[\frac{x}{2} - \frac{x^3}{54}\right]_{.394}^{1.856} = .8096 - .1959 = .6137.$

4. We want the value of x so that $F(x) = .9$

   Since F(x) is an increasing function we can find this by trial and error. The value of x is approximately 2.19.