Free Knot Polynomial Spline Confidence Intervals

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Abstract

We construct approximate confidence intervals for a nonparametric regression function. The construction uses polynomial splines with free knot locations. The number of knots is determined by the GCV criteria. The estimates of knot locations and coefficients are obtained through a nonlinear least square solution that corresponds to the maximum likelihood estimate. Confidence intervals are then constructed based on the asymptotic distribution of the MLE. Average coverage probabilities and accuracy of the estimate are examined via simulation. This includes comparisons between our method and some existing ones such as smoothing spline and variable knots selection as well as a Bayesian version of the variable knots method. Simulation results indicate that our method seems to work well for smooth underlying functions and also reasonably well for unsmooth (discontinuous) functions. It also performs well for fairly small sample sizes. As a practical example we apply the method to study the productivity of US banks. The corresponding analysis supports certain research hypotheses concerning the effect of federal policy on banking efficiency.

Key words: Nonparametric regression; Confidence intervals; MLE; Piecewise polynomials; Free knots; B-splines.