Sharp Adaptive Estimation by a Blockwise Method

T. Tony Cai, Mark G. Low and Linda H. Zhao

Department of Statistics

University of Pennsylvania

Abstract

We consider a blockwise James-Stein estimator for nonparametric function estimation in suitable wavelet or Fourier bases. The estimator can be readily explained and implemented. We show that the estimator is asymptotically sharp-adaptive in minimax risk over any Sobolev ball containing the true function. Further, for a moderately broad range of bounded sets in Besov space our estimator is asymptotically nearly sharp adaptive in the sense that it comes within the Donoho-Liu constant, 1.24, of being exactly sharp adaptive. Other parameter spaces are also considered. The paper concludes with a Monte-Carlo study comparing the performance of our estimator to that of three other popular wavelet estimators. Our procedure generally (but not always) outperforms two of these and is overall comparable, or perhaps slightly superior, to the third.

Keywords: Adaptive estimation; Besov space; Empirical Bayes; Fourier series; Minimax; Nonparametric regression; Sobolev space; Wavelets; White noise.

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