Statistics 430
HW #10 Solutions

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1 Ch 7 Problem 1

Let W denote winnings, H heads, and T tails.

$$\begin{aligned} E[W] &= E[W|H]P(H) + E[W|T]P(T) \\ &= 2 * 3.5 * .5 + .5 * 3.5 * .5 \\ &= \boxed{4.375} \end{aligned}$$

2 Ch 7 Problem 5

Let X be the horizontal distance traveled by the ambulance, and Y the vertical distance. We know that $X \sim Unif(-1.5, 1.5)$, and likewise Y. Both $|X|$ and $|Y|$ are distributed as $Unif(0, 1.5)$, so each has expectation 1.5.

$$\begin{aligned} E[|X| + |Y|] &= E[|X|] + E[|Y|] \\ &= .75 + .75 = \boxed{1.5} \end{aligned}$$

3 Ch 7 Problem 8

Let X be the number of occupied tables. Let

$$X_i = \begin{cases} 1 & \text{the ith arrival sits at a previously unoccupied table} \\ 0 & \text{otherwise} \end{cases}$$

If a friendless person arrives, then he sits at previously unoccupied table, raising the count of occupied tables by 1. So $X = \sum_{i=1}^{N} X_i$.

$$
\begin{aligned}
E[X_i] &= P(X_i = 1) \text{ because } X_i \text{ is an indicator variable} \\
&= (1-p)^{i-1} \quad \text{since he is not friends with any of the } i-1 \text{ previous arrivals} \\
E[X] &= E\left[\sum_{i=1}^{N} X_i\right] = \sum_{i=1}^{N} E[X_i] \\
&= \sum_{i=1}^{N} (1-p)^{i-1} \\
&= \sum_{j=0}^{N-1} (1-p)^{j} \quad \text{reindexing} \\
&= \frac{1-(1-p)^N}{1-(1-p)} = \boxed{\tfrac{1-(1-p)^N}{p}} \quad \text{sum of a geometric series}
\end{aligned}
$$

4 Ch 7 Problem 11

Let X be the number of changeovers.

$$
X_i = \begin{cases} 1 & \text{flip } i \text{ differs from flip } i-1 \\ 0 & \text{otherwise} \end{cases}
$$

Notice that $X = \sum_{i=2}^{n} X_i$, where the count begins at $i = 2$ since the first flip does not represent a changeover.

$$
\begin{aligned}
E[X_i] &= P(X_i = 1) = P(X_i\text{Heads}|X_{i-1}\text{Tails})P(X_{i-1}\text{Tails}) + P(X_i\text{Tails}|X_{i-1}\text{Heads})P(X_{i-1}\text{Heads}) \\
&\qquad = p(1-p) + (1-p)p = 2p(1-p) \\
E[X] &= E\left[\sum_{i=2}^{n} X_i\right] = \sum_{i=2}^{n} E[X_i] \\
&= \boxed{2(n-1)p(1-p)}
\end{aligned}
$$

5 Ch 7 Problem 12

a

$$
X_i = \begin{cases} 1 & \text{the ith man is sitting next to a woman} \\ 0 & \text{otherwise} \end{cases}
$$

$$
\begin{aligned}
E[X_i] &= E[X_i|\text{ith man sits on the end}]P(ithman \text{ sits on the end}) \\
&+ E[X_i|\text{ith man sits in the middle}]P(ithman \text{ sits in the middle}) \\
&= \frac{n}{2n-1} * \frac{2}{2n} + \left(\frac{3n}{4n-2}\right)\left(\frac{2n-2}{2n}\right) \text{Note: the } \frac{3n}{4n-2} \text{ term is calculated in part b} \\
&= \frac{1}{2n-1} + \frac{3n-3}{4n-2} \\
&= \frac{3n-1}{4n-2} \\
E[X] &= n * \frac{3n-1}{4n-2} = \boxed{\frac{3n^2-n}{4n-2}}
\end{aligned}
$$

b

$$
X_i = \begin{cases} 1 & \text{the ith man is sitting next to a woman} \\ 0 & \text{otherwise} \end{cases}
$$

$$
\begin{aligned}
P(X_i = 1) &= 1 - P(X_i = 0) \\
&= 1 - \frac{\binom{n-1}{2}}{\binom{2n-1}{2}} \\
&= 1 - \frac{(n-1)(n-2)}{(2n-1)(2n-2)} = 1 - \frac{(n-1)(n-2)}{(4n-2)(n-2)} \\
&= \frac{3n}{4n-2} \\
E[X] &= n * \frac{3n}{4n-2} = \boxed{\frac{3n^2}{4n-2}}
\end{aligned}
$$