Statistics 430
HW #7 Solutions

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1 Ch 5 Problem 20

In the random sample of 100 people, the number in favor of a proposed rise in school taxes, X, is distributed $Bin(100, .65)$. With $n = 100$, $p = .65$, the mean number of people in favor is $np = 65$; the standard deviation: $\sqrt{np(1-p)} = \sqrt{22.75}$. We apply the normal approximation to the binomial, remaining mindful of the continuity correction.

a

$$\begin{aligned} P(X \geq 49.5) &= P\left(\frac{X-65}{\sqrt{22.75}} \geq \frac{49.5-65}{\sqrt{22.75}}\right) \\ &\approx 1 - \Phi(3.2497) \\ &\approx \boxed{.9994} \end{aligned}$$

b

$$\begin{aligned} P(59.5 \leq X \leq 70.5) &= P\left(\frac{59.5-65}{\sqrt{22.75}} \leq \frac{X-65}{\sqrt{22.75}} \leq \frac{70.5-65}{\sqrt{22.75}}\right) \\ &\approx P(-1.1531 \leq Z \leq 1.1531) \\ &\approx \Phi(1.1531) - \Phi(-1.1531) \\ &\approx \boxed{.7511} \end{aligned}$$

c

$$\begin{aligned} P(X \leq 74.5) &= P\left((\frac{X-65}{\sqrt{22.75}} \leq \frac{74.5-65}{\sqrt{22.75}}\right) \\ &\approx \Phi(1.9917) \\ &\approx \boxed{.9768} \end{aligned}$$

2 Ch 5 Problem 23

The number of times we roll a 6, X, is a $Bin(1000, \frac{1}{6})$ random variable, with mean $np = 166.67$ and standard deviation $\sqrt{np(1-p)} = 11.785$.

i

$$\begin{aligned} P(149.5 \le X \le 200.5) &= P\left(\frac{149.5 - 166.67}{11.785} \le \frac{X - 166.67}{\sqrt{11.785}} \le \frac{200.5 - 166.67}{11.785}\right) \\ &\approx P\left(-1.4566 \le Z \le 2.8709\right) \\ &\approx \Phi(2.8709) - \Phi(-1.4566) \\ &\approx \boxed{.9253} \end{aligned}$$

ii

Conditional on 6 appearing 200 times, the number of 5's rolled, X, is a $Bin(800, \frac{1}{5})$ random variable, with mean $np = 160$ and standard deviation $\sqrt{np(1-p)} = \sqrt{25.6}$.

$$\begin{aligned} P(X \le 149.5) &= P\left(\frac{X - 160}{\sqrt{25.6}} \le \frac{149.5 - 160}{\sqrt{25.6}}\right) \\ &\approx \Phi(-.9281) \\ &\approx \boxed{.1767} \end{aligned}$$

3 Ch 5 Problem 29

Without loss of generality, assume that the stock that your manager – call him "Bernie" - recommends is presently valued at \$1. In order for the stock to rise at least 30 percent over 1000 periods, its price must rise at least 470 of the days, since $1.012^{469} * .99^{531} = 1.294$ and $1.012^{470} * .99^{530} = 1.323$. The number of days in which the stock's price rises, X, is a $Bin(1000, .52)$ random variable, with mean $np = 520$ and standard deviation $\sqrt{np(1-p)} = \sqrt{249.6}$.

$$\begin{aligned} P(X \ge 469.5) &= P\left(\frac{X - 520}{\sqrt{249.6}} \ge \frac{469.5 - 520}{\sqrt{249.6}}\right) \\ &\approx 1 - \Phi(-3.1965) \\ &\approx \boxed{.9993} \end{aligned}$$

4 Ch 5 Problem 32

Let X denote the number of hours required to repair the machine. We are given that $X \sim Expo(\frac{1}{2})$. We also know the CDF of an exponentially distributed random variable with parameter λ: $F(a) = 1 - e^{-\lambda a}$.

a

$$\begin{aligned} P(X > 2) &= 1 - P(X \le 2) \\ &= 1 - (1 - e^{-\frac{1}{2}*2}) \\ &= \boxed{e^{-1} = .368} \end{aligned}$$

b

By the memoryless property of the exponential distribution, the repair time past 9 hours is also distributed as an $Expo(\frac{1}{2})$ random variable. Hence

$$\begin{aligned} P(X > 1) &= 1 - P(X \le 1) \\ &= 1 - (1 - e^{-\frac{1}{2}*1}) \\ &= \boxed{e^{-\frac{1}{2}} = .607} \end{aligned}$$

5 Ch 5 Problem 34

i

Again, because the exponential distribution is memoryless, the number of additional thousands of miles that Jones will squeeze out of the car is still $Expo(\frac{1}{20})$.

$$\begin{aligned} P(X > 20) &= 1 - P(X \le 20) \\ &= 1 - (1 - e^{-\frac{1}{20}*20}) \\ &= \boxed{e^{-1} = .368} \end{aligned}$$

ii

$$\begin{aligned} P(X > 30 | X > 10) &= \frac{P(X > 30 \cap X > 10)}{P(X > 10)} \\ &= \frac{P(X > 30)}{P(X > 10)} \\ &= \tfrac{1}{4} / \tfrac{3}{4} \\ &= \boxed{\tfrac{1}{3}} \end{aligned}$$

6 Ch 5 Problem 37

a

$$\begin{aligned} P(|X| > \frac{1}{2}) &= P(X < -\frac{1}{2} \cup X > \frac{1}{2}) \\ &= \frac{1}{4} + \frac{1}{4} \\ &= \boxed{\frac{1}{2} = .5} \end{aligned}$$

b

For $0 \leq x \leq 1$:

$$\begin{aligned} P(|X| < x) &= P(-x < X < x) \\ F_{|X|}(x) &= P(X < x) - P(x < -x) \\ F_{|X|}(x) &= \frac{1+x}{2} - \frac{1-x}{2} \\ F_{|X|}(x) &= x \end{aligned}$$

$$\boxed{f_{|X|}(x) = \begin{cases} 1 & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}}$$

To wit, $|X|$ is uniformly distributed over $(0, 1)$.

7 Ch 5 Problem 39

Let $X \sim Expo(1)$, and let $Y = \log X$. Then

$$\begin{aligned} P(Y < y) &= P(\log X < y) \\ F_Y(y) &= P(X < e^y) \\ F_Y(y) &= 1 - e^{-e^y} \end{aligned}$$

$$\begin{aligned} f_Y(y) &= -e^{-e^y} * -e^y \\ &= \boxed{\begin{cases} e^{y-e^y} & -\infty < y < \infty \\ 0 & \text{otherwise} \end{cases}} \end{aligned}$$

8 Ch 5 Problem 40

Let $X \sim Unif(0, 1)$, and let $Y = e^X$.

For 1 < y < e :

$$\begin{aligned} P(Y < y) &= P(e^X < y) \\ F_Y(y) &= P(X < \log y) \\ F_Y(y) &= \log y \end{aligned}$$

$$f_Y(y) = \boxed{\begin{cases} \frac{1}{y} & 1 < y < e \\ 0 & \text{otherwise} \end{cases}}$$