Homework 7 Optional Problems Solutions

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1 Ch 5 Problem 26*

Let X be the number of heads in 1000 tosses of the fair coin, and let Y be the number heads in 1000 tosses of the biased coin. X and Y have distributions $Binomial(1000, 0.5)$ and $Binomial(1000, 0.55)$ respectively.

(a) If the coin is actually fair, we shall reach a false conclusion when $X \geq 525$. Since $n = 1000$ is large, the distribution of X can be approximated by $Normal(500, 250)$.

$$\begin{aligned} P(X \geq 525) &= 1 - P(X < 525) \\ &= 1 - \Phi\left(\frac{525.5 - 500}{\sqrt{250}}\right) \\ &= 1 - \Phi(1.61) \\ &= 1 - 0.9464 \\ &= 0.0537 \end{aligned}$$

(b) If the coin is actually biased, we shall reach a false conclusion when $Y < 525$. Since $n = 1000$ is large, the distribution of Y can be approximated by $Normal(550, 247.5)$.

$$\begin{aligned} P(Y < 525) &= \Phi\left(\frac{524.5 - 550}{\sqrt{247.5}}\right) \\ &= \Phi(-1.62) \\ &= 1 - \Phi(1.62) \\ &= 1 - 0.9474 \\ &= 0.0526 \end{aligned}$$

2 Ch 5 Problem 27*

Let's firstly assume that the coin is fair. Then let X be the number of heads in the 10,000 tosses. $X \sim Binomial(10000, 0.5)$. By normal approximation, $X \rightarrow Normal(5000, 2500)$. Under this assumption, calculate the probability of having at least 5800 heads, to see whether it is a reasonably likely event.

$$\begin{aligned} P(X \geq 5800) &= 1 - P(X < 5800) \\ &= 1 - \Phi\left(\frac{5799.5 - 5000}{\sqrt{2500}}\right) \\ &= 1 - \Phi(15.99) \\ &\approx 0 \end{aligned}$$

Hence, it's not reasonable to assume the coin is fair.

3 Ch 5 Problem 38*

The equation having both roots real means

$$\begin{aligned} \Delta &= 16Y^2 - 16Y - 32 \\ &= 16(Y-2)(Y+1) \\ &\geq 0 \end{aligned}$$

So $Y \leq -1$ or $Y \geq 2$. Since $Y \sim Uniform(0, 5)$,

$$P(Y \leq -1 \text{ or } Y \geq 2) = 0 + \frac{3}{5} = \frac{3}{5}$$

4 Ch 5 Problem 41*

$\theta \sim Uniform(-\pi/2, \pi/2)$. $R = A\sin\theta$, where A is a fixed constant.

$$\begin{aligned} P(R \leq r) &= P(A\sin\theta \leq r) \\ &= P(\sin\theta \leq \frac{r}{A}) \\ &= P(\theta \leq \arcsin\frac{r}{A}) \\ &= \frac{\arcsin(\frac{r}{A}) + \frac{2}{\pi}}{\pi} \\ &= \frac{\arcsin(\frac{r}{A})}{\pi} + 2 \end{aligned}$$

So the distribution of R is

$$P(R = r) = \frac{d}{dr}P(R \leq r)$$

$$
\begin{aligned}
&= \frac{1}{\pi}\frac{1/A}{\sqrt{1-r^2/A^2}} \\
&= \frac{1}{\pi\sqrt{A^2-r^2}}
\end{aligned}
$$