Martingale Markets:
Abstracting the Distinguished Asset

J. Michael Steele

November 12, 2008
Appealing Theory, Appalling Facts, and Eternal Hope

▶ Part I: Rubinstein’s Theorem and the CAPM
Appealing Theory, Appalling Facts, and Eternal Hope

- Part I: Rubinstein’s Theorem and the CAPM
- Part II: Those Nasty Little Stylistic Facts
Appealing Theory, Appalling Facts, and Eternal Hope

- Part I: Rubinstein’s Theorem and the CAPM
- Part II: Those Nasty Little Stylistic Facts
- Part III: “Natural” Models where the Market Asset is Special
Appealing Theory, Appalling Facts, and Eternal Hope

- Part I: Rubinstein’s Theorem and the CAPM
- Part II: Those Nasty Little Stylistic Facts
- Part III: “Natural” Models where the Market Asset is Special
- Part IV: Mathematical Features of a Candidate
Appealing Theory, Appalling Facts, and Eternal Hope

- Part I: Rubinstein’s Theorem and the CAPM
- Part II: Those Nasty Little Stylistic Facts
- Part III: “Natural” Models where the Market Asset is Special
- Part IV: Mathematical Features of a Candidate
- Part V: Return to the Menagerie and a Free Snack
Rubinstein’s CAPM Theorem

Consider a one-period investment model. Assume each investor acts to maximize his next period expected utility. Assume that asset returns are jointly normal.

Big CAPM Conclusion: For each asset \( A \),

\[
\begin{align*}
    r_A - r_0 &= \beta (r_M - r_0) + \epsilon \\
    \epsilon &\sim N(0, \sigma)
\end{align*}
\]

where \( r_A \) is the return on the asset, \( r_0 \) is the risk-free rate, \( r_M \) is the market return, and \( \beta \) is a constant that depends on all the utilities and on the distribution of the asset returns.
Rubinstein’s CAPM Theorem

Consider a one-period investment model.

Assume each investor:
- Has a utility function
- Acts to maximize his next period expected utility

Assume that asset returns are jointly normal.

Big CAPM Conclusion: For each asset $A$,

$$r_A - r_0 = \beta (r_M - r_0) + \epsilon$$

where $\epsilon \sim N(0, \sigma)$

$r_A$ is the return on the asset,
$r_0$ is the risk-free rate,
$r_M$ is the market return, and
$\beta$ is a constant that depends on all the utilities and on the distribution of the asset returns.
Rubinstein’s CAPM Theorem

- Consider a one-period investment model
- Assume each investor has a utility function and acts to maximize his next period expected utility.
- Assume that asset returns are jointly normal.
- Big CAPM Conclusion: For each asset $A$
  \[ r_A - r_0 = \beta (r_M - r_0) + \epsilon \]
  where $\epsilon \sim N(0, \sigma)$
  - $r_A$ is the return on the asset,
  - $r_0$ is the risk-free rate,
  - $r_M$ is the market return, and
  - $\beta$ is a constant that depends on all the utilities and on the distribution of the asset returns.
Rubinstein’s CAPM Theorem

- Consider a one-period investment model
- Assume each investor
  - Has a utility function

\[ r_A - r_0 = \beta (r_M - r_0) + \epsilon \]

where \( \epsilon \sim N(0, \sigma) \)

- \( r_A \) is the return on the asset,
- \( r_0 \) is the risk-free rate,
- \( r_M \) is the market return, and
- \( \beta \) is a constant that depends on all the utilities and on the distribution of the asset returns.
Rubinstein’s CAPM Theorem

- Consider a one-period investment model
- Assume each investor
  - Has a utility function
  - Acts to maximize his next period expected utility

Assume that asset returns are jointly normal

\[ r_A - r_0 = \beta (r_M - r_0) + \epsilon \]

where \( r_A \) is the return on the asset, \( r_0 \) is the risk-free rate, \( r_M \) is the market return, and \( \beta \) is a constant that depends on all the utilities and on the distribution of the asset returns.
Rubinstein’s CAPM Theorem

- Consider a one-period investment model
- Assume each investor
  - Has a utility function
  - Acts to maximize his next period expected utility
- Assume that asset returns are jointly normal
Rubinstein’s CAPM Theorem

- Consider a one-period investment model
- Assume each investor
  - Has a utility function
  - Acts to maximize his next period expected utility
- Assume that asset returns are jointly normal
- Big CAPM Conclusion: For each asset A

$$r_A - r_0 = \beta (r_M - r_0) + \epsilon$$

where $\epsilon \sim N(0, \sigma)$

- $r_A$ is the return on the asset,
- $r_0$ is the risk-free rate,
- $r_M$ is the market return, and
- $\beta$ is a constant that depends on all the utilities and on the distribution of the asset returns.
Rubinstein’s CAPM Theorem

- Consider a one-period investment model
- Assume each investor
  - Has a utility function
  - Acts to maximize his next period expected utility
- Assume that asset returns are jointly normal
- Big CAPM Conclusion: For each asset $A$

$$r_A - r_0 = \beta(r_M - r_0) + \epsilon \quad \text{where} \quad \epsilon \sim N(0, \sigma)$$

where $r_A$ is the return on the asset, $r_0$ is the risk free rate, $r_M$ is the market return, and $\beta$ is a constant that depends on all the utilities and on the distribution of the asset returns.
CAPM: Why So Many So Love It

▶ All your investment problems reduce to one problem
▶ You will hold all assets in proportion to their market weight
▶ The "market" and the risk free asset are the only assets you need to consider: Pick your "percent" and you are done.
▶ This conclusion is massively appealing!
▶ Moreover, it is simply mathematics....
▶ .... given our assumptions
▶ WHICH STINK

J. Michael Steele
Martingale Markets: Abstracting the Distinguished Asset
CAPM: Why So Many So Love It

- All your investment problems reduce to one problem
CAPM: Why So Many So Love It

- All your investment problems reduce to one problem
- You will hold all assets in proportion to their market weight
All your investment problems reduce to one problem
You will hold all assets in proportion to their market weight
The “market” and the risk free asset are the only assets you need to consider: Pick your “percent” and you are done.

This conclusion is massively appealing!
Moreover, it is simply mathematics... given our assumptions
WHICH STINK
CAPM: Why So Many So Love It

- All your investment problems reduce to one problem
- You will hold all assets in proportion to their market weight
- The “market” and the risk free asset are the only assets you need to consider: Pick your “percent” and you are done.
- This conclusion is massively appealing!
CAPM: Why So Many So Love It

► All your investment problems reduce to one problem
► You will hold all assets in proportion to their market weight
► The “market” and the risk free asset are the only assets you need to consider: Pick your “percent” and you are done.
► This conclusion is massively appealing!
► Moreover, it is simply mathematics ....
CAPM: Why So Many So Love It

- All your investment problems reduce to one problem
- You will hold all assets in proportion to their market weight
- The “market” and the risk free asset are the only assets you need to consider: Pick your “percent” and you are done.
- This conclusion is massively appealing!
- Moreover, it is simply mathematics ....
- .... given our assumptions
CAPM: Why So Many So Love It

- All your investment problems reduce to one problem
- You will hold all assets in proportion to their market weight
- The “market” and the risk free asset are the only assets you need to consider: Pick your “percent” and you are done.
- This conclusion is massively appealing!
- Moreover, it is simply mathematics ....
- ..... given our assumptions
- WHICH STINK
What Do You Mean by “Distribution of Returns”? 

There is a subtle assumption implicit even in speaking about “the distribution of returns.” It always makes sense to speak of distribution of \( r_t \) given the past \( r_{t-1}, r_{t-2}, \ldots \), but to speak of the distribution of \( \{r_t\} \) by itself, we must assume stationarity. We can’t actually test for stationarity. Example: Consider any deterministic cycle with a random start. As a matter of practice, this doesn’t matter much. As an intellectual matter, there is strangely good news. Common Sense (of Sorts): One should only assume that which one cannot test and reject.

J. Michael Steele
Martingale Markets: Abstracting the Distinguished Asset
What Do You Mean by “Distribution of Returns”? 

- There is a subtle assumption implicit even in speaking about “the distribution of returns.”
What Do You Mean by “Distribution of Returns”? 

▶ There is a subtle assumption implicit even in speaking about “the distribution of returns.”

▶ It always makes sense to speak of distribution of $r_t$ given the past $r_{t-1}, r_{t-2}, ...$ but to speak of the distribution of $\{r_t\}$ by itself, we must assume stationarity.
What Do You Mean by “Distribution of Returns”? 

▶ There is a subtle assumption implicit even in speaking about “the distribution of returns.”

▶ It always makes sense to speak of distribution of $r_t$ given the past $r_{t-1}, r_{t-2}, \ldots$ but to speak of the distribution of $\{r_t\}$ by itself, we must assume stationarity.

▶ We can’t actually test for stationarity. Example: Consider any deterministic cycle with a randomize start.
What Do You Mean by “Distribution of Returns”? 

- There is a subtle assumption implicit even in speaking about “the distribution of returns.”
- It always makes sense to speak of distribution of $r_t$ given the past $r_{t-1}, r_{t-2}, \ldots$ but to speak of the distribution of $\{r_t\}$ by itself, we must assume stationarity.
- We can’t actually test for stationarity. Example: Consider any deterministic cycle with a randomize start.
- As a matter of practice, this doesn’t matter much. As an intellectual matter, there is strangely good news.

"Common Sense (of Sorts): One should only assume that which one cannot test and reject."

J. Michael Steele
Martingale Markets: Abstracting the Distinguished Asset
What Do You Mean by “Distribution of Returns”?

- There is a subtle assumption implicit even in speaking about “the distribution of returns.”
- It always makes sense to speak of distribution of $r_t$ given the past $r_{t-1}, r_{t-2}, ...$ but to speak of the distribution of $\{r_t\}$ by itself, we must assume stationarity.
- We can’t actually test for stationarity. Example: Consider any deterministic cycle with a randomize start.
- As a matter of practice, this doesn’t matter much. As an intellectual matter, there is strangely good news.
- Common Sense (of Sorts): One should only assume that which one cannot test and reject.
Testing the Normality of Asset Returns

Take any decent sized time series of almost any asset — Stock, Bond, Mutual Fund, ETF, or more exotic item.

Take any test of normality: Jarque-Bera, Shapiro-Wilks, even Kolmogorov-Smirnov...

You will almost always strongly reject the normality of the returns. With a test that is tail sensitive, such as Jarque-Bera, rejection is a virtual certainty.

Bottom Line: Asset Returns are not normal.

Asset Returns — The First Stylized Facts:

Fatter Tails — more like a T with 3 to 5 degrees of freedom

Modest Asymmetry — Left tail is fatter than the right tail
Testing the Normality of Asset Returns

Take any decent sized time series of almost any asset — Stock, Bond, Mutual Fund, ETF, or more exotic item.

Bottom Line: Asset Returns are not normal.

Asset Returns - The First Stylized Facts:
- Fatter Tails — more like a T with 3 to 5 degrees of freedom
- Modest Asymmetry — Left tail is fatter than the right tail
Testing the Normality of Asset Returns

- Take any decent sized time series of almost any asset — Stock, Bond, Mutual Fund, ETF, or more exotic item.
- Take any test of normality: Jarque-Bera, Shapiro-Wilks, even Kolmogorov-Smirnov...

Bottom Line: Asset Returns are not normal.

Asset Returns — The First Stylized Facts:
- Fatter Tails — more like a T with 3 to 5 degrees of freedom
- Modest Asymmetry — Left tail is fatter than the right tail

J. Michael Steele
Martingale Markets: Abstracting the Distinguished Asset
Testing the Normality of Asset Returns

- Take any decent sized time series of almost any asset — Stock, Bond, Mutual Fund, ETF, or more exotic item.
- Take any test of normality: Jarque-Bera, Shapiro-Wilks, even Kolmogorov-Smirnov...
- You will almost always strongly reject the normality of the returns. With a test that is tail sensitive, such as Jarque-Bera, rejection is a virtual certainty.
Testing the Normality of Asset Returns

- Take any decent sized time series of almost any asset — Stock, Bond, Mutual Fund, ETF, or more exotic item.
- Take any test of normality: Jarque-Bera, Shapiro-Wilks, even Kolmogorov-Smirnov...
- You will almost always strongly reject the normality of the returns. With a test that is tail sensitive, such as Jarque-Bera, rejection is a virtual certainty.
- **Bottom Line:** Asset Returns are not normal.
Testing the Normality of Asset Returns

- Take any decent sized time series of almost any asset — Stock, Bond, Mutual Fund, ETF, or more exotic item.
- Take any test of normality: Jarque-Bera, Shapiro-Wilks, even Kolmogorov-Smirnov...
- You will almost always strongly reject the normality of the returns. With a test that is tail sensitive, such as Jarque-Bera, rejection is a virtual certainty.
- **Bottom Line:** Asset Returns are not normal.
- Asset Returns — The First Stylized Facts:
Testing the Normality of Asset Returns

- Take any decent sized time series of almost any asset — Stock, Bond, Mutual Fund, ETF, or more exotic item.
- Take any test of normality: Jarque-Bera, Shapiro-Wilks, even Kolmogorov-Smirnov...
- You will almost always strongly reject the normality of the returns. With a test that is tail sensitive, such as Jarque-Bera, rejection is a virtual certainty.
- **Bottom Line:** **Asset Returns are not normal.**
- Asset Returns — The First Stylized Facts:
  - Fatter Tails — more like a T with 3 to 5 degrees of freedom
Testing the Normality of Asset Returns

- Take any decent sized time series of almost any asset — Stock, Bond, Mutual Fund, ETF, or more exotic item.
- Take any test of normality: Jarque-Bera, Shapiro-Wilks, even Kolmogorov-Smirnov...
- You will almost always strongly reject the normality of the returns. With a test that is tail sensitive, such as Jarque-Bera, rejection is a virtual certainty.
- **Bottom Line:** Asset Returns are not normal.
- Asset Returns — The First Stylized Facts:
  - Fatter Tails — more like a T with 3 to 5 degrees of freedom
  - Modest Asymmetry — Left tail is fatter than the right tail
Pondering the Independence of Asset Returns

Take the returns of a common stock and apply a test such as Ljung-Box that measures the distance from white noise. You typically fail to reject the white noise hypothesis. This modestly argues that perhaps the independence assumption of Black-Scholes world is not so bad? Here we come to a strange but creative idea — on a whim, consider the squares of the returns. The tests for linear predictability (ACF tests, LB tests) now show massive predictability — hence massive dependence of the series \( \{ r^2_t \} \).

Second Stylized Fact: Asset returns are not independent. At a minimum their squares show substantial predictability.
Pondering the Independence of Asset Returns

- Take the returns of a common stock and apply a test such as Ljung-Box that measures the distance from white noise.

You typically fail to reject the white noise hypothesis. This modestly argues that perhaps the independence assumption of Black-Scholes world is not so bad?

Here we come to a strange but creative idea —-

- On a whim, consider the squares of the returns.

The tests for linear predictability (ACF tests, LB tests) now show massive predictability — hence massive dependence of the series \( \{r_t^2\} \).

Second Stylized Fact: Asset returns are not independent. At a minimum their squares show substantial predictability.
Pondering the Independence of Asset Returns

- Take the returns of a common stock and apply a test such as Ljung-Box that measures the distance from white noise.
- You typically fail to reject the white noise hypothesis.
Pondering the Independence of Asset Returns

- Take the returns of a common stock and apply a test such as Ljung-Box that measures the distance from white noise.
- You typically fail to reject the white noise hypothesis.
- This modestly argues that perhaps the independence assumption of Black-Scholes world is not so bad?

Second Stylized Fact: Asset returns are not independent. At a minimum their squares show substantial predictability.
Take the returns of a common stock and apply a test such as Ljung-Box that measures the distance from white noise.

You typically fail to reject the white noise hypothesis.

This modestly argues that perhaps the independence assumption of Black-Scholes world is not so bad?

Here we come to a strange but creative idea —-
Pondering the Independence of Asset Returns

- Take the returns of a common stock and apply a test such as Ljung-Box that measures the distance from white noise.
- You typically fail to reject the white noise hypothesis.
- This modestly argues that perhaps the independence assumption of Black-Scholes world is not so bad?
- Here we come to a strange but creative idea —-
  - On a whim, consider the squares of the returns.
Pondering the Independence of Asset Returns

- Take the returns of a common stock and apply a test such as Ljung-Box that measures the distance from white noise.
- You typically fail to reject the white noise hypothesis.
- This modestly argues that perhaps the independence assumption of Black-Scholes world is not so bad?
- Here we come to a strange but creative idea —-
  - On a whim, consider the squares of the returns.
  - The tests for linear predictability (ACF tests, LB tests) now show massive predictability — hence massive dependence of the series \( \{ r_t^2 \} \).
Pondering the Independence of Asset Returns

▶ Take the returns of a common stock and apply a test such as Ljung-Box that measures the distance from white noise.
▶ You typically fail to reject the white noise hypothesis.
▶ This modestly argues that perhaps the independence assumption of Black-Scholes world is not so bad?
▶ Here we come to a strange but creative idea —-
  ▶ On a whim, consider the squares of the returns.
  ▶ The tests for linear predictability (ACF tests, LB tests) now show massive predictability — hence massive dependence of the series \( \{r_t^2\} \).

▶ **Second Stylized Fact:** Asset returns are not independent. At a minimum their squares show substantial predictability
More Stylistic Facts

- High volatility begets high volatility (ARCH effect).
- Large negative shocks tend to produce a greater increase in volatility than positive shocks of comparable size. (Black's "Leverage effect").
- A major portion of individual stocks' movements are explained by the movement of the overall market (CAPM effect).
- Almost ninety percent of a stock's movement can be explained by the market movement and two other factors.
- The change in BMS, a zero cost portfolio of big cap minus small cap stocks (Small Cap Effect).
- The change in HML, a zero cost portfolio of high B/M stocks minus small B/M stocks (Value Effect).
- The stochastic features of asset returns may possess many mysteries, but there are also consistent behaviors found across different nations, across different asset classes, and over many different time periods and time scales.
More Stylistic Facts

- High volatility begets high volatility (ARCH effect)

- Large negative shocks tend to produce a greater increase in volatility than positive shocks of comparable size. (Black’s “Leverage effect”)

- A major portion of individual stocks’ movements are explained by the movement of the overall market (CAPM effect)

- Almost ninety percent of a stock’s movement can be explained by the market movement and two other factors

- The change in BMS, a zero-cost portfolio of big cap minus small cap stocks (Small Cap Effect)

- The change in HML, a zero-cost portfolio of high B/M stocks minus small B/M stocks (Value Effect)

- The stochastic features of asset returns may possess many mysteries, but there are also consistent behaviors found across different nations, across different asset classes, and over many different time periods and time scales.
More Stylistic Facts

- High volatility begets high volatility (ARCH effect)
- Large negative shocks tend to produce a greater increase in volatility than positive shocks of comparable size. (Black’s “Leverage effect”).
More Stylistic Facts

- High volatility begets high volatility (ARCH effect)
- Large negative shocks tend to produce a greater increases in volatility than positive shocks of comparable size. (Black’s “Leverage effect”).
- A major portion of individual stocks movements are explained by the movement of the over all market (CAPM effect)
- The change in BMS, a zero cost portfolio of big cap minus small cap stocks (Small Cap Effect)
- The change in HML, a zero cost portfolio of high B/M stocks minus small B/M stocks (Value Effect)

The stochastic features of asset returns may possess many mysteries, but there are also consistent behaviors that are found across different nations, across different asset classes, and over many different time periods and time scales.

J. Michael Steele
Martingale Markets: Abstracting the Distinguished Asset
More Stylistic Facts

▶ High volatility begets high volatility (ARCH effect)
▶ Large negative shocks tend to produce a greater increases in volatility than positive shocks of comparable size. (Black’s “Leverage effect”).
▶ A major portion of individual stocks movements are explained by the movement of the overall market (CAPM effect)
▶ Almost ninety percent of a stock’s movement can be explained by the market movement and two other factors
More Stylistic Facts

- High volatility begets high volatility (ARCH effect)
- Large negative shocks tend to produce a greater increase in volatility than positive shocks of comparable size. (Black’s “Leverage effect”).
- A major portion of individual stocks movements are explained by the movement of the overall market (CAPM effect)
- Almost ninety percent of a stock’s movement can be explained by the market movement and two other factors
  - The change in BMS, a zero cost portfolio of big cap minus small cap stocks (Small Cap Effect)
More Stylistic Facts

- High volatility begets high volatility (ARCH effect)
- Large negative shocks tend to produce a greater increases in volatility than positive shocks of comparable size. (Black’s “Leverage effect”).
- A major portion of individual stocks movements are explained by the movement of the overall market (CAPM effect)
- Almost ninety percent of a stock’s movement can be explained by the market movement and two other factors
  - The change in BMS, a zero cost portfolio of big cap minus small cap stocks (Small Cap Effect)
  - The change in HML, a zero cost portfolio of high B/M stocks minus small B/M stocks (Value Effect)
More Stylistic Facts

- High volatility begets high volatility (ARCH effect).
- Large negative shocks tend to produce a greater increase in volatility than positive shocks of comparable size. (Black’s “Leverage effect”).
- A major portion of individual stocks movements are explained by the movement of the overall market (CAPM effect).
- Almost ninety percent of a stock’s movement can be explained by the market movement and two other factors:
  - The change in BMS, a zero cost portfolio of big cap minus small cap stocks (Small Cap Effect).
  - The change in HML, a zero cost portfolio of high B/M stocks minus small B/M stocks (Value Effect).
- The stochastic features of asset returns may possess many mysteries, but there are also consistent behaviors that are found across different nations, across different asset classes, and over many different time periods and time scales.
In the Black-Scholes World we assume that the stock price evolves according to
\[ \frac{dS_t}{S_t} = \mu dt + \sigma dB_t \]
This implies that day \( t \) returns \( r_t = \log \left( \frac{S_t}{S_{t-1}} \right) \) are normally distributed and that they are independent.

We're prepared to make assumptions that have weak spots, but we typically expect our models to be approximately realistic at least at some level.

There is much interesting history and sociology in the Black-Scholes trajectory.
In the Black-Scholes World we assume that the stock price evolves according to

\[ dS_t = \mu S_t \, dt + \sigma S_t \, dB_t \]
Sidebar on Black-Scholes World

- In the Black-Scholes World we assume that the stock price evolves according to

\[ dS_t = \mu S_t \, dt + \sigma S_t \, dB_t \]

- This implies that day \( t \) returns \( r_t = \log(S_t/S_{t-1}) \) are normally distributed and that they are independent.
Sidebar on Black-Scholes World

In the Black-Scholes World we assume that the stock price evolves according to

\[ dS_t = \mu S_t \, dt + \sigma S_t \, dB_t \]

This implies that day \( t \) returns \( r_t = \log(S_t/S_{t-1}) \) are normally distributed and that they are independent.

We’re prepared to make assumptions that have weak spots, but we typically expect our models to be approximately realistic at least at some level.
Sidebar on Black-Scholes World

In the Black-Scholes World we assume that the stock price evolves according to

\[ dS_t = \mu S_t \, dt + \sigma S_t \, dB_t \]

This implies that day \( t \) returns \( r_t = \log(S_t/S_{t-1}) \) are normally distributed and that they are independent.

We’re prepared to make assumptions that have weak spots, but we typically expect our models to be approximately realistic at least at some level.

There is much interesting history and sociology in the Black-Scholes trajectory.
Use and Non-Use of Stylized Facts

Suppose we consider a new probabilistic model. We should feel happy when it captures stylized facts—especially critical ones or subtle ones. We should face squarely those facts that are not captured by the model.

News Flash: People are not always forthright in this respect: Essentially all pension funds explicitly or implicitly assume independence of annual returns. They also assume return rates and volatilities are well estimated under the model of IID returns. It is odd that we impute so much "efficiency" to markets where the biggest players are so tangled up in their own pajamas.

J. Michael Steele
Martingale Markets: Abstracting the Distinguished Asset
Use and Non-Use of Stylized Facts

▶ Suppose we consider a new probabilistic model ....
Use and Non-Use of Stylized Facts

- Suppose we consider a new probabilistic model ....
  - We should feel happy when it captures stylized facts — especially critical one or subtle ones.

News Flash: People are not always forthright in this respect:
- Essentially all pension funds explicitly or implicitly assume independence of annual returns.
- They also assume return rates and volatilities are well estimated under the model of IID returns.

It is odd that we impute so much "efficiency" to markets where the biggest players are so tangled up in their own pajamas.

J. Michael Steele
Martingale Markets: Abstracting the Distinguished Asset
Use and Non-Use of Stylized Facts

- Suppose we consider a **new probabilistic model** ....
  - We should feel happy when it captures stylized facts — especially critical one or subtle ones.
  - We should face squarely those facts that are not captured by the model.

---

**News Flash:** People are not always forthright in this respect:

- Essentially all pension funds explicitly or implicitly assume independence of annual returns.
- They also assume return rates and volatilities are well estimated under the model of IID returns.
- It is odd that we impute so much “efficiency” to markets where the biggest players are so tangled up in their own pajamas.
Use and Non-Use of Stylized Facts

- Suppose we consider a **new probabilistic model** ....
  - We should feel happy when it captures stylized facts — especially critical one or subtle ones.
  - We should face squarely those facts that are not captured by the model.
- News Flash: People are not always forthright in this respect:
Use and Non-Use of Stylized Facts

» Suppose we consider a new probabilistic model ....
  » We should feel happy when it captures stylized facts — especially critical one or subtle ones.
  » We should face squarely those facts that are not captured by the model.

» News Flash: People are not always forthright in this respect:
  » Essentially all pension funds explicitly or implicitly assume independence of annual returns.
Use and Non-Use of Stylized Facts

- Suppose we consider a **new probabilistic model** ....
  - We should feel happy when it captures stylized facts — especially critical one or subtle ones.
  - We should face squarely those facts that are not captured by the model.

- **News Flash:** People are not always forthright in this respect:
  - Essentially all pension funds explicitly or implicitly assume independence of annual returns.
  - They also assume return rates and volatilities are well estimated under the model of IID returns.
Use and Non-Use of Stylized Facts

- Suppose we consider a new probabilistic model ....
  - We should feel happy when it captures stylized facts — especially critical one or subtle ones.
  - We should face squarely those facts that are not captured by the model.

- News Flash: People are not always forthright in this respect:
  - Essentially all pension funds explicitly or implicitly assume independence of annual returns.
  - They also assume return rates and volatilities are well estimated under the model of IID returns.

- It is odd that we impute so much “efficiency” to markets where the biggest players are so tangled up in their own pajamas.
What Is the Fundamental Question?

We love the CAPM, but we certainly can’t buy Rubinstein’s assumptions, and we are unhappy with the myriad of CAPM tests. Still, we have some faith. The market asset really is special, by golly. It may be possible to extract this experience from a model that does not violate a horrible list of stylistic facts. We can hunt for this model by leaning hard on generality and abstraction.
What Is the Fundamental Question?

- We love the CAPM, but...
What Is the Fundamental Question?

- We love the CAPM, but
  - we certainly can’t buy Rubinstein’s assumptions,
What Is the Fundamental Question?

- We love the CAPM, but
  - we certainly can’t buy Rubinstein’s assumptions,
  - and we are unhappy with the myriad of CAPM tests.

J. Michael Steele
Martingale Markets: Abstracting the Distinguished Asset
What Is the Fundamental Question?

- We love the CAPM, but
  - we certainly can’t buy Rubinstein’s assumptions,
  - and we are unhappy with the myriad of CAPM tests.
- Still, we have some faith.

J. Michael Steele
Martingale Markets: Abstracting the Distinguished Asset
What Is the Fundamental Question?

- We love the CAPM, but
  - we certainly can’t buy Rubinstein’s assumptions,
  - and we are unhappy with the myriad of CAPM tests.
- Still, we have some faith.
  - The market asset really is special, by golly.
What Is the Fundamental Question?

- We love the CAPM, but
  - we certainly can’t buy Rubinstein’s assumptions,
  - and we are unhappy with the myriad of CAPM tests.
- Still, we have some faith.
  - The market asset really is special, by golly.
  - It may be possible to extract this experience from a model that does not to violate a horrible list of stylistic fact.
What Is the Fundamental Question?

- We love the CAPM, but
  - we certainly can’t buy Rubinstein’s assumptions,
  - and we are unhappy with the myriad of CAPM tests.
- Still, we have some faith.
  - The market asset really is special, by golly.
  - It may be possible to extract this experience from a model that does not to violate a horrible list of stylistic fact.
- We can hunt for this model by leaning hard on generality and abstraction
Martingale Markets, or *MarketGales*

We take as given a collection $T$ of assets (stochastic processes). We assume there is a distinguished asset $V = \{V_t\} \in T$. This could be the market asset, but it need not be.

Given a constant $0 \leq k < \infty$, we say the triple $(T, V, k)$ is a martingale market provided that for each $S = \{S_t\} \in T$ the process $J_t(S, V, k) = S_t - k \int_0^t 1_{V_u} d\langle S, V \rangle_u$, $0 \leq t \leq T$, is an $F_t$ martingale.

Admittedly, this is strange, but bear with me. I’ll at least show it is interesting.
Martingale Markets, or MarketGales

- We take as given a collection $\mathcal{T}$ of assets (stochastic processes).
Martingale Markets, or *MartGales*

- We take as given a collection $\mathcal{I}$ of assets (stochastic processes).
- We assume there is a *distinguished asset* $V = \{V_t\} \in \mathcal{I}$. This could be the *market asset*, but it need not be.
We take as given a collection $\mathcal{T}$ of assets (stochastic processes).

We assume there is a distinguished asset $V = \{V_t\} \in \mathcal{T}$. This could be the market asset, but it need not be.

Given a constant $0 \leq k < \infty$, we say the triple $(\mathcal{T}, V, k)$ is a martingale market provided that for each $S = \{S_t\} \in \mathcal{T}$ the process $J_t(S, V, k)$ defined by

$$J_t(S, V, k) = S_t - k \int_0^t \frac{1}{V_u} d\langle S, V \rangle_u, \quad 0 \leq t \leq T,$$

is an $\mathcal{F}_t$ martingale.
Martingale Markets, or MarketGales

- We take as given a collection $\mathcal{T}$ of assets (stochastic processes).
- We assume there is a distinguished asset $V = \{V_t\} \in \mathcal{T}$. This could be the market asset, but it need not be.
- Given a constant $0 \leq k < \infty$, we say the triple $(\mathcal{T}, V, k)$ is a martingale market provided that for each $S = \{S_t\} \in \mathcal{T}$ the process $J_t(S, V, k)$ defined by

$$J_t(S, V, k) = S_t - k \int_0^t \frac{1}{V_u} d\langle S, V \rangle_u, \quad 0 \leq t \leq T,$$

is an $\mathcal{F}_t$ martingale.

- Admittedly, this is strange, but bear with me. I’ll at least show it is interesting.
MarketGales: Some Natural Properties

Definition Reminder: for each $S_t = \{S_t\}_{t \in T}$ the process $J_t(S, V, k) = S_t - k \int_0^t 1 V_u d\langle S, V \rangle_u$, $0 \leq t \leq T$.

Three Nice Properties

$V$, the distinguished asset, is a submartingale.

$\log V$ is also submartingale.

Uniqueness: If $(T, V)$ and $(T', V')$ are MarketGales, then there is a constant $c$ such that $V_t = cV'_t$ with probability one for all $0 \leq t \leq T$. 
MarketGales: Some Natural Properties

MarketGale (Definition Reminder): for each $S = \{S_t\} \in \mathcal{T}$ the process $J_t(S, V, k)$ is a martingale where

$$J_t(S, V, k) = S_t - k \int_0^t \frac{1}{V_u} d\langle S, V \rangle_u, \quad 0 \leq t \leq T.$$
MarketGales: Some Natural Properties

- **MarketGale (Definition Reminder):** for each $S = \{S_t\} \in \mathcal{T}$, the process $J_t(S, V, k)$ is a martingale where

\[
J_t(S, V, k) = S_t - k \int_0^t \frac{1}{V_u} d \langle S, V \rangle_u, \quad 0 \leq t \leq T.
\]

- **Three Nice Properties**
MarketGales: Some Natural Properties

- MarketGale (Definition Reminder): for each $S = \{S_t\} \in \mathcal{T}$ the process $J_t(S, V, k)$ is a martingale where

  $$J_t(S, V, k) = S_t - k \int_0^t \frac{1}{V_u} d\langle S, V \rangle_u, \quad 0 \leq t \leq T.$$

- Three Nice Properties
  - $V$, the distinguished asset, is a submartingale.
MarketGales: Some Natural Properties

MarketGale (Definition Reminder): for each \( S = \{S_t\} \in \mathcal{T} \)
the process \( J_t(S, V, k) \) is a martingale where

\[
J_t(S, V, k) = S_t - k \int_0^t \frac{1}{V_u} d\langle S, V \rangle_u, \quad 0 \leq t \leq T.
\]

Three Nice Properties

- \( V \), the distinguished asset, is a submartingale.
- \( \log V \) is also submartingale.
MarketGales: Some Natural Properties

MarketGale (Definition Reminder): for each $S = \{S_t\} \in \mathcal{T}$ the process $J_t(S, V, k)$ is a martingale where

$$J_t(S, V, k) = S_t - k \int_0^t \frac{1}{V_u} d\langle S, V \rangle_u, \quad 0 \leq t \leq T.$$ 

Three Nice Properties

- $V$, the distinguished asset, is a submartingale.
- $\log V$ is also submartingale
- Uniqueness: If $(\mathcal{T}, V)$ and $(\mathcal{T}, V')$ are MarketGales, then there is a constant $c$ such that $V_t = cV'_t$ with probability one for all $0 \leq t \leq T$. 

First Interpretation of $k$

Consider the special case $dV_t = \mu_t V_t dt + V_t \sigma_t \cdot dB_t$ and calculate

$$dJ_t(V, V, k) = dV_t - k \frac{1}{V_t} d \langle V, V \rangle_t$$

$$= \left( \mu_t V_t - k V_t \sum_{i=1}^{d} \sigma_t^2(i) \right) dt + V_t \sigma_t \cdot dB_t.$$

For $J_t$ to be a martingale we need to have with probability one that

$$\mu_t = k \sum_{i=1}^{d} \sigma_t^2(i).$$

There are three interesting consequences of this identity:
First Interpretation of $k$

Consider the special case $dV_t = \mu_t V_t dt + V_t \sigma_t \cdot dB_t$ and calculate

$$dJ_t(V, V, k) = dV_t - k \frac{1}{V_t} d \langle V, V \rangle_t$$

$$= \left( \mu_t V_t - k V_t \sum_{i=1}^{d} \sigma_t^2(i) \right) dt + V_t \sigma_t \cdot dB_t.$$

For $J_t$ to be a martingale we need to have with probability one that

$$\mu_t = k \sum_{i=1}^{d} \sigma_t^2(i).$$

There are three interesting consequences of this identity:

- The instantaneous drift is determined by the market risk aversion and the instantaneous volatility,
First Interpretation of $k$

Consider the special case $dV_t = \mu_t V_t dt + V_t \sigma_t \cdot dB_t$ and calculate

$$dJ_t(V, V, k) = dV_t - k \frac{1}{V_t} d\langle V, V \rangle_t$$

$$= \left( \mu_t V_t - k V_t \sum_{i=1}^{d} \sigma_t^2(i) \right) dt + V_t \sigma_t \cdot dB_t.$$

For $J_t$ to be a martingale we need to have with probability one that

$$\mu_t = k \sum_{i=1}^{d} \sigma_t^2(i).$$

There are three interesting consequences of this identity:

- The instantaneous drift is determined by the market risk aversion and the instantaneous volatility,
- Instantaneous drift increases is linear in the risk aversion $k$
First Interpretation of $k$

Consider the special case $dV_t = \mu_t V_t dt + V_t \sigma_t \cdot dB_t$ and calculate

$$dJ_t(V, V, k) = dV_t - k \frac{1}{V_t} d \langle V, V \rangle_t$$

$$= \left( \mu_t V_t - k V_t \sum_{i=1}^{d} \sigma^2_t(i) \right) dt + V_t \sigma_t \cdot dB_t.$$

For $J_t$ to be a martingale we need to have with probability one that

$$\mu_t = k \sum_{i=1}^{d} \sigma^2_t(i).$$

There are three interesting consequences of this identity:

- The instantaneous drift is determined by the market risk aversion and the instantaneous volatility,
- Instantaneous drift increases is linear in the risk aversion $k$
- The instantaneous drift increases as the the volatility increases.
Volatility Discounted Utility: Second View of $k$

For $1 < k < \infty$, the classical isoelastic utility function

$$U_k(w) \equiv \frac{w^{1-k}}{1-k}$$

has Arrow-Pratt relative risk aversion $-wU'_k(w)/U''_k(w) = k$.

**Definition (Volatility Discounted Utility)**

For a martingale market $(\mathcal{T}, V, k)$ and $1 < k < \infty$ the volatility discounted utility, $D(w) \equiv D_{k,V,t}(w)$, is the map from $\mathbb{R}^+$ to $\mathbb{R}$ that is defined by

$$D_{k,V,t}(w) \equiv U_k \left( w \exp\left( -\frac{k}{2} \langle \log V, \log V \rangle_t \right) \right).$$
The Martingale Properties of Volatility Discounted Utilities

Theorem (Discounted Martingale Theorem)

If the triple \((T, V, k)\) is a martingale market, then for each \(S \in T\),

the process \(\{D(S_t) : 0 \leq t \leq T\}\) is a supermartingale and

the process \(\{D(V_t) : 0 \leq t \leq T\}\) is a martingale.
The Martingale Properties of Volatility Discounted Utilities

Theorem (Discounted Martingale Theorem)

If the triple \((T, V, k)\) is a martingale market, then for each \(S \in T\),

the process \(\{D(S_t) : 0 \leq t \leq T\}\) is a supermartingale and

the process \(\{D(V_t) : 0 \leq t \leq T\}\) is a martingale.

This is just what we want: The un-distinguished assets are not so good (in this particular sense)
The Martingale Properties of Volatility Discounted Utilities

Theorem (Discounted Martingale Theorem)

If the triple \((T, V, k)\) is a martingale market, then for each \(S \in T\),

the process \(\{D(S_t) : 0 \leq t \leq T\}\) is a supermartingale and

the process \(\{D(V_t) : 0 \leq t \leq T\}\) is a martingale.

- This is just what we want: The un-distinguished assets are not so good (in this particular sense)
- The distinguished asset holds its own against the ravages of time and risk
Transition: On to Dominated Assets and the Free Snack

There is much more to be said about the newly introduced Martingale Markets. The theory is far from proving its worth, but it has nice properties and it at least makes some non-CAPM steps toward understanding what is special about the market asset (or distinguished asset).

But there are two useful items left to cover:

My favorite argument against "market efficiency"

A Small inefficiency that yields a Free Snack

J. Michael Steele
Martingale Markets: Abstracting the Distinguished Asset
Transition: On to Dominated Assets and the Free Snack

There is much more to be said about the newly introduced *Martingale Markets*

A Small inefficiency that yields a Free Snack
There is much more to be said about the newly introduced *Martingale Markets*

The theory is far from proving its worth, but it has nice properties and it at least makes some non-CAPM steps toward understanding what is special about the market asset (or distinguished asset).
Transition: On to Dominated Assets and the Free Snack

- There is much more to be said about the newly introduced *Martingale Markets*
- The theory is far from proving its worth, but it has nice properties and it at least makes some non-CAPM steps toward understanding what is special about the market asset (or distinguished asset).
- But there are two useful items left to cover:
There is much more to be said about the newly introduced *Martingale Markets.*

The theory is far from proving its worth, but it has nice properties and it at least makes some non-CAPM steps toward understanding what is special about the market asset (or distinguished asset).

But there are two useful items left to cover:

- My favorite argument against “market efficiency”
There is much more to be said about the newly introduced *Martingale Markets*. The theory is far from proving its worth, but it has nice properties and it at least makes some non-CAPM steps toward understanding what is special about the market asset (or distinguished asset).

But there are two useful items left to cover:

- My favorite argument against “market efficiency”
- A Small inefficiency that yields a **Free Snack**
Market Inefficiency: Tales From the Dark Side

It's maddenly difficult to show market inefficiency by exhibiting a “superior strategy” to holding the total market. Ironically, it is easy to go the other direction — many billions are invested in provably inferior strategies.

A New Notion: The Dominated Asset

In theory, dominated assets cannot exist.

Poster Child of Dominated Asset: An S&P500 Index fund with 2.75% expense ratio.

Many large firms exist that offer only dominated assets.

“Favorite” Horror Story: Cornerstone Total Return Fund

Criminal Theme: Managed Distributions
Market Inefficiency: Tales From the Dark Side

- It’s maddenly difficult to show market inefficiency by exhibiting a “superior strategy” to holding the total market.
Market Inefficiency: Tales From the Dark Side

- It’s maddeningly difficult to show market inefficiency by exhibiting a “superior strategy” to holding the total market.
- Ironically, it is easy to go the other direction — many billions are invested in provably inferior strategies.
Market Inefficiency: Tales From the Dark Side

- It’s maddeningly difficult to show market inefficiency by exhibiting a “superior strategy” to holding the total market.
- Ironically, it is easy to go the other direction — many billions are invested in provably inferior strategies.
- A New Notion: The Dominated Asset
Market Inefficiency: Tales From the Dark Side

- It’s maddeningly difficult to show market inefficiency by exhibiting a “superior strategy” to holding the total market.
- Ironically, it is easy to go the other direction — many billions are invested in provably inferior strategies.
- A New Notion: The Dominated Asset
- In theory, dominated assets cannot exist.
Market Inefficiency: Tales From the Dark Side

- It’s maddeningly difficult to show market inefficiency by exhibiting a “superior strategy” to holding the total market.
- Ironically, it is easy to go the other direction — many billions are invested in provably inferior strategies.
- A New Notion: The Dominated Asset
- In theory, dominated assets cannot exist.
- Poster Child of Dominated Asset: An S&P500 Index fund with 2.75% expense ratio.
It’s maddeningly difficult to show market inefficiency by exhibiting a “superior strategy” to holding the total market.

Ironically, it is easy to go the other direction — many billions are invested in provably inferior strategies.

A New Notion: The Dominated Asset

In theory, dominated assets cannot exist.

Poster Child of Dominated Asset: An S&P500 Index fund with 2.75% expense ratio.

Many large firms exist that offer only dominated assets.
It’s maddeningly difficult to show market inefficiency by exhibiting a “superior strategy” to holding the total market. Ironically, it is easy to go the other direction — many billions are invested in provably inferior strategies.

A New Notion: The Dominated Asset

In theory, dominated assets cannot exist.

Poster Child of Dominated Asset: An S&P500 Index fund with 2.75% expense ratio.

Many large firms exist that offer only dominated assets.

“Favorite” Horror Story: Cornerstone Total Return Fund
Market Inefficiency: Tales From the Dark Side

▶ It’s maddeningly difficult to show market inefficiency by exhibiting a “superior strategy” to holding the total market.
▶ Ironically, it is easy to go the other direction — many billions are invested in provably inferior strategies.
▶ A New Notion: The Dominated Asset
▶ In theory, dominated assets cannot exist.
▶ Poster Child of Dominated Asset: An S&P500 Index fund with 2.75% expense ratio.
▶ Many large firms exist that offer only dominated assets.
▶ “Favorite” Horror Story: Cornerstone Total Return Fund
▶ Criminal Theme: Managed Distributions
A Dominated Asset Closer to Home: TIAA-CREF

The CREF Equity Index Fund has a 0.50% expense ratio and Vanguard 500 Index Fund has only a 0.07% expense ratio, so CREF Equity Index has a 43 basis point disadvantage compared to Vanguard.

If even the rosy results of long term history were to prevail, this would be like a one-time fee of 0.43/6 or .078% of your wealth — $78,000 for each million dollars of asset.

If a gloomier future is in store for us where we only get a 3% real return, then this is like giving away $156,000 for each million dollars of current wealth.

There are other ways to do this "arithmetic"

Less inclusive way: Extra Annual Fee of $4,300 for each million. Perhaps "not much" but why not keep it?
A Dominated Asset Closer to Home: TIAA-CREF

- The CREF Equity Index Fund has a 0.50% expense ratio and Vanguard 500 Index Fund has only a 0.07% expense ratio, so CREF Equity Index has a 43 basis point disadvantage compared to Vanguard.

If even the rosy results of long term history were to prevail, this would be like a one-time fee of 0.43/6 or .078% of your wealth — $78,000 for each million dollars of asset.

If a gloomier future is in store for us where we only get a 3% real return, then this is like giving away $156,000 for each million dollars of current wealth.

There are other ways to do this "arithmetic"

- Less inclusive way: Extra Annual Fee of $4,300 for each million. Perhaps "not much" but why not keep it?
A Dominated Asset Closer to Home: TIAA-CREF

- The CREF Equity Index Fund has a 0.50% expense ratio and Vanguard 500 Index Fund has only a 0.07% expense ratio, so CREF Equity Index has a 43 basis point disadvantage compared to Vanguard.

- If even the rosy results of long term history were to prevail, this would be like a one-time fee of 0.43/6 or 0.078% of your wealth — $78,000 for each million dollars of asset.

- There are other ways to do this "arithmetic"
  - Less inclusive way: Extra Annual Fee of $4,300 for each million. Perhaps "not much" but why not keep it?
A Dominated Asset Closer to Home: TIAA-CREF

- The CREF Equity Index Fund has a 0.50% expense ratio and Vanguard 500 Index Fund has only a 0.07% expense ratio, so CREF Equity Index has a 43 basis point disadvantage compared to Vanguard.

- If even the rosy results of long term history were to prevail, this would be like a one-time fee of $0.43/6 or 0.078% of your wealth — $78,000 for each million dollars of asset.

- If a gloomier future is in store for us where we only get a 3% real return, then this is like giving away $156,000 for each million dollars of current wealth.
A Dominated Asset Closer to Home: TIAA-CREF

- The CREF Equity Index Fund has a 0.50% expense ratio and Vanguard 500 Index Fund has only a 0.07% expense ratio, so CREF Equity Index has a 43 basis point disadvantage compared to Vanguard.

- If even the rosy results of long term history were to prevail, this would be like a one-time fee of $0.43/6 or 0.078% of your wealth — $78,000 for each million dollars of asset.

- If a gloomier future is in store for us where we only get a 3% real return, then this is like giving away $156,000 for each million dollars of current wealth.

- There are other ways to do this “arithmetic”
The CREF Equity Index Fund has a 0.50% expense ratio and Vanguard 500 Index Fund has only a 0.07% expense ratio, so CREF Equity Index has a 43 basis point disadvantage compared to Vanguard.

If even the rosy results of long term history were to prevail, this would be like a one-time fee of 0.43/6 or 0.078% of your wealth — $78,000 for each million dollars of asset.

If a gloomier future is in store for us where we only get a 3% real return, then this is like giving away $156,000 for each million dollars of current wealth.

There are other ways to do this “arithmetic”

Less inclusive way: Extra Annual Fee of $4,300 for each million. Perhaps “not much” but why not keep it?
Finally, Your Free Snack: Odd-Lot Preference Game

In most situations the non-institutional investor is at a disadvantage — most notably with short sales or leveraged transactions.

In a few situations, the individual does have an advantage, e.g. there is usually no market impact cost to trades.

There is one situation that is much more concretely juicy: Tender Offers with the “Odd Lot Preference.”

Many times per year there are tender offers for defined quantities of shares (i.e. not “all shares”)

Usually these are over subscribed and those who tender only get the deal on pro-rated fraction of their shares

There is usually (check!) a preference given to odd lot holders

This produces a “game” where the small investor has a concrete advantage over bigger players.

Implementation (Google alerts, Sec.gov, no-fee-for-tenders broker, clock awareness)

Hence Your Free Snack — worth perhaps a $2K-$3K “bonus” year, just a nibble, but still, why not have a snack?

J. Michael Steele
Finally, Your Free Snack: Odd-Lot Preference Game

- In most situations the non-institutional investor is at a disadvantage — most notably with short sales or leveraged transactions.

- There is one situation that is much more concretely juicy: Tender Offers with the "Odd Lot Preference."

- Many times per year there are tender offers for defined quantities of shares (i.e. not "all shares")

- Usually these are oversubscribed and those who tender only get the deal on pro-rated fraction of their shares

- There is usually (check!) a preference given to odd lot holders

- This produces a "game" where the small investor has a concrete advantage over bigger players.

- Implementation (Google alerts, Sec.gov, no-fee-for-tenders broker, clock awareness)

- Hence Your Free Snack — worth perhaps a $2K-$3K "bonus" year, just a nibble, but still, why not have a snack?
Finally, Your Free Snack: Odd-Lot Preference Game

- In most situations the non-institutional investor is at a disadvantage — most notably with short sales or leveraged transactions.
- In a few situations, the individual does have an advantage, e.g. there is usually no market impact cost to trades.
Finally, Your Free Snack: Odd-Lot Preference Game

- In most situations the non-institutional investor is at a disadvantage — most notably with short sales or leveraged transactions.
- In a few situations, the individual does have an advantage, e.g. there is usually no market impact cost to trades.
- There is one situation that is much more concretely juicy: Tender Offers with the “Odd Lot Preference.”

- Implementation (Google alerts, Sec.gov, no-fee-for-tenders broker, clock awareness)
- Hence Your Free Snack — worth perhaps a $2K-$3K “bonus” year, just a nibble, but still, why not have a snack?
Finally, Your Free Snack: Odd-Lot Preference Game

▶ In most situations the non-institutional investor is at a disadvantage — most notably with short sales or leveraged transactions.

▶ In a few situations, the individual does have an advantage, e.g. there is usually no market impact cost to trades.

▶ There is one situation that is much more concretely juicy: Tender Offers with the “Odd Lot Preference.”
  ▶ Many times per year there are tender offers for defined quantities of shares (i.e. not “all shares”)
Finally, Your Free Snack: Odd-Lot Preference Game

▶ In most situations the non-institutional investor is at a disadvantage — most notably with short sales or leveraged transactions.
▶ In a few situations, the individual does have an advantage, e.g. there is usually no market impact cost to trades.
▶ There is one situation that is much more concretely juicy: **Tender Offers with the “Odd Lot Preference.”**
  ▶ Many times per year there are tender offers for defined quantities of shares (i.e. not “all shares”)
  ▶ Usually these are over subscribed and those who tender only get the deal on pro-rated fraction of their shares
Finally, Your Free Snack: Odd-Lot Preference Game

- In most situations the non-institutional investor is at a disadvantage — most notably with short sales or leveraged transactions.
- In a few situations, the individual does have an advantage, e.g. there is usually no market impact cost to trades.
- There is one situation that is much more concretely juicy: **Tender Offers with the “Odd Lot Preference.”**
  - Many times per year there are tender offers for defined quantities of shares (i.e. not “all shares”)
  - Usually these are over subscribed and those who tender only get the deal on pro-rated fraction of their shares
  - There is usually (check!) a preference given to odd lot holders
Finally, Your Free Snack: Odd-Lot Preference Game

- In most situations the non-institutional investor is at a disadvantage — most notably with short sales or leveraged transactions.
- In a few situations, the individual does have an advantage, e.g. there is usually no market impact cost to trades.
- There is one situation that is much more concretely juicy: **Tender Offers with the “Odd Lot Preference.”**
  - Many times per year there are tender offers for defined quantities of shares (i.e. not “all shares”)
  - Usually these are over subscribed and those who tender only get the deal on pro-rated fraction of their shares
  - There is usually (check!) a preference given to odd lot holders
  - This produces a “game” where the small investor has a concrete advantage over bigger players.

Implementation (Google alerts, Sec.gov, no-fee-for-tenders broker, clock awareness)

Hence Your Free Snack — worth perhaps a $2K-$3K “bonus” year, just a nibble, but still, why not have a snack?

J. Michael Steele
Martingale Markets: Abstracting the Distinguished Asset
Finally, Your Free Snack: Odd-Lot Preference Game

- In most situations the non-institutional investor is at a disadvantage — most notably with short sales or leveraged transactions.
- In a few situations, the individual does have an advantage, e.g. there is usually no market impact cost to trades.
- There is one situation that is much more concretely juicy: **Tender Offers with the “Odd Lot Preference.”**
  - Many times per year there are tender offers for defined quantities of shares (i.e. not “all shares”)
  - Usually these are over subscribed and those who tender only get the deal on pro-rated fraction of their shares
  - There is usually (check!) a preference given to odd lot holders
  - This produces a “game” where the small investor has a concrete advantage over bigger players.
- Implementation (Google alerts, Sec.gov, no-fee-for-tenders broker, clock awareness)
Finally, Your Free Snack: Odd-Lot Preference Game

- In most situations the non-institutional investor is at a disadvantage — most notably with short sales or leveraged transactions.
- In a few situations, the individual does have an advantage, e.g. there is usually no market impact cost to trades.
- There is one situation that is much more concretely juicy: Tender Offers with the “Odd Lot Preference.”
  - Many times per year there are tender offers for defined quantities of shares (i.e. not “all shares”)
  - Usually these are over subscribed and those who tender only get the deal on pro-rated fraction of their shares
  - There is usually (check!) a preference given to odd lot holders
  - This produces a “game” where the small investor has a concrete advantage over bigger players.
  - Implementation (Google alerts, Sec.gov, no-fee-for-tenders broker, clock awareness)
  - Hence Your Free Snack — worth perhaps a $2K-$3K “bonus” year, just a nibble, but still, why not have a snack?