Statistics 433: Stochastic Processes and Applications
Homework No. 4

READING. Read Chapter 4, Poisson Processes, in Cinlar. It overlaps with Ross's Chapter 2, but it goes a bit further; it's also more polished in some ways. Be attentive to the differences in the two treatments. In particular, in his Lemma 1.5 can be used to drop our assumption that $P(N(h) \geq 2) = o(h)$ in our proof that the qualitative definition of the Poisson process implies the quantitative definition of the Poisson process. This is not huge deal, but it has philosophical content and it is worth some contemplation.

PROBLEM 1. Consider the M/G/∞ queue (Ross page 19). As before, assume that the service time S_i of the i'th customer has the same distribution G for each i. We assume also that the customers arrive according to a Poisson process $N(t)$ with arrival rate λ, and the service random variables $\{S_1, S_2, ...\}$ are independent of each other and independent of the arrival process. Consider the random variable

$$M(t) = \max\{S_i : i \leq N(t)\}.$$

That is, $M(t)$ is the maximum of the service times of each of the customers who have arrived by time t. Here if $N(t) = 0$ then $M(t) = 0$, and if $N(t) = 3$ then we have $M(t) = \max(S_1, S_2, S_3)$. Some of these customers may complete their service after time t; that's perfectly OK.

Calculate distribution function $P(M(t) \leq x)$. Your answer should be a formula that involves the exponential function, G, t, and λ. Check that your answer is reasonable by considering the following questions: Does your candidate have the right value at $x = 0$? Does it have the right value as $x \to \infty$? Can you see that your answer is a monotone increasing function of x?

PROBLEM 2. Do Cinlar's problem (8.7) page 102-103. . [PS: An earlier version of this problem mentioned the conditional variance formula. That comment is relevant to Cinlar's problem (8.3) or (8.9) which I decided not to assign this time. It will appear next time, after we have discussed the conditional variance formula in class. Apologies for any confusion... actually confusions. It was also a cut and past error that led to the statement "Note that Cinlar uses the short hand $d\phi(t)$ for what one would also write in a traditional calculus class as $\phi'(x)\,dx$" This is relevant to problem (8.3) which was not assigned — but should have been.]

PROBLEM 3. Do Cinlar's problem (8.8) page 103. The type setting of the expression in part (b) is not perfectly clear. What you are to compute is $E[\alpha^Z]$, where α is a real number in the interval $[0, 1]$ and Z is the random variable defined in part (a).