

# ARCH modeling in finance\*

## A review of the theory and empirical evidence

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Although volatility clustering has a long history as a salient empirical regularity characterizing high-frequency speculative prices, it was not until recently that applied researchers in finance have recognized the importance of explicitly modeling time-varying second-order moments. Instrumental in most of these empirical studies has been the Autoregressive Conditional Heteroskedasticity (ARCH) model introduced by Engle (1982). This paper contains an overview of some of the developments in the formulation of ARCH models and a survey of the numerous empirical applications using financial data. Several suggestions for future research, including the implementation and tests of competing asset pricing theories, market microstructure models, information transmission mechanisms, dynamic hedging strategies, and the pricing of derivative assets, are also discussed.

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## 1. Introduction

Uncertainty is central to much of modern finance theory. According to most asset pricing theories the risk premium is determined by the covariance between the future return on the asset and one or more benchmark portfolios; e.g., the market portfolio or the growth rate in consumption. In option pricing the uncertainty associated with the future price of the underlying asset is the most important determinant in the pricing function. The construction of hedge portfolios is another example where the conditional future variances and covariances among the different assets involved play an important role.

While it has been recognized for quite some time that the uncertainty of speculative prices, as measured by the variances and covariances, are changing through time [see, e.g., Mandelbrot (1963) and Fama (1965)], it was not until recently that applied researchers in financial and monetary economics have started explicitly modeling time variation in second- or higher-order moments. One of the most prominent tools that has emerged for characterizing such changing variances is the Autoregressive Conditional Heteroskedasticity (ARCH) model of Engle (1982) and its various extensions. Since the introduction of the ARCH model several hundred research papers applying this modeling strategy to financial time series data have already appeared. In this paper we survey those contributions that we consider to be the most important and promising in the formulation of ARCH-type models and their applications in the modeling of speculative prices. Several interesting topics in empirical finance awaiting future research are also discussed.

The plan of this paper is as follows. We begin in section 2 with a brief overview of some of the important theoretical developments in the parameterization and implementation of ARCH-type models, and continue in section 3 with applications of the ARCH methodology to stock return data. Sections 4 and 5 cover the modeling of interest rates and foreign exchange rates, respectively. A detailed bibliography is given at the end of the paper.

## 2. ARCH

Following the seminal paper by Engle (1982) we shall refer to all discrete time stochastic processes  $\{\varepsilon_t\}$  of the form

$$\varepsilon_t = z_t \sigma_t, \quad (1)$$

$$z_t \text{ i.i.d., } E(z_t) = 0, \quad \text{var}(z_t) = 1, \quad (2)$$

with  $\sigma_t$  a time-varying, positive, and measurable function of the time  $t - 1$  information set, as an ARCH model. For now  $\varepsilon_t$  is assumed to be a

univariate process, but extensions to multivariate settings are straightforward as discussed below. By definition  $\varepsilon_t$  is serially uncorrelated with mean zero, but the conditional variance of  $\varepsilon_t$  equals  $\sigma_t^2$ , which may be changing through time. In most applications  $\varepsilon_t$  will correspond to the innovation in the mean for some other stochastic process, say  $\{y_t\}$ , where

$$y_t = g(x_{t-1}; b) + \varepsilon_t, \quad (3)$$

and  $g(x_{t-1}; b)$  denotes a function of  $x_{t-1}$  and the parameter vector  $b$ , where  $x_{t-1}$  is in the time  $t-1$  information set. To simplify the exposition, in most of the discussion below we shall assume that  $\varepsilon_t$  is itself observable.

Let  $f(z_t)$  denote the density function for  $z_t$ , and let  $\theta$  be the vector of all the unknown parameters in the model. By the prediction error decomposition, the log-likelihood function for the sample  $\varepsilon_T, \varepsilon_{T-1}, \dots, \varepsilon_1$  becomes, apart from initial conditions,<sup>1</sup>

$$L(\theta) = \sum_{t=1}^T [\log f(\varepsilon_t \sigma_t^{-1}) - \log \sigma_t]. \quad (4)$$

The second term in the summation is a Jacobian term arising from the transformation from  $z_t$  to  $\varepsilon_t$ . Note that (4) also defines the sample log-likelihood for  $y_T, y_{T-1}, \dots, y_1$  as given by (3). Given a parametric representation for  $f(z_t)$ , maximum likelihood estimates for the parameters of interest can be computed directly from (4) by a number of different numerical optimization techniques.

The setup in eqs. (1) and (2) is extremely general and allows for a wide variety of models. At the same time, the economic theory explaining temporal variation in conditional variances is very limited. Consequently, in the remainder of this section we shall concentrate on some of the more successful time series techniques that have been developed for modeling  $\sigma_t^2$ . These models for the temporal dependence in conditional second moments bear much resemblance to the time series techniques for conditional first moments popularized in the early seventies. Just as the integration of time series techniques for the conditional mean into structural econometric model building has led to a much deeper and richer understanding of the underlying dynamics, similar results have already started to emerge in the modeling of conditional variances and covariances.

<sup>1</sup>Throughout this paper, the dependence of  $\varepsilon_t$  and  $\sigma_t$  on the parameter vector  $\theta$  are suppressed for notational convenience.

### 2.1. The linear ARCH( $q$ ) model

As Engle (1982) suggests in his seminal paper, one possible parameterization for  $\sigma_t^2$  is to express  $\sigma_t^2$  as a linear function of past squared values of the process,

$$\sigma_t^2 = \omega + \sum_{i=1}^q \alpha_i \varepsilon_{t-i}^2 = \omega + \alpha(L) \varepsilon_t^2, \quad (5)$$

where  $\omega > 0$  and  $\alpha_i \geq 0$ , and  $L$  denotes the lag operator. This model is known as the linear ARCH( $q$ ) model. With financial data it captures the tendency for volatility clustering, i.e., for large (small) price changes to be followed by other large (small) price changes, but of unpredictable sign. In order to reduce the number of parameters and ensure a monotonic declining effect of more distant shocks, an ad hoc linearly declining lag structure was often imposed in many of the earlier applications of the model; i.e.,  $\alpha_i = \alpha(q+1-i)/(q+1)$  as in Engle (1982, 1983).

For  $z_t$  normally distributed, the conditional density entering the likelihood function in (4) takes the form

$$\log f(\varepsilon_t \sigma_t^{-1}) = -0.5 \log 2\pi - 0.5 \varepsilon_t^2 \sigma_t^{-2}. \quad (6)$$

Maximum likelihood (ML) based inference procedures for the ARCH class of models under this distributional assumption are discussed in Engle (1982) and Pantula (1985). Although the likelihood function is highly nonlinear in the parameters, a simple scoring algorithm is available for the linear ARCH( $q$ ) model defined in (5). Furthermore, a Lagrange Multiplier (LM) test for  $\alpha_1 = \dots = \alpha_q = 0$  can be calculated as  $TR^2$  from the regression of  $\varepsilon_t^2$  on  $\varepsilon_{t-1}^2, \dots, \varepsilon_{t-q}^2$ , where  $T$  denotes the sample size. This same test is generally valid using consistently estimated residuals from the model given in (3). An alternative but asymptotically equivalent testing procedure is to subject  $\varepsilon_t^2$  to standard tests for serial correlation based on the autocorrelation structure, including conventional portmanteau tests as in Ljung and Box (1978). In addition, Gregory (1989) suggests a nonparametric test for ARCH( $q$ ) derived from a finite state Markov chain approximation, while Robinson (1991) presents an LM test for very general serially dependent heteroskedasticity. The small sample performance of some of these estimators and test statistics have been analyzed by Engle, Hendry, and Trumble (1985), Diebold and Pauly (1989), Bollerslev and Wooldridge (1991), and Gregory (1989). Interestingly, the well-known small sample downward bias for the parameter estimates in autoregressive models for the mean carries over to the estimates for  $\alpha_1, \dots, \alpha_q$  also.<sup>2</sup>

<sup>2</sup>It is also worth noting that in the presence of ARCH( $q$ ) effects, standard tests for serial correlation in the mean will lead to overrejections; see Weiss (1984), Taylor (1984), Milhøj (1985), Diebold (1987), and Domowitz and Hakkio (1987) for further discussion.

As an alternative to ML estimation, ARCH-type models can also be estimated directly with Generalized Method of Moments (GMM). This was suggested and implemented by Mark (1988), Bodurtha and Mark (1991), Glosten, Jagannathan, and Runkle (1991), Simon (1989), and Rich, Raymond, and Butler (1990a, b), and in a closely related context by Harvey (1989) and Ferson (1989). A comparison of the efficiency of exact ML, Quasi Maximum Likelihood (QML), and GMM estimates using different instrument sets would be interesting. Bayesian inference procedures within the ARCH class of models are developed in a series of papers by Geweke (1988, 1989a, b), who uses Monte Carlo methods to determine the exact posterior distributions.

An observationally equivalent representation for the model in (1), (2), and (5) is given by the time-varying parameter MA( $q$ ) model,

$$\varepsilon_t = w_t + \sum_{i=1}^q a_{ti} \varepsilon_{t-i},$$

where  $w_t, a_{t1}, \dots, a_{tq}$  are i.i.d. with mean zero and variance  $\omega, \alpha_1, \dots, \alpha_q$ , respectively. This relationship between the time-varying parameter class of models and the linear ARCH( $q$ ) model has been further studied by Tsay (1987), Bera and Lee (1989, 1991), Kim and Nelson (1989), Wolff (1989), Cheung and Pauly (1990), and Bera, Higgins, and Lee (1991). Similarly, in Weiss (1986b) and Higgins and Bera (1989a) comparisons to the bilinear time series class of models are considered.

## 2.2. The linear GARCH( $p, q$ ) model

In many of the applications with the linear ARCH( $q$ ) model a long lag length  $q$  is called for. An alternative and more flexible lag structure is often provided by the Generalized ARCH, or GARCH( $p, q$ ), model in Bollerslev (1986),<sup>3</sup>

$$\sigma_t^2 = \omega + \sum_{i=1}^q \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^p \beta_i \sigma_{t-i}^2 = \omega + \alpha(L) \varepsilon_t^2 + \beta(L) \sigma_t^2. \quad (7)$$

To ensure a well-defined process all the parameters in the infinite-order AR representation  $\sigma_t^2 = \phi(L) \varepsilon_t^2 = (1 - \beta(L))^{-1} \alpha(L) \varepsilon_t^2$  must be nonnegative, where it is assumed that the roots of the polynomial  $\beta(\lambda) = 1$  lie outside the unit circle; see Nelson and Cao (1991) and Drost and Nijman (1991). For a GARCH(1, 1) process this amounts to ensuring that both  $\alpha_1$  and  $\beta_1$  are nonnegative. It follows also that  $\varepsilon_t$  is covariance stationary if and only if

<sup>3</sup>The simple GARCH(1, 1) model was independently suggested by Taylor (1986).

$\alpha(1) + \beta(1) < 1$ .<sup>4</sup> Of course, in that situation the GARCH( $p, q$ ) model corresponds exactly to an infinite-order linear ARCH model with geometrically declining parameters.

An appealing feature of the GARCH( $p, q$ ) model concerns the time series dependence in  $\varepsilon_t^2$ . Rearranging terms, (7) is readily interpreted as an ARMA model for  $\varepsilon_t^2$  with autoregressive parameters  $\alpha(L) + \beta(L)$ , moving average parameters  $-\beta(L)$ , and serially uncorrelated innovation sequence  $\{\varepsilon_t^2 - \sigma_t^2\}$ . Following Bollerslev (1988), this idea can be used in the identification of the orders  $p$  and  $q$ , although in most applications  $p = q = 1$  is found to suffice.<sup>5</sup>

Much of modern finance theory is cast in terms of continuous time stochastic differential equations, while virtually all financial time series are available at discrete time intervals only. This apparent gap between the empirically motivated ARCH models and the underlying economic theory is the focus of Nelson (1990b), who shows that the discrete time GARCH(1, 1) model converges to a continuous time diffusion model as the sampling interval gets arbitrarily small.<sup>6</sup> Along similar lines, Nelson (1992) shows that if the true model is a diffusion model with no jumps, then the discrete time variances are consistently estimated by a weighted average of past residuals as in the GARCH(1, 1) formulation. Another possible reason for the success of the GARCH( $p, q$ ) models in estimating conditional variances is discussed in Brock, Hsieh, and LeBaron (1991). They show that if  $\varepsilon_t^2$  is linear in the sense of Priestley (1981), the GARCH( $p, q$ ) representation may be seen as a parsimonious approximation to the possibly infinite Wold representation for  $\varepsilon_t^2$ .

While aggregation in conventional ARMA models for the conditional mean is straightforward, temporal aggregation within the ARCH class of models is not obvious. However, in an insightful recent paper, Drost and Nijman (1991) show that the class of GARCH( $p, q$ ) models is closed under temporal aggregation, appropriately defined in terms of best linear projections. Also, Diebold (1986b, 1988), using a standard central limit theorem type argument, shows convergence towards normality of a martingale process with ARCH errors under temporal aggregation.

### 2.3. Nonnormal conditional densities

At the same time that high-frequency financial data exhibit volatility clustering, it is also widely recognized that the unconditional price or return

<sup>4</sup>In an interesting recent paper, Hansen (1990) derives sufficient conditions for near epoch dependence and the application of standard asymptotic theory in a GARCH(1, 1) model.

<sup>5</sup>As pointed out in Milhøj (1990) within the context of the ARCH(1) model, the asymptotic standard error for the autocorrelations and the partial autocorrelations for  $\varepsilon_t^2$  exceeds  $1/\sqrt{T}$  in the presence of ARCH, thus leading to potentially lower power of such tests.

<sup>6</sup>See also the comparison in Taylor (1990a) of the statistical properties of the GARCH(1, 1) and autoregressive random variable (ARV) models motivated by diffusion formulations.

distributions tend to have fatter tails than the normal distribution; for some of the earliest evidence see Mandelbrot (1963) and Fama (1965). Although the unconditional distribution for  $\varepsilon_t$  in the GARCH( $p, q$ ) model with conditional normal errors as given by (1), (2), (6), and (7) have fatter tails than the normal distribution [see Milhøj (1985) and Bollerslev (1986)], for many financial time series it does not adequately account for the leptokurtosis. That is, the standardized residuals from the estimated models,  $\hat{z}_t = \hat{\varepsilon}_t \hat{\sigma}_t^{-1}$ , often appear to be leptokurtic.<sup>7</sup>

Following White (1982), asymptotic standard errors for the parameters in the conditional mean and variance functions that are robust to departures from normality have been derived by Weiss (1984, 1986a). Bollerslev and Wooldridge (1991) present a consistent estimator for this robust variance-covariance matrix in an ARCH framework that requires only first derivatives, together with an illustration of the small sample performance of the estimator and the properties of the robust  $TR^2$  Lagrange Multiplier tests in Wooldridge (1988, 1990). It is found that the conventional standard errors based on the outer product of the quasi-gradient obtained under the assumption of conditional normality tend to understate the true standard errors for the parameters in the conditional variance equation when conditional leptokurtosis is present. These ideas are also illustrated empirically in Baillie and Bollerslev (1991).<sup>8</sup>

While the QML based inference procedures are straightforward to implement, fully efficient maximum likelihood estimates may be preferred in some situations. In addition to the potential gains in efficiency, the exact form of the error distribution also plays an important role in several important applications of the ARCH model, such as option pricing and the construction of optimal forecast error intervals; see Engle and Mustafa (1992) and Baillie and Bollerslev (1992). Bollerslev (1987) suggests using the standardized Student- $t$  distribution with the degrees of freedom being estimated.<sup>9</sup> Other parametric densities that have been considered in the estimation of ARCH models include the normal-Poisson mixture distribution in Jorion (1988), the power exponential distribution in Baillie and Bollerslev (1989), the normal-lognormal mixture distribution in Hsieh (1989a), and the generalized exponential distribution in Nelson (1990c). In a related context, McCulloch (1985) suggests the use of an infinite variance leptokurtic stable Paretian

<sup>7</sup>It follows from Jensen's inequality that with a correctly specified conditional variance, the excess kurtosis in  $\varepsilon_t \sigma_t^{-1}$  cannot exceed the excess kurtosis in  $\varepsilon_t$ ; see Hsieh (1989a).

<sup>8</sup>At the same time, abstracting from any inference, Nelson (1990d) has shown that the normal quasi-likelihood increases with more precise volatility estimates (appropriately defined), while this is not generally true for nonnormal likelihood functions.

<sup>9</sup>In the continuous time conditionally normal GARCH(1, 1) diffusion approximation discussed in Nelson (1990b), the innovations observed over short time intervals are approximately  $t$ -distributed.

distribution in the maximum likelihood estimation of the so-called Adaptive Conditional Heteroskedasticity, or ACH, model.

As an alternative to maximum likelihood, a semiparametric density estimation technique could be used in approximating  $f(z_t)$ . Following Gallant and Nychka (1987), in Gallant and Tauchen (1989), Gallant, Hsieh, and Tauchen (1989), and Gallant, Rossi, and Tauchen (1990),  $f(z_t)$  is replaced by a polynomial expansion, whereas Engle and Gonzalez-Rivera (1991) suggest a density estimator originally developed by Tapia and Thompson (1978).<sup>10</sup> By avoiding any specific distributional assumption, semiparametric density estimation gives an added flexibility in the specification. Of course, compared to full information maximum likelihood with a correctly specified density, the semiparametric approach invariably involves a loss in asymptotic efficiency. However, with markedly skewed distributions the efficiency of the semiparametric estimator compares favorably with the QML estimates obtained under the assumption of conditional normality; see Engle and Gonzalez-Rivera (1991).

#### 2.4. Nonlinear and nonparametric ARCH

In the GARCH( $p, q$ ) model (7) the variance only depends on the magnitude and not the sign of  $\varepsilon_t$ . As discussed in section 3.3 below, this is somewhat at odds with the empirical behavior of stock market prices where leverage effects may be present. In the Exponential GARCH( $p, q$ ), or EGARCH( $p, q$ ), model introduced by Nelson (1990c),  $\sigma_t^2$  is an asymmetric function of past  $\varepsilon_t$ 's as defined by (1), (2), and

$$\log \sigma_t^2 = \omega + \sum_{i=1}^q \alpha_i (\phi z_{t-i} + \gamma [|z_{t-i}| - E|z_{t-i}|]) + \sum_{i=1}^p \beta_i \log \sigma_{t-i}^2. \quad (8)$$

Unlike the linear GARCH( $p, q$ ) model in (7), there are no restrictions on the parameters  $\alpha_i$  and  $\beta_i$  to ensure nonnegativity of the conditional variances. Thus, the representation in (8) resembles an unrestricted ARMA( $p, q$ ) model for  $\log \sigma_t^2$ . If  $\alpha_i \phi < 0$ , the variance tends to rise (fall) when  $\varepsilon_{t-i}$  is negative (positive) in accordance with the empirical evidence for stock returns discussed below. Assuming  $z_t$  is i.i.d. normal, it follows that  $\varepsilon_t$  is covariance stationary provided all the roots of the autoregressive polynomial  $\beta(\lambda) = 1$  lie outside the unit circle. The EGARCH model is closely related to

<sup>10</sup>The fact that the information matrix is not block diagonal between the 'density parameters' and the ARCH parameters complicates the adaptive estimation of the ARCH parameters, as suggested by Engle and Gonzalez-Rivera (1991).

the logarithmic parameterization discussed by Geweke (1986) and Pantula (1986) and the Multiplicative ARCH model suggested by Milhøj (1987b, c),

$$\log \sigma_t^2 = \omega + \sum_{i=1}^q \alpha_i \log z_{t-i}^2 + \sum_{i=1}^p \beta_i (\log z_{t-i}^2 - \log \sigma_{t-i}^2).$$

Many other alternative parametric ARCH formulations have been considered in the literature, including power transformations of  $\varepsilon_t^2$  as in the Nonlinear ARCH model of Higgins and Bera (1989b) and Bera and Higgins (1991) and a threshold ARCH model as in Zakoian (1990); see also Engle and Bollerslev (1986). In the threshold model, the  $\sigma_t^2$  is a linear piecewise function, thereby allowing different reactions of volatility to different signs and magnitudes of the shocks. A related model, based on a Markov chain approach, is proposed by Gouriéroux and Monfort (1992). Also, Harvey, Ruiz, and Sentana (1992) have recently proposed an unobserved components time series model in which ARCH disturbances are placed on both the state and updating equations. A more formal comparison of these many alternative formulations would be informative.

Instead of relying on a parametric representation for  $\sigma_t^2$ , a nonparametric estimation technique could be used in approximating the conditional variance. Following Robinson (1987a, b), several authors, including Pagan and Ullah (1988), Robinson (1988), Whistler (1988), Pagan and Hong (1990), and Sentana and Wadhvani (1989), have advocated kernel methods in which  $\sigma_t^2$  is estimated as a weighted average of  $\varepsilon_t^2$ ,  $t = 1, 2, \dots, T$ , such that  $\varepsilon_t$ 's for which the conditioning set (defined in terms of lagged information) 'close' to that of  $\varepsilon_t$  receives the highest weight.<sup>11</sup> Several different weighting schemes are possible, but the most frequent in the estimation of ARCH models have been Gaussian kernels. Very little is known about the infinite sample properties of these techniques in the present context.

An alternative nonparametric estimation strategy involves approximating the unknown conditional variance function by a series expansion where the number of terms in the expansion is an increasing function of the sample size. For instance, in the Flexible Fourier Form introduced by Gallant (1981),  $\sigma_t^2$  is approximated by a function of polynomial and trigonometric terms in lagged values of  $\varepsilon_t$ . In the seminonparametric approach in Gallant and Nychka (1987) the basic idea is to multiply the normal density in (6) by a polynomial expansion. For increasing orders of this expansion, under weak conditions the maximum likelihood estimates found using this method have the same classical properties as nonparametric estimates. In applications of this idea very-low-order polynomial approximations have typically been used;

<sup>11</sup>To guard against the influence of outliers, the 'leave-one-out' estimator is often adopted; i.e.,  $\tau \neq t$ .

see, e.g., Pagan and Hong (1988), Gallant and Tauchen (1989), and Gallant, Hsieh, and Tauchen (1989).

A good illustration and comparison of some of the different parametric and nonparametric methods discussed in this subsection is given in Pagan and Schwert (1990).

### 2.5. ARCH-in-Mean models

In the ARCH-in-Mean, or ARCH-M, model introduced by Engle, Lilien, and Robins (1987), the conditional mean is an explicit function of the conditional variance of the process, as given by (1), (2), and

$$y_t = g(x_{t-1}, \sigma_t^2; b) + \varepsilon_t. \quad (9)$$

In this model, an increase in the conditional variance will be associated with an increase or a decrease in the conditional mean of  $y_t$  depending on the sign of the partial derivative of  $g(x_{t-1}, \sigma_t^2; b)$  with respect to  $\sigma_t^2$ . Many theories in finance involve an explicit tradeoff between the risk and the expected return. The ARCH-M model is ideally suited to handling such questions in a time series context where the conditional variance may be time-varying. The most common choices for the functional form of  $g(x_{t-1}, \sigma_t^2; b)$  have involved linear or logarithmic functions of  $\sigma_t^2$  or  $\sigma_t$ .

Formally, estimation of the ARCH-M model poses no added difficulties. However, in the absence of ARCH-M effects, the information matrix obtained under the auxiliary assumption of conditional normality is block diagonal between the parameters in the conditional mean and variance functions of the model. This is no longer true for the ARCH-M model. Thus, unlike the linear GARCH model in (7) where consistent estimates of the parameters in the function  $g(x_{t-1}; b)$  can be obtained even in the presence of misspecification in  $\sigma_t^2$ , consistent estimation in the ARCH-M model requires that the full model be correctly specified. This parallels the results for asymmetric variance formulations such as the EGARCH model in (8), where correct specification of the full model is generally required in order to guarantee consistency; see Pagan and Sabau (1987a) and Pagan and Hong (1990). Diagnostic tests for the variance specification therefore become very important before interpretations are made about the parameter estimates. The consistency tests outlined in Pagan and Sabau (1987b) form the basis for one such set of diagnostics.

### 2.6. Persistence in variance

A common finding in much of the empirical literature using high-frequency financial data concerns the apparent persistence implied by the estimates for

the conditional variance functions. In the linear GARCH( $p, q$ ) model in (7) this is manifested by the presence of an approximate unit root in the autoregressive polynomial; i.e.,  $\alpha_1 + \dots + \alpha_q + \beta_1 + \dots + \beta_p = 1$ . Engle and Bollerslev (1986) refer to this class of models as Integrated in variance, or IGARCH.<sup>12</sup> As in the martingale model for conditional means, current information remains important for forecasts of the conditional variance for all horizons.<sup>13</sup> To illustrate, in the simple IGARCH(1, 1) model with  $\alpha_1 + \beta_1 = 1$ , the minimum mean square error forecast for the conditional variance  $s$  steps ahead is equal to  $\omega(s - 1) + \sigma_{t+1}^2$ .<sup>14</sup>

Consequently, the unconditional variance for the IGARCH(1, 1) and the general IGARCH( $p, q$ ) model does not exist. The idea of an infinite unconditional variance distribution in characterizing financial data is not new to the IGARCH class of models, however. Mandelbrot (1963) and Fama (1965) both suggest the stable Paretian class of distributions with characteristic exponent less than two as providing a good description of the distributional properties of speculative prices.

While the IGARCH class of models bears much resemblance to the well-known ARIMA class of models for conditional first moments in terms of the optimal forecasts of the process, the analogy is far from complete. As shown in Nelson (1990a) and Bougerol and Picard (1992), the IGARCH model is strictly stationary and ergodic, though not covariance stationary. Asymptotic theory for ARCH models is notoriously difficult. However, in an important paper, Lumsdaine (1991) shows that standard asymptotically based inference procedures are generally valid even in the presence of IGARCH effects, though the Monte Carlo evidence presented in Hong (1988) suggests that the sample sizes must be quite large for the asymptotic distributions to provide good approximations.

Whereas many financial time series may exhibit a high degree of persistence in the variance of their univariate time series representations, this persistence is likely to be common among different series, so that certain linear combinations of the variables show no persistence. In that situation the variables are defined to be co-persistent in variance. This has important implications for the construction of optimal long-term forecasts for the conditional variances and covariances which are essential in many asset pricing relationships; see Bollerslev and Engle (1990) for further discussion along these lines.

<sup>12</sup>One possible explanation for the empirical IGARCH behavior is provided by the diffusion approximations in Nelson (1990b). In the diffusion limit for the GARCH(1, 1) model,  $\alpha_1 + \beta_1$  converges to one as the sampling frequency diminishes.

<sup>13</sup>An unobserved component alternative to the IGARCH model has recently been proposed by Shephard (1990).

<sup>14</sup>The IGARCH(1, 1) model with  $\omega = 0$  is closely related to the ACH formulation in terms of absolute errors proposed by McCulloch (1985).

## 2.7. Multivariate ARCH

The models discussed in the preceding sections have all been univariate. However, many issues in asset pricing and portfolio allocation decisions can only be meaningfully analyzed in a multivariate context. Thus, now let  $\{\varepsilon_t\}$  denote an  $N \times 1$  vector stochastic process. Then any process that permits the representation

$$\varepsilon_t = z_t \Omega_t^{1/2}, \quad (10)$$

$$z_t \text{ i.i.d.}, \quad E(z_t) = 0, \quad \text{var}(z_t) = I, \quad (11)$$

where the time-varying  $N \times N$  covariance matrix  $\Omega_t$  is positive definite and measurable with respect to the time  $t-1$  information set, is naturally referred to as a multivariate ARCH model. Inference in the multivariate ARCH model proceeds as for the univariate model in (1) and (2) discussed above.<sup>15</sup>

The general multivariate definition in (10) and (11) opens up a wide variety of possible representations, but only a few parameterizations have been found particularly useful. In the multivariate linear ARCH( $q$ ) model in Kraft and Engle (1983),  $\Omega_t$  is given by a linear function of the contemporaneous cross-products in the past squared errors; i.e.,  $\text{vech}(\varepsilon_{t-1}\varepsilon'_{t-1}), \dots, \text{vech}(\varepsilon_{t-q}\varepsilon'_{t-q})$  where  $\text{vech}(\cdot)$  denotes the operator that stacks the lower portion of an  $N \times N$  matrix as an  $(N(N+1)/2) \times 1$  vector. This model was subsequently generalized to the multivariate linear GARCH( $p, q$ ) model in Bollerslev, Engle, and Wooldridge (1988),

$$\text{vech}(\Omega_t) = W + \sum_{i=1}^q A_i \text{vech}(\varepsilon_{t-i}\varepsilon'_{t-i}) + \sum_{i=1}^p B_i \text{vech}(\Omega_{t-i}). \quad (12)$$

Here  $W$  denotes an  $(N(N+1)/2) \times 1$  vector, and  $A_i$  and  $B_i$  are  $(N(N+1)/2) \times (N(N+1)/2)$  matrices. Several properties of this model, including sufficient conditions for this parameterization to ensure that  $\Omega_t$  are positive definite, have been derived in Baba, Engle, Kraft, and Kroner (1991). The number of unique parameters in (12) equals  $\frac{1}{2}N(N+1)[1 + N(N+1)(p+q)/2]$ , so in practice for moderately sized  $N$  some simplifying assumptions must be imposed. For instance, in the diagonal GARCH( $p, q$ ) model employed by Bollerslev, Engle, and Wooldridge (1988), the  $A_i$  and  $B_i$  matrices are all taken to be diagonal. A simple parameterization for the diagonal GARCH( $p, q$ ) model guaranteed to be positive definite is given in Attanasio and Edey (1988) and in Baba, Engle, Kraft, and Kroner (1991).

<sup>15</sup>In terms of the log-likelihood function in (4), the Jacobian term  $-\log|\Omega_t^{1/2}|$  replaces  $-\log \sigma_t$  and  $\varepsilon_t \Omega_t^{-1/2}$  replaces  $\varepsilon_t \sigma_t^{-1}$ .

Motivated by the commonality in volatility clustering across different assets, Diebold and Nerlove (1989) have proposed a multivariate latent factor ARCH model. Identification in the context of this model is discussed in Sentana (1990), but the presence of an unobserved latent variable in the covariance matrix renders exact inference in the latent factor ARCH model extremely complicated. Alternatively, in the  $K$ -factor ARCH representation suggested by Engle (1987),

$$\text{vech}(\Omega_t) = W + \sum_{k=1}^K \text{vech}(f_k f_k') \sigma_{tk}^2, \quad (13)$$

where the  $f_k$ 's denote  $N \times 1$  vectors and  $\sigma_{tk}^2$  the time-varying variance of the  $k$ th factor. Of course, to complete this representation an explicit form for the  $\sigma_{tk}^2$ 's is needed. In the  $K$ -factor GARCH( $p, q$ ) model, the  $\sigma_{tk}^2$ 's are given by the conditional variance of  $K$  independent linear combinations of  $\varepsilon_t$ , each of which has a univariate GARCH( $p, q$ ) representation; i.e.,  $\sigma_{tk}^2 = \omega_k + \alpha_k(L)(g'_k \varepsilon_t)^2 + \beta_k(L)\sigma_{tk}^2$ , where  $g'_i g_j = g'_i f_j = 0$  for  $i \neq j$ . Low-order factor GARCH models have been estimated by Engle, Granger, and Kraft (1984), Kroner (1987), Engle, Ng, and Rothschild (1989, 1990), and Engle and Ng (1990). The small sample behavior of various estimators in the one-factor GARCH(1, 1) model has been analyzed by Lin (1991).

Other multivariate representations include the constant conditional correlation model put forward in Bollerslev (1990). In this representation the conditional covariance matrix  $\Omega_t$  is time-varying, but the conditional correlations are assumed to be constant. This assumption greatly simplifies the inference procedures, and several studies have found it to be a reasonable empirical working hypothesis; see, for instance, Cecchetti, Cumby, and Figlewski (1988), Kroner and Claessens (1991), McCurdy and Morgan (1991a), Ng (1991), Kroner and Lastrapes (1991), Brown and Ligeralde (1990), Baillie and Bollerslev (1990), and Schwert and Seguin (1990).

## 2.8. Alternative measures of uncertainty

Several alternative measures to the ARCH model defined above have been employed in characterizing volatility in speculative prices. One such alternative involves the construction of variance estimates by averaging the squared errors obtained with models for the conditional mean estimated over finer horizons. For instance, following Merton (1980), several authors, including Poterba and Summers (1986), French, Schwert and Stambaugh (1987), Schwert (1989a, 1990a, b), and Schwert and Seguin (1990), construct monthly stock return variance estimates by taking the average of the squared daily

returns within the month,<sup>16</sup> whereas Pindyck (1988) and Harris (1989) use this idea in calculating annual standard deviations of returns. To assess the temporal dependence, standard time series models are subsequently estimated for these variance estimates. Obviously, this procedure does not make efficient use of all the data. Furthermore, following Pagan (1984, 1986), the conventional standard errors from the second-stage estimation may not be appropriate. If the information matrix for the full model is not block diagonal between the parameters in the mean and variance, the actual parameter estimates may also be inconsistent; see Pagan and Ullah (1988). Further, as argued by Chou (1988) and Attanasio (1988), the two-stage procedure may result in misleading conclusions about the true underlying dependence in the second-order moments of the data. This is formally documented by the Monte Carlo evidence in Attanasio and Wadhvani (1989), where the estimates for the ARCH-in-Mean parameters are found to be biased towards zero. Nonetheless, the computational simplicity of the two-stage procedure makes it an appealing tool for preliminary data analysis.

In a second alternative to the ARCH models discussed above, ARMA-type models are estimated for the conditional standard deviation, as measured by the absolute value of the errors from some first-step estimates for the conditional mean. Schwert (1989a) uses this strategy in determining the underlying causes of movements in conditional stock market volatility. This two-stage alternative remains subject to the same errors-in-variables type problem discussed by Pagan and Ullah (1988) and, as argued by Schwert (1989a), the resulting test statistics, including tests for persistence in the variance, have to be carefully interpreted.

Another popular method for assessing volatility in financial data is based on the implied volatility from options prices. Under the assumption of a constant variance, the lack of arbitrage leads to the well-known Black and Scholes (1973) formula; for some of the first empirical evidence along these lines, see Black and Scholes (1972). The possibility of a stochastic volatility process within this framework is explicitly considered by Hsieh and Manaster (1988), Jorion (1988), Lyons (1988), Engle and Mustafa (1992), Day and Lewis (1992), and Engle, Hong, and Kane (1990). Even though this method potentially could lead to estimates superior to ARCH-type alternatives, not all assets of interest have actively traded options. Also, several complications arise in the theory of option pricing with stochastic volatility; see, for instance, Melino and Turnbull (1990) among others.

The use of inter-period high and low prices constitutes yet another method for assessing the variability. Under the assumption that the logarithm of

<sup>16</sup>The monthly variance estimate is often adjusted by adding two times the first-order daily sample serial correlation coefficient in order to account for any negative autocorrelation possibly induced by nonsynchronous trading.

speculative prices takes the form of a continuous time random walk with a constant instantaneous variance, the exact distribution of the high/low price ratio follows from the theory of range statistics. As shown by Parkinson (1980), the moments of this high/low price distribution are functions of the underlying variance of the process, suggesting an estimator of the variance based on the realized interperiod highs and lows; see Taylor (1987) and Schwert (1990a). With a time-invariant conditional variance, the efficiency of this alternative estimator compares favorably with the conventional estimator given by the sample analogue of the mean adjusted squared returns over fixed time intervals. The generalization of these ideas to other stochastic processes allowing for time-varying variances is not straightforward, although the ARCH diffusion approximation in Nelson (1990b) may prove constructive. Furthermore, high and low prices are not readily available for many assets.

Building on the theoretical developments in Glosten and Milgrom (1985), where the variance of the asset price is proportional to the square of the bid-ask spread, the magnitude in the spread could also be used in extracting estimates of volatility. This idea has been pursued recently by Brock and Kleidon (1990).

Cross-sectional dispersion in survey data also forms the basis for a measure of uncertainty. For example, the dispersion in experts earnings forecasts has been used as an estimate of the systematic risk for a stock by Cragg and Malkiel (1982) and Weston (1986) among others. Similarly, the dispersion in forecasts among experts has been suggested as a measure of inflation uncertainty by Carlson (1977) and Levi and Makin (1979), among others, though Rich, Raymond, and Butler (1990b) find only limited evidence of a relationship between forecast dispersion and ARCH measures of uncertainty. Also, Frankel and Froot (1987) and Froot and Frankel (1989) provide an interesting use of survey data in modeling expectations formation and in interpreting tests of the unbiasedness hypothesis in the forward foreign exchange rate market. However, survey measures only provide an indicator of the heterogeneity in expectations, which may not be a good approximation of the fundamental underlying uncertainty depending on the homogeneity of expectations. In addition, the availability of survey data pertaining to speculative prices is very limited.

A related measure often used in quantifying the uncertainty of inflation is derived from relative contemporaneous prices. As discussed by Pagan, Hall, and Trevedi (1983), the validity of this method depends on the homogeneity of the different markets, assets or agents; see also Pagan and Ullah (1988). This is the same idea underlying the relative return dispersion measure across different stocks recently put forward by Amihud and Mendelson (1989) and Cutler (1989) as a means for quantifying overall market volatility, and is related to the use of cross-sectional dispersion of asset returns as a

measure of uncertainty in the traditional event studies literature. A formal characterization of conditions under which heterogeneity in cross-sectional returns could be used as a measure of return uncertainty would be interesting.

A utility-based comparison of some of these alternative statistical models for volatility in terms of the willingness of an investor with a mean–variance utility function to pay for one variance forecast rather than another is given in West, Edison, and Cho (1991). Very interestingly, on using weekly exchange rate data the authors find that the GARCH(1,1) formulation in (7) tends to outperform the alternative methods investigated, and they argue ‘... that an investment advisor whose only specialized tool is GARCH may be as worthy of her hire as are professionals currently on Wall Street’. Alternative profit driven evaluations have been suggested by Brock, Lakonishok, and LeBaron (1990) and Engle, Hong, and Kane (1990). Engle, Hong, and Kane, for example, compare the difference in profits arising from pricing one-day options on the NYSE portfolio on the basis of alternative variance forecasts. The GARCH(1,1) model is again found to outperform MA and ARMA formulations for the squared residual returns. This is consistent with the direct forecast-based comparison of implied options variances and GARCH(1,1) estimates in Lamoureux and Lastrapes (1991) and Day and Lewis (1992).

A more conventional statistically-based mean squared error comparison of statistical volatility models is conducted by Pagan and Schwert (1990), who find that nonparametric methods may be superior for stock returns. They conclude that the extension of parametric models in the nonparametric direction (e.g., by adding on Fourier terms) is likely to be the best modeling strategy. Further comparisons of these many alternatives to ARCH-based methods for assessing the uncertainty in different speculative prices seem worthwhile.

### 2.9. Sources of ARCH

While serial correlation in conditional second moments is clearly a property of speculative prices, a systematic search for the causes of this serial correlation has only recently begun. One possible explanation for the prominence of ARCH effects is of course the presence of a serially correlated news arrival process, as discussed by Diebold and Nerlove (1989) and Gallant, Hsieh, and Tauchen (1989).<sup>17</sup> In a detailed empirical analysis, Engle, Ito, and Lin (1990a,b) find support for this hypothesis, although any satisfactory explanation for this dependence in the underlying news arrival process is

<sup>17</sup>Bookstaber and Pomerantz (1989) arrive at a compound Poisson process for volatility by assuming a linear relation to the underlying information arrival process.

notably lacking.<sup>18</sup> This is also related to Stock's (1987, 1988) notion of time deformation in which economic and calendar time proceed at different speeds. In a related context Bollerslev and Domowitz (1991) have shown how the actual market mechanisms may themselves result in very different temporal dependence in the volatility of transactions prices, with a particular automated trade execution system inducing a very high degree of persistence in the variance process. Some other preliminary theoretical results on the foundation of ARCH models together with some interesting empirical evidence for the significance of various exogenous forcing variables in the variance equation have been obtained by Domowitz and Hakkio (1985), Smith (1987), Diebold and Pauly (1988a), Hsieh (1988b), Lai and Pauly (1988), Ng (1988), Thum (1988), Backus, Gregory, and Zin (1989), Giovannini and Jorion (1989), Hodrick (1989), Schwert (1989a), Attanasio and Wadhvani (1989), Engle and Susmel (1990), and Brock and Kleidon (1990) among others. Further developments concerning the identification and formulation of equilibrium models justifying empirical specifications for the observed heteroskedasticity remains a very important area for future research. At the same time, the direct implementation of such more structural models over short time intervals, e.g., daily or weekly, is likely to be hindered by the unavailability of data.

### 3. Applications of ARCH to stock return data

Volatility clustering in stock return series has many important theoretical implications, so it is not surprising that numerous empirical applications of the ARCH methodology in characterizing stock return variances and covariances have already appeared. In the following sections, a review of this literature will be presented.

#### 3.1. ARCH effects and model specification

ARCH effects have generally been found to be highly significant in equity markets. For example, highly significant test statistics for ARCH have been reported for individual stock returns by Engle and Mustafa (1992), for index returns by Akgiray (1989), and for futures markets by Schwert (1990a).<sup>19</sup> As in the specification of ARMA( $p, q$ ) models for the conditional mean, most empirical implementations of GARCH( $p, q$ ) models adopt low orders for the

<sup>18</sup>Following Tauchen and Pitts (1983) such serial correlation in the news arrival process would likely induce a strong contemporaneous relationship between volume and volatility as well, thus explaining the Lamoureux and Lastrapes (1990a) results.

<sup>19</sup>Schwert (1990a) finds that futures returns tend to be slightly more volatile than cash returns, possibly because futures react more quickly to news due to lower transactions costs and because they price the underlying bundle of securities simultaneously.

lag lengths  $p$  and  $q$ . Typically, GARCH(1, 1), GARCH(1, 2), or GARCH(2, 1) models are adopted. It is interesting to note that such small numbers of parameters seem sufficient to model the variance dynamics over very long sample periods. For instance, French, Schwert, and Stambaugh (1987) analyze daily S&P stock index data for 1928–1984 for a total of 15,369 observations and require only four parameters in the conditional variance equation.

Exceptions to this low-order rule in the ARCH specification include Bodurtha and Mark (1991) and Attanasio (1991) where ARCH(3) models are employed in analyzing portfolios of monthly NYSE stock returns and monthly excess returns on the S&P 500 index, respectively. It is possible that this Ken Kroner and Ray Chou would like to thank the Economic Science Lab at the University of Arizona and the Georgia Tech Foundation, respectively, for financial support seasonality may be explained by the clustering effect for firms in the quarterly announcements of dividends and earnings. If variances of stock returns are systematically evoked by these announcements, then a monthly stock index return may exhibit such a seasonal pattern in conditional variances. This hypothesis is yet to be tested, but it illustrates the importance of understanding the generating forces behind the ARCH effects, as discussed further in sections 2.9 and 3.8. Similarly, the well-known weekend effect, according to which the variance of returns tends to be higher on days following closures of the market, could also lead to the finding of high-order ARCH models. This effect, as documented by French and Roll (1986) using daily unconditional variances, remains significant in the low-order ARCH models for daily index returns presented in French, Schwert, and Stambaugh (1987), Nelson (1989, 1990c), and Connolly (1989), and a failure to take proper account of such deterministic influences might lead to a spurious seasonal ARCH effect.<sup>20</sup>

The importance of adjusting for ARCH effects in the residuals from conventional market models has been analyzed in a series of papers by Morgan and Morgan (1987), Bera, Bubnys, and Park (1988), Diebold, Im, and Lee (1989), Connolly (1989), and Schwert and Seguin (1990), where it is argued that inferences can be seriously affected by ignoring the ARCH error structure. For instance, Morgan and Morgan (1987), in a study of the small firm effect, find that correcting for the conditional variance in returns from portfolios long in small and short in large firms reduces the estimate of market risk and increases the estimate of abnormal return.

Related to the specification of ARCH models, it is also worth noting the results in the recent literature on deterministic chaos as a form of nonlinearities in stock returns. On applying the correlation integral-based test statistic in Brock, Dechert, and Scheinkman (1987) (BDS), which has power against both deterministic chaos and nonlinear dependencies, most studies tend to find that once ARCH effects are removed the BDS test on standardized

<sup>20</sup> Interestingly, Baillie and DeGennaro (1989) have argued that by including a proxy for variations in delivery and payment terms, the effect of the holding period in the conditional variance becomes much less important.

residuals exhibits very little evidence of nonlinear dependence.<sup>21</sup> Hence most of the seemingly chaotic nonlinearities work through the conditional variance. For examples, see Schwert (1989b) or Scheinkman and LeBaron (1989) for individual firms' returns, LeBaron (1988) for daily returns of the NYSE value-weighted index, LeBaron (1989) for daily and weekly returns of the S&P 500 index, and Hsieh (1990) for a series of different weekly returns, including size-ranked portfolios. The BDS statistic has also been used as a diagnostic tool in the specification of multivariate ARCH models for equity returns by McCurdy and Stengos (1992).

### 3.2. Nonnormal conditional densities

Stock returns tend to exhibit nonnormal unconditional sampling distributions, in the form of skewness but more pronounced in the form of excess kurtosis [see, e.g., Fama (1965)]. As described in section 2.3, the conditional normality assumption in ARCH generates some degree of unconditional excess kurtosis, but typically less than adequate to fully account for the fat-tailed properties of the data. One solution to the kurtosis problem is the adoption of conditional distributions with fatter tails than the normal distribution. In Baillie and DeGennaro (1990) and de Jong, Kemna, and Kloeck (1990), the assumption of conditionally  $t$ -distributed errors together with a GARCH(1, 1) model for the conditional variance is adopted, and it is found that failure to model the fat-tailed property can lead to spurious results in terms of the estimated risk–return tradeoff. Other attempts to model the excess conditional kurtosis in stock return indices include the estimates of the EGARCH model with a generalized exponential distribution in Nelson (1989) and the jump-diffusion process with ARCH errors in Jorion (1988).

An alternative to the explicit assumption of conditionally leptokurtic distributions is the seminonparametric method discussed in section 2.4. Using this method, Gallant and Tauchen (1989) report significant evidence of both conditional heteroskedasticity in the direction of ARCH and conditional nonnormality for the daily NYSE value-weighted index for two separate periods, 1959–1978 and 1959–1984. A variant of the seminonparametric method in which the leading term is an ARCH-type formulation is also used in Gallant, Hansen, and Tauchen (1989) in estimating the density function for monthly stock returns. Similarly, Engle and Gonzalez-Rivera (1991) employ nonparametric density estimation with a GARCH(1, 1) specification for the conditional variance to model the daily stock returns for some small firms. They note that the skewness as well as kurtosis are important in

<sup>21</sup>As shown in Brock, Hsieh, and LeBaron (1990) both analytically and through Monte Carlo methods, a correction factor is required when applying the BDS statistic to standardized residuals from estimated GARCH models. Failure to do so would lead to overrejections, i.e., finding nonlinear dependence, too often.

characterizing the conditional density function of returns on many small firm stocks.

### 3.3. Nonlinear ARCH and the leverage effect

In addition to the leptokurtic distribution of stock return data, Black (1976) has noted a negative correlation between current returns and future volatility. A plausible economic explanation suggested by Black (1976) and further investigated by Christie (1982) is the so-called leverage effect. According to the leverage effect, a reduction in the equity value would raise the debt-to-equity ratio, hence raising the riskiness of the firm as manifested by an increase in future volatility. As a result, the future volatility will be negatively related to the current return on the stock. The linear GARCH( $p, q$ ) model is not able to capture this kind of dynamic pattern since the conditional variance is only linked to past conditional variances and squared innovations, and hence the sign of returns plays no role in affecting the volatilities. This limitation of the standard ARCH formulation is one of the primary motivations for the EGARCH model in (8) developed by Nelson (1990c). In this class of ARCH models, the volatility depends not only on the magnitude of the past surprises in returns but also on their corresponding signs. Empirical support for this specification of the ARCH model is documented in Nelson (1989, 1990c).

Discussion of the leverage effect can also be found in Kupiec (1990) where the leverage effect is tested within the context of a linear GARCH( $p, q$ ) model by introducing a stock price level in the variance equation. The coefficient is insignificant though this may be a result of a failure to adjust for the strong trend in the price level. However, recent empirical evidence in Gallant, Rossi, and Tauchen (1990) using semiparametric estimation techniques suggest that when conditioning on past trading volume together with past returns, the leverage effect in the daily NYSE index is no longer statistically significant. One possible explanation for this finding could be that the estimated leverage effect is attributable to a few outliers which become less influential in a bivariate setting or with a fat-tailed distribution; see also French (1990). Further empirical work along these lines, including individual stock returns, could be very informative.

It is also worth noting that the leverage effect can only partially explain the strong negative correlation between current return and current volatility in the stock market; e.g., Black (1976) and Christie (1982). In contrast to the causal linkage of current return and future volatility explained by the leverage effect, the fundamental risk–return relation predicts a positive correlation between future returns and current volatilities in stock prices. This issue is discussed in the following subsection. However, an alternative explanation is the volatility feedback effect, studied in French, Schwert, and Stambaugh (1987) and Campbell and Hentschel (1990).

### 3.4. ARCH-M and asset pricing models

The importance of ARCH models in finance comes partly from the direct association of variance and risk and the fundamental tradeoff relationship between risk and return. Three of the most prominent theories in asset pricing, the CAPM of Sharpe (1964), Lintner (1965), Mossin (1966), and Merton (1973), the consumption-based CAPM of Breeden (1979) and Lucas (1978), and the APT of Ross (1976) and Chamberlain and Rothschild (1983), have all found empirical implementations using ARCH. We will discuss these empirical papers in this and the next subsection.

Building on the intertemporal CAPM in Merton (1973), Merton (1980) provides an approximate linear relationship between the return and variance of the market portfolio. The ARCH-M model developed by Engle, Lilien, and Robins (1987) discussed in section 2.5 above, provides a natural tool for estimation of this linear relationship. The parameter measuring the impact of the conditional variances on the excess returns corresponds to the coefficient of relative risk aversion.

Applications of this model to different stock index returns have been reported by numerous authors. Examples include French, Schwert, and Stambaugh (1987) for the daily S&P index, Chou (1988) for the weekly NYSE value-weighted returns along with different temporal aggregations of the daily returns, Attanasio and Wadhvani (1989) for monthly and annual returns for both U.S. and U.K. stock indices, and Friedman and Kuttner (1988) for quarterly U.S. stock indices; see also Pindyck (1984, 1988) and Poterba and Summers (1986). In a related study on dividend-price ratio and volatility-measured discount factors, Campbell and Shiller (1989) estimate the relative risk aversion parameter using annual data on the Cowles/S&P for 1871–1986 and a value-weighted index for the NYSE for the 1926–1986 period.

Interestingly, in all the above papers the estimates of the risk aversion parameter are unanimously positive and fall within a fairly small range, from 1 to 4.5. Furthermore, with the exception of Campbell and Shiller (1989), all of these point estimates are significantly different from zero at the usual 5% level. This is in sharp contrast to the literature reporting the many alternative structural-based estimates of the risk aversion parameter, where very imprecise and often implausible point estimates are reported; see, e.g., Grossman, Melino, and Shiller (1987) in the context of a Consumption CAPM and Engle and Rodrigues (1989) using international data in a multivariate CAPM model.

Some evidence of the sensitivity of the parameter estimate in the ARCH-M model with respect to different model specifications is given in Baillie and DeGennaro (1990) using both daily and monthly portfolio returns. By changing the conditional distribution from normal to Student- $t$ , the parameter for the conditional variance entering the mean equation changes from signifi-

cantly positive at the 5% level to insignificant and of either sign. Similar results are found in Bollerslev and Wooldridge (1991) using robust standard errors; see also French, Schwert, and Stambaugh (1987) and Cocco and Paruolo (1990). Furthermore, Glosten, Jagannathan, and Runkle (1991) show that the sign of the ARCH-M coefficient is sensitive to the instruments which are added to the mean and variance equations of the model; see also Gallant, Rossi, and Tauchen (1990) for similar results in a seminonparametric framework.

The constancy of the linear relationship between the expected return and the conditional variance in the simple ARCH-M model has also been called into question by various authors. For example, on introducing additional instruments over the past squared residuals in estimating the conditional variance, Harvey (1989) reports the coefficient to be significantly time-varying of either sign, depending on the stage of the business cycle. This constancy is also challenged by Chou, Engle, and Kane (1992), who generalize the standard ARCH-M model to allow the parameter of the conditional variance in the mean equation to be time-varying through a state-space formulation. They also find instability, which they credit to Roll's (1977) critique; see also Ferson, Kandel, and Stambaugh (1987) and Ng (1991). Including various proxies for the omitted 'nonstock' risky assets is found to help establish the constancy of the parameter. This empirical evidence against the validity of a simple linear relationship between the expected return and the volatility of stock indices are corroborated by the theoretical results in Backus and Gregory (1988) and Gennotte and Marsh (1987).<sup>22</sup>

In a related context, Attanasio and Wadhvani (1989) find that the predictability of stock returns given lagged dividend yields reported in Fama and French (1988), among others, can be explained by a risk measure using ARCH. The evidence of this finding is stronger for the U.S. than for the U.K. However, other explanatory variables, including lagged nominal interest rates and inflation rates, remain significant in explaining the movement of expected returns in addition to the influence of the own conditional variance. Attanasio (1991) extends the ARCH-M model to incorporate both the static and the consumption CAPM in a nested formulation. His result confirms the evidence in Mankiw and Shapiro (1986) and many others, that the static CAPM performs better, from an empirical point of view, than the consumption CAPM. Of course, this might be attributed to aggregate consumption providing a poor measure of the fundamental consumption risk. See also Lee and Yoon (1990) and Sentana and Wadhvani (1989).

<sup>22</sup> In discussing the ARCH-in-Mean relationship it is also worth noting the recent empirical findings in Sentana and Wadhvani (1991) where, motivated by a noise trading model in which some traders follow feedback strategies, it is found that the constancy of the serial correlation parameter is affected by the level of volatility. Similarly, LeBaron (1989) argues that the magnitude of the serial correlation is inversely related to the volatility, consistent with nonsynchronous trading being more severe when volatility and volume are both low.

It is apparent that the final words have not been said on the empirical relationship between expected market return and volatility. However, it is also clear that in the framework of conditional models, any satisfactory model must incorporate the temporal variation in volatility. Empirically the explicit ARCH-M formulations or the seminonparametric methods both hold promise of further interesting insights into this important issue. Nonetheless, the use of the ARCH-M model as an implementation of Merton's (1973) CAPM is not without criticism. As noted by Pagan and Ullah (1988) and discussed in section 2.5, in the ARCH-M model the estimates for the parameters in the conditional mean equation are not asymptotically independent of the estimates of the parameters in the conditional variance, hence any misspecification in the variance equation generally leads to biased and inconsistent estimates of the parameters in the mean equation.

In a related context the implications of most contingent claims pricing models also depend crucially on the variance of the underlying asset; see Rubinstein (1987) for a review of theoretical models for the pricing of derivative assets. We shall not attempt a detailed survey of the empirical literature here, but as discussed in section 2.8 above, the ARCH methodology has already been successfully applied to the pricing of individual stocks and stock index options by Jorion (1988), Engle, Hong, and Kane (1992), Day and Lewis (1992), Lamoureux and Lastrapes (1991), and Choi and Wohar (1990) among others.

### *3.5. Multivariate ARCH, factor ARCH, and asset pricing models*

While the papers discussed in the previous section use univariate analysis, many interesting questions in finance can only be meaningfully answered within a multivariate framework. In Bollerslev, Engle, and Wooldridge (1988) a multivariate GARCH(1, 1)-M formulation is used in the implementation of a CAPM model for a market portfolio consisting of three assets – stocks, bonds, and bills. The model estimates suggest a significant positive mean variance tradeoff among the three broadly defined asset categories. However, while the trivariate model seems superior to the corresponding three univariate GARCH(1, 1)-M models, there is also some evidence that the growth rate in aggregate consumption expenditures and lagged excess returns may have additional explanatory power over the nondiversifiable risk as measured by the time-varying conditional covariance with the market.

A similar approach has been used in a series of papers in analyzing the mean–variance tradeoff across both domestic and international equity markets. A partial list of these studies includes Bodurtha and Mark (1991), Hall, Miles, and Taylor (1988), Kaplan (1988), Engel, Frankel, Froot, and Rodrigues (1989), Engel and Rodrigues (1989), Giovannini and Jorion (1989), Ng (1991), De Santis and Sbordone (1990), French (1990), Giovannini and Jorion (1990), Harvey (1991), and McCurdy and Stengos (1992). Without attempting a

detailed discussion of this extensive literature, a common thread in most of these studies concerns the finding of a time-varying risk premium, while at the same time the restrictions implied by the CAPM are formally rejected. It is important to recognize that the relationships that hold with the conditional CAPM will not hold with unconditional moments; see Bollerslev, Engle, and Wooldridge (1988). Thus, earlier rejections of the unconditional CAPM do not have any direct bearing on these results.

As discussed in section 2.7, computational difficulties are of major concern in applications of multivariate ARCH models. In addition to the diagonal parameterization and the constant correlations structure used in the applications above, the factor ARCH model in (13) provides an alternative simplifying structure on the covariance matrix. A factor ARCH model is used by Engle, Ng, and Rothschild (1989) for ten size-ranked portfolios. Interestingly, one of the empirically identified factors is found to load onto a January dummy variable, while the other is related to the bond risk premium. Hence the well-known small firm effect is explained in this model as a response to time-varying covariances.

A factor ARCH approach is also employed in King, Sentana, and Wadhvani (1990) in an international asset pricing model to study the link between international stock markets. This linkage is investigated further by Hamao, Masulis, and Ng (1990a), who examine the issue of volatility spillovers among international stock markets using an ARCH-M model on daily open and close prices. Some evidence is provided for spillovers of volatility from New York to Tokyo and London to Tokyo, but not from Tokyo to either New York or London. Using cross-correlations among standardized residuals from GARCH(1,1) models, these results are confirmed in Cheung and Ng (1990). Interestingly, these spillovers are shown in Hamao, Masulis, and Ng (1990b) to have been magnified following the October 1987 crash. A similar approach is taken in Engle and Susmel (1990), where a significant spillover between the U.S. and U.K. stock markets is found, and in Ng, Chang, and Chou (1991), where spillovers are found among the Pacific Rim countries.

Other studies concerning the transmission of volatility include Chan, Chan, and Karolyi (1990), where the relationship between the S&P 500 stock index and the stock index futures market is investigated using five-minute data from 1984–1986 for a total of 36,500 observations. Consistent with the notion that futures trading tends to increase the volatility in the cash market, a causal relationship from the futures market to the cash market is documented. Interestingly, however, a reverse transmission of volatility from the cash market to the futures market is also evident. The transmission of volatility within the stock market is studied by Conrad, Gultekin, and Kaul (1990), who find that volatility, or news, is incorporated into security prices in a unidirectional manner from the largest to smallest firms.

Future work along these lines seems promising and might help in further understanding the linkage and transmission of stock return volatility.

### 3.6. Volatility persistence

An interesting property of stock market volatility relates to the persistence of shocks to the variance. Poterba and Summers (1986) argue that for multiperiod assets like stocks shocks have to persist for a long time for a time-varying risk premium to be able to explain the large fluctuations observed in the stock market. If volatility changes are only transitory, no significant adjustments to the risk premium will be made by the market; hence no significant changes in the discount factor or the price of a stock as determined by the net present value of the future expected cash flow will occur.<sup>23</sup>

Poterba and Summers (1986), on using a two-step procedure, argue that shocks to the U.S. stock market are only short-lived, with a half-life of less than six months. As a result, they reject Malkiel's (1979) and Pindyck's (1984) hypothesis that shocks to the investment environment during the early and mid-seventies were the most important factor in explaining the market plunge during the mid-seventies. However, on using a GARCH(1,1)-M model, Chou (1988) reports a very different result on the persistence of volatility, with the average half-life for volatility shocks being about one year, consistent with the changing risk premium hypothesis; see also Campbell and Hentschel (1990). These markedly different findings are most likely due to the difference in estimation methodology; see section 2.8 above for a critique of the two-step estimation method.

Indeed, formal tests for a unit root in variance have been performed by several authors, and the null hypothesis of a unit root is typically not rejected. For example, French, Schwert, and Stambaugh (1987) find a unit root in the variance of the S&P daily index, Chou (1988) finds one in the variance of the NYSE value-weighted index, Pagan and Schwert (1990) find one in the variance of U.S. stocks, and Schwert and Seguin (1990) find one in the variance of monthly size-ranked portfolios. Interestingly, Schwert and Seguin also find evidence of a common source of time-varying volatility across the disaggregated stock portfolios, suggesting the portfolios might be co-persistent in the sense of Bollerslev and Engle (1990); cf. section 2.6. Furthermore, this finding of a unit root seems robust to the parameterization of the ARCH model chosen. For example, Nelson (1989, 1990c) finds evidence of persistence using an EGARCH formulation, and Gallant, Rossi, and Tauchen (1990) find evidence using seminonparametric methods.

The degree of persistence in volatility shocks is also investigated in Engle and Mustafa (1992), who combine the Black-Scholes option pricing formula with a stochastic variance process modeled by an ARCH process. The GARCH(1,1) model for the volatility of the underlying security, inferred from the observed option prices written on the security, indicates very strong

<sup>23</sup>Attanasio (1991), however, argues that if volatility is high enough, then it does not have to persist in order to affect returns.

persistence of the conditional variances. However, a markedly lower persistence is reported after the October 1987 crash. A qualitatively similar result is given by Schwert (1990a), who finds that the stock volatility returned to pre-crash levels by early 1988. However, this short-lived property for volatility shocks due to a market crash is not observed for any of the smaller market downturns prior to 1987. Along these lines, Friedman and Laibson (1989) modify the ARCH model such that outliers, or extremely large shocks, are allowed to have different dynamic effects than 'ordinary' shocks. Interestingly, and in contrast to the Kearns and Pagan (1990) results on Australian data, 'ordinary' shocks tend to persist longer than outliers, so GARCH models, which do not distinguish outliers from ordinary shocks, therefore tend to underestimate the persistence of 'ordinary' shocks.<sup>24</sup> Also, Engle and Gonzalez-Rivera (1991) report that the persistence in variance seems to be related to the size of the firm, with small firms having a lower persistence than the larger firms studied in the paper by Engle and Mustafa (1992). This is also in accordance with the results for size ranked portfolios reported in Schwert and Seguin (1990).

Lamoureux and Lastrapes (1990a) argues that the high degree of persistence in GARCH models might be due to a misspecifications of the variance equation. By introducing dummy variables for deterministic shifts in the unconditional variances, they discover that the duration of the volatility shocks is substantially reduced. A similar point is raised by Diebold (1986a), who conjectures that the apparent existence of a unit root as in the IGARCH class of models may be the result of shifts in regimes which affect the level of the unconditional variances. The same critique of standard tests for unit roots in the conditional mean has recently been put forward by Perron (1989). The identification of the timing of shifts in the unconditional variance and the degree of dependence of conditional variances remain areas for useful research. Generalizations of Hamilton's (1989) model of stochastic regime shifting may prove helpful along these lines; see Schwert (1989b) and Pagan and Schwert (1990). However, it should be noted that even if IGARCH with a constant  $\omega$  is generating the data, then dummy variables for 'deterministic' shifts in regime will probably show up as being significant.

These somewhat mixed empirical results, together with the important economic implications of the volatility persistence issue, suggest the need for further research in this area. Such investigations may shed light on linkages between the different modeling dichotomies employed in previous studies, the outliers versus ordinary shocks as in Friedman and Laibson (1989), the distinction between recession association and nonrecession associated or financial crises associated and nonfinancial crises associated persistence in

<sup>24</sup>This could be because large outliers might be the result of large doses of measurement error, which would not be expected to persist.

Schwert (1989a), large versus small firm persistence as in Engle and Gonzalez-Rivera (1991), the identification of deterministic shifts in the unconditional variances versus persistent conditional variances as in Lamoureux and Lastrapes (1990a), and the possible co-persistence in variance across different stocks and portfolios in Schwert and Seguin (1990). Further, a distinction between permanent versus temporary shocks, as is common in the literature about issues pertaining to unit roots in the mean, may also help in analyzing whether conditional variances are positively related to the expected stock returns. It is possible that the overall variance can be decomposed into two components, one of which is 'priced' and the other of which is 'non-priced'.

It would also be interesting to use different data sets to further assess the degree of persistence in stock return volatility. With very few exceptions, most current studies use data from the U.S. stock market. More insights may be provided by using data from other U.S. markets, e.g., options market and futures markets, together with international stock market data. Some existing examples are given by Attanasio's (1988) study on the U.K. market, Kearns and Pagan's (1990) study on Australian data, Hamao, Masulis, and Ng's (1990a) study on volatility spillovers among three international stock markets, de Jong, Kemna, and Kloeck's (1990) study on the Dutch stock market, and the international CAPM model by Engel and Rodrigues (1989).

### 3.7. ARCH and event studies

The significant ARCH effects in individual firm's stock returns has important implications for the conventional event study methodology frequently applied in empirical studies of corporate finance. In fact, the importance of recognizing time-varying variances in the context of event studies has already been pointed out by many researchers in finance; see, e.g., Brown and Warner (1985). It is intuitively clear that in assessing the abnormal returns, it is essential to get a correct estimate of the standard error for the purpose of statistical inferences. This is especially true, since it is frequently documented that 'events' are associated with changes in the variabilities of the underlying stock returns. However, the current treatment of changing variances in the literature is mostly ad hoc, and a systematic approach using the ARCH methodology seems clearly attractive.

Several empirical works have appeared which apply the ARCH methodology to event studies; e.g., Connolly and McMillan (1988) on capital structure changes, Poon (1988) on stock splits, and de Jong, Kemna, and Kloeck (1990) on the option expiration effect. De Jong, Kemna, and Kloeck (1990), for example, show that ignoring the fat tails and the time-varying variances could lead to spurious detection of abnormal returns.

In all of the above studies, the dynamic patterns of the conditional variances and the betas have not been modeled simultaneously. However, the link between time-variation in beta and the time-varying conditional variance of a firm can further be exploited as in the CAPM model of Bollerslev, Engle, and Wooldridge (1988) discussed in section 3.5 in which the beta is given by the ratio of the time-varying covariance of the individual firm's return to the variance of the market return. In particular, by assuming a constant conditional correlation structure as in Bollerslev (1990), the dynamics of beta are completely specified by the firm's own variances and the variance of the market. This model seems more plausible than an ARCH variance coupled with a constant beta or a beta process independent of the error variances.

### 3.8. *The ARCH effect and economic interpretations*

The widespread existence of ARCH effects and the persistence of stock return volatility have led researchers to search for its origin(s). The GARCH( $p, q$ ) model can be viewed as a reduced form of a more complicated dynamic structure for the time-varying conditional second-order moments. Thus interpretations and explanatory variables for the observed ARCH effects have been proposed both on the micro and the macro level. On the micro level, Lamoureux and Lastrapes (1990b) argue that the ARCH effect is a manifestation of clustering in trading volumes. By introducing the contemporaneous trading volumes in the variance equation of a GARCH(1, 1) model for individual firm's returns, they discover that the lagged squared residuals are no longer significant. A simultaneity problem may seriously bias their results, as contemporaneous correlations between volume and price data have been documented by various authors, e.g., Karpoff (1987) among others. Indeed, using lagged volume as an instrument for the contemporaneous volume does not 'remove' the standard ARCH effect. This joint relation of lagged volume and lagged returns to stock return volatility is explored using seminonparametric results in Gallant, Rossi, and Tauchen (1990) for the value-weighted NYSE index. In addition to the positive correlation between conditional volatility and volume, the Gallant, Rossi, and Tauchen study also finds that large price movements are followed by high volume.

On the macroeconomic level, relevant economic variables driving stock volatilities have also been proposed by various researchers. For example, both Campbell (1987) and Glosten, Jagannathan, and Runkle (1991) have found that nominal interest rates are significant determinants of volatility. In addition, Glosten, Jagannathan, and Runkle (1991) show that entering the interest rate into the GARCH formulation leads to a decrease in persistence as measured by the conventional linear GARCH parameters, suggesting cointegration between the interest rate and returns. Other related studies include Attanasio (1991) and Attanasio and Wadhvani (1989), who report a

significant role for dividend yields in driving stock volatilities. Engel and Rodrigues (1989) show that the variance of stock returns depends on the M1 money supply and an oil price index, while Schwert (1989a) identifies a linkage to the business cycle and financial crises. By using U.S. stock returns for 1834–1987, Schwert finds that stock volatility tends to be higher during recessions and reacts strongly to banking crises.

A related and much debated issue concerns the impact of changes in margin requirements on stock volatilities. Hardouvelis (1990) and Hardouvelis and Peristiani (1990) find a significant negative relationship between return volatility and margin requirements in the U.S. and Japanese markets, respectively. However, Hsieh and Miller (1990) and Schwert (1989b,c) argue that this result is likely to be spurious because of the high degree of persistence in volatility shocks; see also Kupiec (1990) and Seguin (1990). In fact, these studies find that changes in margin requirements tend to follow increases in volatility, but not vice versa.

It is unlikely that the determinants of the ARCH effect, or more generally the duration of fluctuations, is exhausted by the variables suggested in the above list of studies. While exploring a larger set of variables is certainly a worthwhile exercise, a more fruitful strategy for future research in this area might involve the construction of structural models that can explain the empirical findings. The recent evidence in Brock and Kleidon (1990) documenting the widening bid–ask spread around opening and closing, possibly related to peak load pricing, might be interesting. Also, further developments along the lines of Admati and Pfleiderer (1988) among others, that simultaneously determine the price and the volume of stock returns in accordance with the documented empirical regularities, could prove informative.

#### **4. Applications of ARCH to interest rate data**

The relationship between long- and short-term interest rates and the importance of a risk premium in explaining the term structure have received much attention during the last decade. For instance, Shiller (1979) and Singleton (1980) have both argued that long-term interest rates are too volatile to be established by the rational expectations theory of the term structure and a constant liquidity premium. This is also consistent with other studies that have found the estimators of future interest rates derived from the term structure under the assumption of rational expectations and a time-invariant risk premium to be biased. Subsequent attempts by Shiller, Campbell, and Schoenholtz (1983) and Mankiw and Summers (1984) among others to model particular forms of irrational expectations have largely been unsuccessful. However, as the degree of uncertainty for the different rates varies through time, so will the compensation required by a risk-averse investor, and a time-varying risk premium might therefore reconcile these

findings with market efficiency.<sup>25</sup> In the following sections, we shall discuss some of the papers which use ARCH techniques to model time-varying conditional second-order moments and risk premia in the term structure of interest rates.

#### 4.1. Model specification and volatility persistence

Modeling volatility clustering in interest rate data goes back at least to Fama (1976). However, the first explicit ARCH formulation is given in Weiss (1984), who estimates ARCH models on a set of sixteen different macroeconomic time series, including monthly data on AAA corporate bond yields. Very significant ARCH effects are evident. These findings have been confirmed in many subsequent studies, and as for stock returns the actual parameter estimates obtained from many of these models are indicative of high persistence in the volatility shocks, or IGARCH behavior. For instance, Hong (1988), on estimating a GARCH(1,1) model on the excess return of three-month Treasury bills over one-month Treasury bills, finds  $\hat{\alpha}_1 + \hat{\beta}_1 = 1.073$ . Similar results are reported in Engle, Lilien, and Robins (1987) using a linear ARCH(12) specification on quarterly data for the excess holding yield of six-month Treasury bills over three-month Treasury bills. At the same time, the estimates for twenty-year AAA corporate bonds suggest that for the longer end of the term structure, volatility shocks may be somewhat less persistent. A formal investigation of this issue would be interesting. Note also that the results in Engle, Ng, and Rothschild (1990) indicate that the underlying forces behind the volatility shocks for the shorter end of the term structure are common across the different rates, indicative of co-persistence in variance.

Whereas the simple ARCH models with conditionally normal errors have been found inadequate in capturing all the excess kurtosis for stock return and foreign exchange rates, less evidence along these lines is currently available for interest rates. Some exceptions include the studies by Lee and Tse (1991), who find significant evidence against conditional normality in the Singapore Asian dollar market using conditional  $t$  and Gram–Charlier distributions, and McCulloch (1985), who finds significant departures from conditional normality in U.S. data using the Adaptive Conditional Heteroskedasticity formulation. Also, most studies involving interest rates have adopted linear GARCH( $p, q$ ) specifications. However, as with the leverage effect for stock return data discussed in section 3.3, it is certainly possible that nonlinear dependencies exist in the conditional variance for interest

<sup>25</sup>Allowing for a unit root in the short rate could also explain the apparent excess volatility; see Campbell and Shiller (1991).

rates. A more systematic investigation of both of these issues would be interesting.

#### 4.2. ARCH-M and time-varying risk premia

In Engle, Lilien, and Robins (1987) the ARCH-M model is applied to quarterly data on the excess holding yield of six-month over three-month Treasury bills from 1960 to 1984. After experimenting with different functional forms a significant time-varying risk premium as proxied by the logarithm of the conditional variance is found to provide the best fit for the data. On average the term premium is only 0.14 quarterly percent, but it varies in a systematic way through the sample.<sup>26</sup> Interestingly, with the notable exception of the yield spread, variables which had previously been found successful in forecasting excess returns generally are no longer significant when a function of the conditional variance is included as a regressor. Similar results are reported in Baba (1984). However, the empirical findings for the six-month Treasury bill data have been called into question by Pagan and Sabau (1987b) who, on using several different tests for consistency, argue that the ARCH(12) variance equation is misspecified, resulting in inconsistent parameter estimates for the risk premium term.

The usefulness of the ARCH-M model for providing a good measure of risk has also been challenged on more theoretical grounds by Backus, Gregory, and Zin (1989). By generating data from an artificial Mehra and Prescott (1985) representative agent dynamic exchange economy in which the risk premia are known functions of the state, it is shown that in this economy the ARCH effects are more closely related to forecast errors than to the risk premium. This issue is pursued further in Backus and Gregory (1988), who show that there need be no relationship between the risk premium and conditional variances in their theoretical economy. In contrast, Morgan and Neave (1989) derive a theoretical model in which the return of a futures contract is linearly determined by its own conditional standard deviation. Using Treasury bill futures and Eurodollar futures contracts, the explicit ARCH-M specification suggested by the theory is generally supported empirically, although other variables such as day-of-the-week effects and the level of short-term interest rates are also found to be important. Among other extensions, more theoretical work along these lines could prove insightful. Also, the application of an ARCH-M framework might help shed light on the recently debated issue of the relationship between term structure and the

<sup>26</sup>Amsler (1985) uses the liquidity premium estimated by Engle, Lilien, and Robins (1987) in deriving the implied variance bound for the long versus short rate. Including this time-varying liquidity premium is found to widen the variance bound and weaken Shiller's (1979) conclusion of excess volatility.

Federal Reserve System; see Mankiw, Miron, and Weil (1987, 1990), Hardouvelis (1988), and Fische and Wohar (1990).

#### 4.3. Multivariate ARCH and the term structure

As discussed above, the theoretical motivation for the significant univariate ARCH-M relationships observed with short-term interest rates is somewhat lacking. Most asset pricing theories call for an explicit tradeoff between the expected returns and the conditional covariance(s) with some benchmark portfolio(s). For instance, according to the standard CAPM the expected returns are proportional to the covariance of the returns with the market portfolio. As discussed in section 3.5 above, Bollerslev, Engle, and Wooldridge (1988) use a trivariate GARCH(1,1)-M model to implement a CAPM with time-varying covariances, assuming the market consists of only bills, bonds, and stocks. Interestingly, the nondiversifiable risk as measured by the time-varying conditional covariance with the market is found to provide a better explanation for the term premia than does the own conditional variance from the corresponding univariate GARCH(1,1)-M models. The implied betas for both bills and bonds are also found to be time-varying and forecastable. Similarly, Evans (1989) employs a multivariate ARCH-M approach in estimating and testing an intertemporal CAPM in which the betas are allowed to change through time, and finds that the ICAPM is not rejected if the benchmark portfolio is taken to include both stocks and real estate.

The pricing of the short end of the term structure is studied in Engle, Ng, and Rothschild (1990) using data on two-month through twelve-month Treasury bills. Interestingly, on applying both one- and two-factor versions of the factor ARCH model, an equally-weighted bill portfolio is found to be effective in predicting both the volatility and the risk premium across the different maturities. Engle and Ng (1990) use a similar model to study the shape of the yield curve through time and the effect of yield shocks on volatility; see also the empirical evidence in Steeley (1990) pertaining to U.K. data. Among many other promising extensions, future work for the longer end of the term structure would be desirable.

In a different application, Evans and Wachtel (1990) investigate the effects of movements of output and inflation on interest rates based on a generalized Fisher equation derived from the consumption CAPM. Using monthly data and an indirect two-step estimation procedure, Evans and Wachtel (1990) argue that, in contrast to the standard Fisher equation, the consumption-based CAPM generalization with time-varying conditional covariances and time-varying coefficients adequately explains the dynamics of short-term interest rates.

#### 4.4. Dynamic hedging

The traditional estimate of the risk-minimizing hedge ratio is found by regressing the instrument being hedged on the hedging instrument, corresponding to an estimate of the unconditional covariance divided by the unconditional variance. This is also the approach taken in the study by Park and Bera (1987), where estimates for the risk-minimizing hedge ratios with spot and futures mortgage rates (GNMA) are presented. Park and Bera (1987) find that, when cross-hedging is involved, the regression residuals are characterized by ARCH, and more efficient estimates of the hedge ratio are obtained by explicitly modeling the heteroskedasticity using a simple linear ARCH(1) model; see also Bera, Park, and Bubnys (1987).

However, the systematic temporal variation observed in the conditional second-order moments for most high-frequency financial time series, including interest rates and interest rate futures, means that the hedge ratios which involve functions of the conditional variances and covariances will generally not be time-invariant. The multivariate ARCH model is ideally suited to addressing this question. This is the approach taken by Cecchetti, Cumby, and Figlewski (1988), where a bivariate linear ARCH(3) model with constant conditional correlations is estimated for monthly twenty-year Treasury bonds and Treasury bonds futures. Both the estimates for the risk-minimizing hedge ratio and the utility optimizing hedge ratio, obtained under the assumption of log utility, are found to exhibit substantial variation through the sample period, ranging between 0.52 and 0.91. Among many other interesting extensions, the same ideas could be used in the analysis of immunization and portfolio insurance strategies.

### 5. Applications of ARCH to foreign exchange rate data

The characterization of exchange rate movements, including second-order dynamics, have important implications for many issues in international finance. In addition to international asset pricing theories along the lines discussed in the previous two sections for domestic assets, international portfolio management obviously depends on expected exchange rate movements through time. Several policy-oriented questions relating to the impact of the exchange rate on different macroeconomic variables also require an understanding of the exchange rate dynamics.

#### 5.1. ARCH effects and model specification

As for other speculative prices, traditional time series models have not been able to capture the stylized facts of short-run exchange rate movements,

such as their contiguous periods of volatility and stability together with their leptokurtic unconditional distributions; see, e.g., Mussa (1979) and Friedman and Vandersteel (1982). As discussed above, the ARCH class of models is ideally suited to modeling such behavior. Whereas stock returns have been found to exhibit some degree of asymmetry in their conditional variances, the two-sided nature of the foreign exchange market makes such asymmetries less likely. In the absence of any structural model for the conditional variances, the linear GARCH( $p, q$ ) model in (7) therefore is a natural candidate for modeling exchange rate dynamics.

For example, using daily data on five different nominal U.S. dollar rates, Hsieh (1988a) argues that the conditional distributions of the daily nominal returns are changing through time, as evidenced by highly significant autocorrelations for the squared returns, but that an ARCH(12) model with linearly declining lag structure captures most of the nonlinear stochastic dependencies present; see also Milhøj (1987a), Diebold (1988), and Diebold and Nerlove (1989).<sup>27</sup> These findings are corroborated in the later papers by Hsieh (1989a, b) using GARCH(1, 1) type formulations.<sup>28</sup> Interestingly, judged on the basis of the BDS test for nonlinear dependencies discussed in section 3.1 and the Ljung–Box test for the standardized squared residuals, the simple GARCH(1, 1) model does better in describing the data than the ARCH(12) model estimated in Hsieh (1988a). Similar conclusions are reached in the studies by Taylor (1986), McCurdy and Morgan (1988), Kugler and Lenz (1990), and Papell and Sayers (1990).

Of course, as for other speculative prices, it is possible that the significant ARCH effects could be due to misspecified first-order dynamics resulting in dependence in the higher-order conditional moments. However, if such nonlinear dependence is present in the conditional mean it should be exploitable for forecasting purposes. Interestingly, in a detailed nonparametric analysis using locally-weighted regression techniques for ten weekly U.S. dollar exchange rates, Diebold and Nason (1990) find that forecasts based on these nonparametric estimates lead to no improvement in forecast accuracy when compared to the forecasts from a simple martingale model, consistent with the idea that any significant dependencies in short-run exchange rate movements work through the conditional variance and higher even-ordered conditional moments only. Similar conclusions are reached in the studies by

<sup>27</sup>Tsay (1987), on using a generalization of the time-varying parameter formulation of the standard linear ARCH( $q$ ) model as discussed in section 2.1, finds that when allowing for cross-parameter correlations the estimates from this model with weekly data on the British pound/U.S. dollar exchange rate are very close to the results obtained with a conventional linear ARCH(12) model.

<sup>28</sup>Only for the British pound, as analyzed further in Gallant, Hsieh, and Tauchen (1989) using seminonparametric methods, is there any substantial evidence against the GARCH(1, 1) model including deterministic vacation effects in the conditional variance as a simple parsimonious representation of the daily nominal rates.

Meese and Rose (1991) and Kim (1989), though Taylor (1990b) surprisingly argues that the conditional mean can be predicted well enough to obtain net trading profits.

While ARCH effects are highly significant with daily and weekly data, both Diebold (1988) and Baillie and Bollerslev (1989) have noted that ARCH effects tend to weaken with less frequently sampled data. For example, in Baillie and Bollerslev (1989) the average Ljung–Box portmanteau test for the first ten autocorrelations for the squared logarithmic first difference of the exchange rates averaged across the six currencies decreases gradually from a highly significant 130.6 for daily data to an insignificant 10.6 for data sampled monthly. This is in accordance with the asymptotic results in Diebold (1986b, 1988), and as shown in Drost and Nijman (1991) the actual parameter estimates obtained by Baillie and Bollerslev (1989) for the GARCH(1,1) models with less frequently sampled data may also be explained by aggregation effects. For most domestic assets the empirical evidence pertaining to temporal aggregation is less clear, possible due to compounding higher-order nonlinear dependencies. A detailed empirical study of these issues across different asset categories seems worthwhile.

## 5.2. Nonnormal conditional densities

While the simple symmetric linear GARCH(1,1) model may provide a good description of the second-order dynamics for most exchange rate series over the post-1973 free float, the assumption of conditional normality does not capture all the excess kurtosis observed in daily or weekly data; see McCurdy and Morgan (1987), Milhøj (1987a), Hsieh (1989a), and Baillie and Bollerslev (1989). As discussed in section 2.3 the resulting QML estimates obtained under the assumption of conditional normality are generally consistent and asymptotically normally distributed but the asymptotic covariance matrix of the parameter estimates will have to be appropriately modified. However, in many applications, including options pricing, a complete characterization of the distribution for the spot rates and not just the conditional variance are of interest.

Following the discussion in the previous sections, several alternative conditional error distributions have consequently been employed in the literature. Baillie and Bollerslev (1989) find that the Student- $t$  distribution compares favorably to the power exponential and captures the excess kurtosis for most of the rates. The Student- $t$  distribution is also estimated by Hsieh (1989a), together with the generalized error distribution, a normal-Poisson, and a normal-lognormal mixture distribution. It is also worth noting the results in Jorion (1988), where the jump-diffusion ARCH(1) model discussed in section 3.2 above is estimated for weekly data on the Deutschemark/U.S. dollar rate for the 1974–1985 period. Based on a standard likelihood ratio test, both the

jump process parameters and the ARCH parameters are jointly significant, consistent with the presence of excess kurtosis in the standardized residuals from conventional ARCH models.

In a related context, Lastrapes (1989) finds, not surprisingly, that including dummy variables in the conditional variance to allow for changes in the policy of the FED reduces the degree of leptokurtosis in the standardized residuals.<sup>29</sup> Similarly, McCurdy and Morgan (1988) find that departures from conditional normality tend to be associated with a few specific policy events. Further work trying to endogenously determine the timing of major exchange rate movements and changes in regimes would be interesting and could help explain part of the remaining leptokurtosis; see also Engel and Hamilton (1990).

### 5.3. Nonlinear and nonparametric ARCH

As discussed in the previous section, several authors have noted deviations from normality in the standardized residuals from estimated linear GARCH( $p, q$ ) models, and successfully proceeded to characterize these deviations by some parametric leptokurtic density. Alternatively, following the discussion in section 2.4, a nonparametric procedure could be employed. This is the approach taken by Gallant, Hsieh, and Tauchen (1989), where the seminonparametric technique of Gallant and Tauchen (1989) is used in estimating a model for the 'recalcitrant' British pound/U.S. dollar rate analyzed in Hsieh (1989a). The leading term in the expansion for the conditional density resembles the conventional linear ARCH model, and contrary to other speculative prices, the response of the conditional variance to negative and positive surprises is virtually symmetric. However, the estimated conditional density has interesting hump-shaped tails. This same shape is also evident in the results reported in Engle and Gonzalez-Rivera (1991), where nonparametric density estimation is used in characterizing the distribution of the standardized residuals from a GARCH(1, 1) model for the same rate and sample period. It is likely that this particular pattern is influenced by a few observations, and therefore peculiar to the given period. In fact, Bollerslev (1987) and Baillie and Bollerslev (1989) on analyzing data for the British pound for 1980–1985, i.e., excluding data from the 1970's, find little evidence against the simple GARCH(1, 1) model with  $t$ -distributed errors.

### 5.4. Sources of intermarket and intramarket volatility

Maintaining market efficiency, the pronounced ARCH effects present with high-frequency data could be due to the amount of information or the quality

<sup>29</sup>Also, the degree of persistence in the conditional variance is diminished.

of the information reaching the market in clusters, or from the time it takes market participants to fully process the information; see, e.g., Diebold and Nerlove (1989) and Gallant, Hsieh, and Tauchen (1989). In order to show that information processing is the source of the volatility clustering, Engle, Ito, and Lin (1990a) define four separate market locations: Europe, New York, Pacific, and Tokyo. If the information arrivals in one market are uncorrelated with the information arrivals in any other market, a test of whether increased volatility in one market causes an increase in volatility in another market is in effect a test of information processing as the source of volatility clustering. The results in Engle, Ito, and Lin (1990a) with intraday observations on the Japanese yen/U.S. dollar rate show that, except for the Tokyo market, each market's volatility is significantly affected by changes in volatility in the other markets, so that volatility is transmitted through time and different market locations as a 'meteor shower', lending support to the information processing hypothesis. Information processing as the main determinant behind the volatility spillovers is also consistent with the evidence reported in Engle, Ito, and Lin (1990b), who rule out the influence of stochastic policy coordination on the basis of equally important volatility spillovers in the early 1980's, a period known for little international policy coordination. Lin (1989) on applying a multivariate factor ARCH model reaches a similar conclusion.

Using hourly data on four major U.S. currencies during the first half of 1986, Baillie and Bollerslev (1991) also examine the causal relationship between returns and volatility. Significant evidence for the 'meteor shower' hypothesis is again evident. Interestingly, however, Baillie and Bollerslev (1991) also report some evidence for market-specific volatility, after taking account of deterministic patterns across the trading day. Furthermore, the volatility during the day is found to exhibit a very distinct and remarkably similar pattern for all four rates, with increases occurring around the opening and closing of each of the three major world markets, i.e., London, New York, and Tokyo. Consistent with the findings in Whistler (1988), the U.S. market is overall the most volatile, followed by the European market.

The implementation and tests of more structural models consistent with the empirical findings discussed above would clearly be of interest and could help in gaining some further understanding about the underlying market micro structure theories at work. The empirical analysis of higher-frequency data, such as the continuously recorded bid and ask quotations described in Goodhart (1990), also hold the promise of important insights along these lines.

### 5.5. *Volatility persistence*

In accordance with the findings for stock returns and interest rates, the persistence of volatility shocks in the foreign exchange market is also very

high. For instance, Engle and Bollerslev (1986), on estimating a GARCH(1, 1) model for weekly data on the U.S. dollar vis-a-vis the Swiss franc, finds  $\alpha_1 + \beta_1 = 0.996$ , providing a motivation for the Integrated GARCH, or IGARCH, class of models discussed above. Very similar results are reported in Bollerslev (1987), McCurdy and Morgan (1987, 1988), Hsieh (1988a), Kim (1989), Baillie and Bollerslev (1989), Hsieh (1989a), and Taylor (1990a).<sup>30</sup>

Even though many different currencies may exhibit IGARCH-type behavior, it is certainly possible that this persistence is common across different rates.<sup>31</sup> The presence of such co-persistence among the variances has many important practical implications (e.g., in optimal portfolio allocation decisions involving a trade-off between future expected returns and the associated risk). The empirical relevance of this idea has been illustrated within the context of a bivariate GARCH(1, 1) model by Bollerslev and Engle (1990), where it is found that most of the persistence in the conditional covariance matrix for the Deutschmark and the British pound/U.S. dollar rates derives from some common set of underlying forcing variables, and that the corresponding bilateral Deutschmark/British pound rate has much less persistent volatility shocks. In addition to further theoretical work along these lines, extensions of the limited empirical evidence to other currencies and asset categories would be desirable.

### 5.6. ARCH-M models and the risk premium

A growing body of literature has found that the forward rate is not an unbiased predictor of the corresponding future spot rate; see, e.g., Hakkio (1981), Hsieh (1984), Baillie (1989), and McCurdy and Morgan (1991a). Assuming that expectations are rational, a risk premium can reconcile this observation with market efficiency, and several theoretical models have been formulated which generate risk premia in foreign exchange markets.<sup>32</sup> Examples include Hodrick and Srivastava (1984), Domowitz and Hakkio (1985), Diebold and Pauly (1988a), and Kendall (1989). According to most of these theories, the risk premium depends on some function of the conditional probability distribution of the future spot rate. Given the evidence in the previous sections pertaining to the time-varying nature of the conditional

<sup>30</sup>Somewhat puzzling, for the hourly GARCH(24, 1) models with hourly dummy variables in the conditional variances reported by Baillie and Bollerslev (1991), the estimates indicate much less persistence, with  $\hat{\alpha}_1 + \hat{\alpha}_{24} + \hat{\beta}_1$  between 0.374 and 0.771 only.

<sup>31</sup>For example, in a study similar to Lamoureux and Lastrapes (1990a), Connolly (1990) finds that using contemporaneous volume in the conditional variance equation for the yen/dollar spot rate tends to decrease the measured persistence, suggesting a common forcing variable for volume and volatility.

<sup>32</sup>An alternative explanation consistent with market efficiency would be the restriction imposed by limit moves. However, in a detailed empirical analysis, Kodres (1990) finds little support for this hypothesis.

distribution of spot exchange rates, this may therefore result in a time-varying risk premium. Several different specifications and proxies for this risk premium have been used empirically, many of which depend directly on the conditional variance of the spot rate; see Hodrick (1987) for an excellent survey of this literature.

The first attempts by Domowitz and Hakkio (1985) and Diebold and Pauly (1988a) at modeling such a time-varying risk premium in the forward foreign exchange market within a univariate ARCH-M framework were largely unsuccessful. Several explanations for this are possible. For example, the problem of determining who is compensated for risk in an exchange economy might argue against the constancy of the ARCH-M parameter, leading to insignificant results.<sup>33</sup> An alternative explanation is that both studies use monthly data, which as noted in section 5.1 generally shows only minimal ARCH effects, thereby leading to insignificant findings.<sup>34</sup>

Indeed, Kendall and McDonald (1989) on using weekly data for the Australian/U.S. dollar and a GARCH(1,1)-M model obtain a significant estimate for the ARCH-M parameter. Conversely, the results in McCurdy and Morgan (1988) with daily and weekly futures data, and in Kendall (1989) with weekly spot data do not support a significant simple mean–variance tradeoff. However, since the conditional variance merely serves as a proxy for the risk premium, a more structural based multivariate approach is likely to be superior from a theoretical perspective.

Before we turn to a discussion of the implementation of such multivariate models, it is also worth noting the analysis in Hodrick (1989), where an ARCH-M-type model is used to examine how the exchange rate is affected by the uncertainty in the inflation rate, monetary policy, and income growth. A two-step procedure is employed in which the exogenous conditional variances are estimated from a set of linear ARCH(1) models, and subsequently used as regressors to explain the monthly movements in the U.S. exchange rate for Japan, West Germany, and the United Kingdom. As the conditional variance estimates again show little temporal variation on a monthly basis, the results for the formal monetary cash-in-advance model are somewhat disappointing, but holds the promise of important future insight.

<sup>33</sup>Also, Frankel (1986) argues that the risk premium must be small because it is determined by the conditional variance of the difference between the change in the spot rate and the forward discount, which is bounded by the unconditional variance. However, as Pagan (1988) points out, this argument is not true if the conditional variance is changing through time. Thus as noted in Frankel (1988), only the average risk premium must be small.

<sup>34</sup>In contrast, Pagan and Ullah (1988) find strong support for the presence of a time-varying risk premium in the Canadian/U.S. dollar market with monthly data over the earlier time period from 1970–1978. The risk premium here is proxied by a simple linear function of a nonparametric estimate for the conditional variance obtained from a normal kernel; cf. section 2.4. However, this might be driven by the influence of the Quebec crisis.

Of course, from a more technical point of view, the indirect two-step procedure is subject to the same criticism as discussed in section 2.8 above.

### 5.7. Multivariate ARCH models and asset pricing

Several authors have speculated that the weak results that have been found in the foreign exchange market using univariate ARCH-M models to estimate time-varying risk premia might be due to the conditional variances being poor proxies for risk; see, e.g., Domowitz and Hakkio (1985), McCurdy and Morgan (1987, 1988), Diebold and Pauly (1988a), Lee (1988), Thomas and Wickens (1989), and Baillie and Bollerslev (1990). In particular, the premium might be better approximated by a function of the time-varying cross-currency conditional covariances and not just the own conditional variance.

Indirect support for this hypothesis is provided by Lee (1988), who finds that the conditional covariance between the Deutschmark and the Japanese yen/U.S. dollar spot rates, as modeled by a bivariate ARCH(12) model, helps explain the weekly movements in the yen/U.S. dollar rate. The results in Baillie and Bollerslev (1990) with weekly data and a four-dimensional GARCH(1, 1) model for the one-month-forward rate forecast error for four European currencies also indicate highly significant contemporaneous correlations. However, the time-varying conditional covariances do not yield any improvement in forecast accuracy beyond the MA(4) correlation structure in the overlapping forward rate forecast errors implied from a simple martingale model for the spot rates.

More formal tests for mean-variance efficiency and alternative pricing formulations have also found their implementation in the foreign exchange market. Attanasio and Edey (1988), Mark (1988), Engel and Rodrigues (1989), and Giovannini and Jorion (1989) all estimate and test specifications of the international CAPM in Frankel (1982), while explicitly allowing for a time-varying conditional covariance matrix. Modeling the temporal dependence in the second-order moments generally leads to significantly better performance of the model and a more precise estimate of the coefficient of relative risk aversion. Nonetheless, both Engel and Rodrigues (1989) using five monthly U.S. exchange rates and Giovannini and Jorion (1989) with weekly data on three U.S. currencies and a stock market index, formally reject the restrictions implied by the CAPM. An alternative structural based approach is taken by Kaminsky and Peruga (1990), who estimate a version of the intertemporal consumption-based CAPM in which the risk premium is a function of the time-varying conditional covariances between the future spot rate and consumption. Using monthly data together with a multivariate ARCH(1) formulation little support for the model is forthcoming. Of course, the model may still provide a good description over shorter time intervals

than one month, but the availability of data complicates such an analysis; see McCurdy and Morgan (1991a). In fact, using weekly foreign exchange rates, McCurdy and Morgan (1991b) find evidence of a significant time-varying risk premium in deviations from uncovered interest rate parity, where the risk premium is given by the conditional covariance with a benchmark portfolio set equal to the return on a worldwide equity index.

While the studies discussed above have highlighted the importance of accounting for short-lived temporal variation in both conditional variances and covariances, a completely satisfactory model for the time-varying risk premium in the forward foreign exchange market has yet to be formulated.

### 5.8. Multivariate ARCH models, policy analysis, and dynamic hedging

Multivariate ARCH models have also been useful in addressing various policy issues related to the foreign exchange market. For instance, Diebold and Pauly (1988b) and Bollerslev (1990) study the effect on short-run exchange rate volatility following the creation of the European Monetary System (EMS). Both studies find an increase in the conditional variances and covariances among the different European rates after the 1979 inception of the EMS. At the same time, Bollerslev (1990), on estimating a multivariate GARCH(1, 1) model with constant conditional correlations, argues that the coherence also increased over the EMS period, possibly as a result of the increased policy coordination among the member countries.

In a series of recent papers, the effect of central bank interventions on foreign exchange dynamics have been analyzed in the context of a GARCH formulation by Connolly and Taylor (1990), Humpage and Osterbert (1990), and Mundaca (1990). A common finding across these studies concerns the positive correlation between current intervention and exchange rate volatility. However, further analysis regarding the simultaneous determination of exchange rates and intervention policies seems warranted.

Other macroeconomic motivated applications include Kroner and Lastrapes (1991), who use a multivariate GARCH(1, 1)-M model to show that exchange rate uncertainty significantly affects the level and the price of trade in the economy. In a related context, Kroner and Claessens (1991) present a dynamic multiple hedging model based on the intertemporal CAPM in which the optimal hedging portfolio is a function of the time-varying variances and covariances. Using a multivariate GARCH(1, 1) model, the optimal debt portfolios for Indonesia are estimated.

Given the substantial increase in international portfolio diversification by many investors and institutions in recent years coupled with the complex second-order dynamics of short-run exchange rate movements, it would be very interesting to extend the analysis in Kroner and Claessens (1991) and Cecchetti, Cumby, and Figlewski (1988), discussed in section 4.3 above, to

optimal dynamic hedging strategies for the currency risk involved with direct short-term investment in foreign assets. The results in Kroner and Sultan (1991) pertaining to the yen and Baillie and Myers (1991) for different commodities are encouraging.

## 6. Conclusion

Volatility is a key variable which permeates most financial instruments and plays a central role in many areas of finance. For example, volatility is crucially important in asset pricing models and dynamic hedging strategies as well as in the determination of options prices. From an empirical standpoint, it is therefore of utmost importance to carefully model any temporal variation in the volatility process. The ARCH model and its various extensions have proven very effective tools along these lines. Indeed, by any yardstick, the literature on ARCH has expanded dramatically since the seminal paper by Engle (1982). However, many interesting research topics remain to be examined, some of which are discussed above and others of which the reader will undoubtedly glean upon reading this survey. It is our hope that this overview of the extensive ARCH literature may serve as a catalyst in fostering further research in this important area.

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