

WHY WE SHOULD NOT MAKE MEAN LOG OF WEALTH BIG THOUGH YEARS TO ACT ARE LONG

Paul A. SAMUELSON*

Massachusetts Institute of Technology, Cambridge, MA 02139, USA

He who acts in N plays to make his mean log of wealth as big as it can be made will, with odds that go to one as N soars, beat me who acts to meet my own tastes for risk.

Who doubts *that*? What we do doubt¹ is that *it* should make us change our views on gains and losses – should taint our tastes for risk.

To be clear is to be found out. Know that life is not a game with net stake of one when you beat your twin, and with net stake of nought when you do not. A win of ten is not the same as a win of two. Nor is a loss of two the same as a loss of three. *How much* you win by counts. *How much* you lose by counts.

As soon as we see *this* clear truth, we are back to our own tastes for risk. Mean log of wealth then bores those of us with tastes for risk not real near to one odd (thin!) point on the line of *all* the tastes for risk – and this holds for each N , with N as big as you like.

Why then do some still think they should want to make mean log of wealth big? They nod. They feel ‘That way I *must* end up with more. More sure beats less’. But they err. What they do not see is this:

When you lose – and you *sure can* lose – with N large, you can lose real big. Q.E.D.

Long since, in Samuelson (1963, p. 4), I had to prove what is not hard to grasp:

If it does not pay to do an act once, it will not pay to do it twice, thrice, . . . , or at all.

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¹Cf. the views of Ophir (1978, 1979) and Latané (1978).

Can we bring the dead rule back to life by what it tells us about the mean growth rate? No. Here's why not.

For large N , when you act at each turn to make the mean of log of wealth big, you will make your *mean growth rate* big in this sense:

As N grows large, the odds go to one that my mean growth rate (per turn) will end up real close to a rate less than that which you (with big odds) *end up* close to.

Who doubts *that* truth? But *it* does not rule out this clear truth.

For N as large as one likes, your growth rate can well (and at times must) turn out to be less than mine – and turn out so much less that my tastes for risk will force me to shun your mode of play. To make N large will *not* (say it again, *not*) make me change my mind so as to tempt me to your mode of play. Q.E.D.

No doubt some will say: 'I'm not sure of my taste for risk. I lack a rule to act on. So I grasp at one that at least ends doubt: better to act to make the odds big that I win than to be left in doubt?' Not so. There is more than one rule to end doubt. Why pick on one odd one? Why not try to come a bit more close to that which is not clear but which you ought to try to make more clear?

No need to say more.² I've made my point.³ And, save for the last word, have done so in prose of but one syllable.

²We should spare the dead. When a chap has said he now doubts that '...this same rule [of max of mean of log of wealth] is approximately valid for all utility functions [...insofar as certain approximations are permissible...]', we should take him at his word and free his shade of all guilt. For a live friend to still say: 'given the qualifications it seems to me that this [above quoted] statement of Savage is very difficult to refute', as the French say, gives one to cry. Those key words are false when we make them clear. When we don't make them clear, there is nought to talk about (to say Yes or No to). As the French say too, it is a case of put up or ... For more on this, see Latané (1959, p. 151; 1978, p. 397) and Samuelson (1959, p. 245).

³Let me tie down one loose end. Look at this Odd Rule:

From Acts $A(N)$ and $B(N)$ pick Act $A(N)$ if, for their two end wealths, $W_A(N)$ and $W_B(N)$, with odds of more than one half (or more than $1 - \varepsilon_N$, $0 < \varepsilon_N \ll 1$), $W_A(N) > W_B(N)$.

This Odd Rule is odd since it can put you in this Fix:

You may well pick $A(N)$ from $A(N)$ and $B(N)$, and pick $B(N)$ from $B(N)$ and $C(N)$, and yet still pick $C(N)$ from $A(N)$ and $C(N)$.

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This Fix *can* come for *all* N , as large as we choose to make N . It can do so though the truth of Thorp (1971, p. 603) holds,

$$\operatorname{plim}_{N \rightarrow \infty} W_A(N) = \bar{W}_A > \bar{W}_B = \operatorname{plim}_{N \rightarrow \infty} W_B(N), \quad \bar{W}_B > \bar{W}_C = \operatorname{plim}_{N \rightarrow \infty} W_C(N),$$

rules out

$$\operatorname{plim}_{N \rightarrow \infty} W_C(N) > \operatorname{plim}_{N \rightarrow \infty} W_A(N).$$

That is so since $\bar{W}_A > \bar{W}_B$ and $\bar{W}_B > \bar{W}_C$ means $\bar{W}_A > \bar{W}_C$.

But I'd not said: 'When you act to make mean of log of wealth large, you could get in the Fix'. To see why you can't, we need only note that

$$\text{mean of log of } W_A(N) = L_A(N) > L_B(N) = \text{mean of log of } W_B(N),$$

and

$$\text{mean of log of } W_B(N) = L_B(N) > L_C(N) = \text{mean of log of } W_C(N),$$

rules out

$$L_C(N) > L_A(N),$$

and it does so for *all* N , as when N is as small as one.

Some goals are strange, but need not be as bad as the odd rules some seek to base them on.