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The St. Petersburg Paradox: A Discussion of Some Recent Comments*

The recent notes by Brito [1] and by Shapley [3] on the St. Petersburg paradox are provocative and certainly shed new light on this old topic, but they miss the point of the paradox. Essentially, Brito's argument is that the bound on the duration of an individual's life imposes a bound on the amount of commodities that he can consume. The utility of this maximal consumption then provides a bound on the range of the utility function, and with a bounded utility there is no paradox.[1] Shapley, though he agrees that utility may in principle be unbounded, argues that no person or institution is in a position to guarantee payment of arbitrarily large sums, so that in fact there can exist no "reliable" lottery ticket with arbitrarily large payoffs. Effectively, therefore, the utilities that appear in lottery tickets are bounded, and again the paradox disappears.

Neither argument is convincing.[2] Brito assumes that the payoffs to lottery tickets are expressible in terms of commodity bundles of a fixed finite dimension l, and that the utility function is defined and finite on the entire non-negative orthant of l space.[3] Shapley assumes that the lottery ticket represents an obligation to the individual in question—let's call him Paul—undertaken by some other individual or institution, whom we may call Peter. But these assumptions, though they are part of the classical presentation of the paradox, seem irrelevant and artificial. The payoffs need not be expressible in terms of a fixed finite number of commodities, or in terms of commodities at all; and the probabilities in the "lottery ticket" need not represent the promises of Peter, but could simply correspond to Paul's own evaluation of the

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[1] Menger [2, pp. 217 ff.], has pointed out that the paradox is constructible if and only if utility is unbounded. See also [1, p. 123].

[2] Shapley concentrates his fire not so much on the paradox itself, but on the view that it constitutes an argument against risk neutrality for money. In this he is right; though as he himself acknowledges, one does not need the St. Petersburg paradox to reject risk neutrality. But it appears from his paper that he thinks that he has actually resolved the whole paradox; and in this we think he is wrong.

[3] Or alternatively, that the consumption space is compact.

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situation.[4] If this is the case there is nothing to prevent the construction of a lottery ticket with an infinite expected utility, and then the full force of the paradox is upon us.

For example, the lottery ticket that Paul is considering might be some kind of open-ended activity—one that could lead to sensations that he has not heretofore experienced. Examples might be religious, aesthetic, or emotional experiences, like entering a monastery, climbing a mountain, or engaging in research with possibly spectacular results. It seems reasonable to suppose that before engaging in such an activity, Paul would perceive the utility of the resulting sensation as a random variable, and there is no particular reason to assume that this random variable is bounded. Shapley's credibility issue would of course not arise in such a situation.

In my opinion the simplest, most straightforward, and most natural resolution of the paradox lies in the conclusion that utility must be bounded. Unbounded utility would lead to counterintuitive conclusions even without the St. Petersburg paradox. If Paul's utility were unbounded, then for any fixed prospect x (e.g., a long, happy, and useful life), there would be a prospect y with the property that Paul would prefer a lottery yielding y with probability $1/10^{100}$ and death with the complementary probability to the prospect x. This, I think, is about as hard to swallow as the idea of infinite utility.[5]

References

1. D. L. Brito, Becker's theory of the allocation of time and the St. Petersburg paradox, *J. Econ. Theory* **10** (1975), 123–126.
2. K. Menger, The role of uncertainty in economics, *in* "Essays in Mathematical Economics in Honor of Oskar Morgenstern" (Martin Shubik, Ed.), pp. 211–231, Princeton Univ.

[4] It is conceivable that Shapley is merely saying in a roundabout way that resources are bounded; that Paul cannot reasonably assign positive probability to receiving more than the world's GNP, whether he is buying a lottery ticket from Peter or whether he is entering business on his own. In that case, of course, the argument is essentially the same as Brito's.

[5] Basically, this is merely a restatement of the St. Petersburg argument. It is essentially due to Menger [2, p. 221], though he himself appears to have misunderstood its implications. He writes that "most people would refuse to risk one dollar in order to obtain a probability of 1/10,000,000 of winning an amount which has even a subjective value of $10 million." It is of course not clear what is meant by "an amount with a subjective value of $10,000,000." One interpretation might be an amount x whose utility $u(x)$ is 10,000,000 $u(\$1)$. But by the definition of utility, Paul *would* then be willing to risk $1 in order to get a 1/10,000,000 chance at x. One can make sense of the above quotation from Menger only by turning it around to say that for most people there is *no* prospect x for which they would risk $1, when the probability of winning is only 1/10,000,000. But this simply means that utility is bounded by 10,000,000 $u(\$1)$.

Press, Princeton, N. J., 1967. (Translated by Wolfgang Schoellkopf from the *Z. Nationalokonomie* **5** (1934), 349–485.)
3. L. S. SHAPLEY, The St. Petersburg paradox: A con game?, *J. Econ. Theory* **14** (1977), 439–442.

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