# We Don't Quite Know What We Are Talking About When We Talk About Volatility

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ABSTRACT

Finance professionals, who are regularly exposed to notions of volatility, seem to confuse ***mean absolute deviation*** with ***standard deviation***, causing an underestimation of 25% with theoretical Gaussian variables. In some “fat tailed” markets the underestimation can be up to 90%. The mental substitution of the two measures is consequential for decision making and the perception of market variability.

## INTRODUCTION

There is no particular normative reason to express or measure volatility in one of several possible ways, provided one remains consistent. However, once it is expressed, substituting one measure for another will lead to a consequential mistake. Suppose one measures "volatility" in root-mean-square deviations from the mean, as used by conventional statistics. It would be an error to substitute the definition and consider it mean deviation in the activity of decision-making, opinion formation, or verbal descriptions of the property of the process. Yet, to preview what we find in a survey of over 87 people trained to know the difference, people make this mistake.

This brief note provides experimental evidence that participants, with varied backgrounds in financial markets (whether traders, quantitative analysts, graduate students in financial engineering, or portfolio managers) make the mistake of interpreting a physical description (in mean absolute returns per day) as a calculated measure (standard deviation). We illustrate the confusion experimentally and conclude by discussing implications for financial decision making and portfolio risk management.

## EXPERIMENTS

To investigate common understanding of mean absolute deviation, we asked professionals and students of finance the following question:

> ***A stock (or a fund) has an average return of 0%. It moves on average 1% a day in absolute value; the average up move is 1% and the average down move is 1%. It does not mean that all up moves are 1%--some are .6%, others 1.45%, etc. Assume that we live in the Gaussian world in which the returns (or daily percentage moves) can be safely modeled using a Normal Distribution. Assume that a year has 256 business days. The following questions concern the standard deviation of returns (i.e., of the percentage moves), the "sigma" that is used for volatility in financial applications. What is the daily sigma? What is the yearly sigma?***

Our suspicion that there would be considerable confusion was fed by years of hearing options traders make statements of the kind, "an instrument that has a daily standard deviation of 1% should move 1% a day on average". Not so. In the Gaussian world, where x is a random variable, assuming a mean of 0, in expectation, the ratio of standard deviation to mean deviation should satisfy the following equality $\frac{\sum |\boldsymbol{x}|}{\sqrt{\sum \boldsymbol{x}^2}} = \sqrt{\frac{2}{\pi}}$. Since mean absolute deviation is about .8 times the standard deviation, in our problem the daily sigma should be 1.25% and the yearly sigma should be 20% (which is the daily sigma annualized by multiplying by 16, the square root of the number of business days).

To test the hypothesis that mean absolute deviation is being confused with standard deviation, we ran three studies. We first posed this question to 97 portfolio managers, assistant portfolio managers, and analysts employed by investment management companies who were taking part in a professional seminar. The second group of participants comprised 13 Ivy League graduate students preparing for a career in financial engineering. The third group consisted of 16 investment professionals working for a major bank. The question was presented in writing and explained verbally to make sure definitions were clear. All respondents in the latter two groups handed in responses, but only 58 did so in the first group. One might expect this sort of self-selection to improve accuracy.

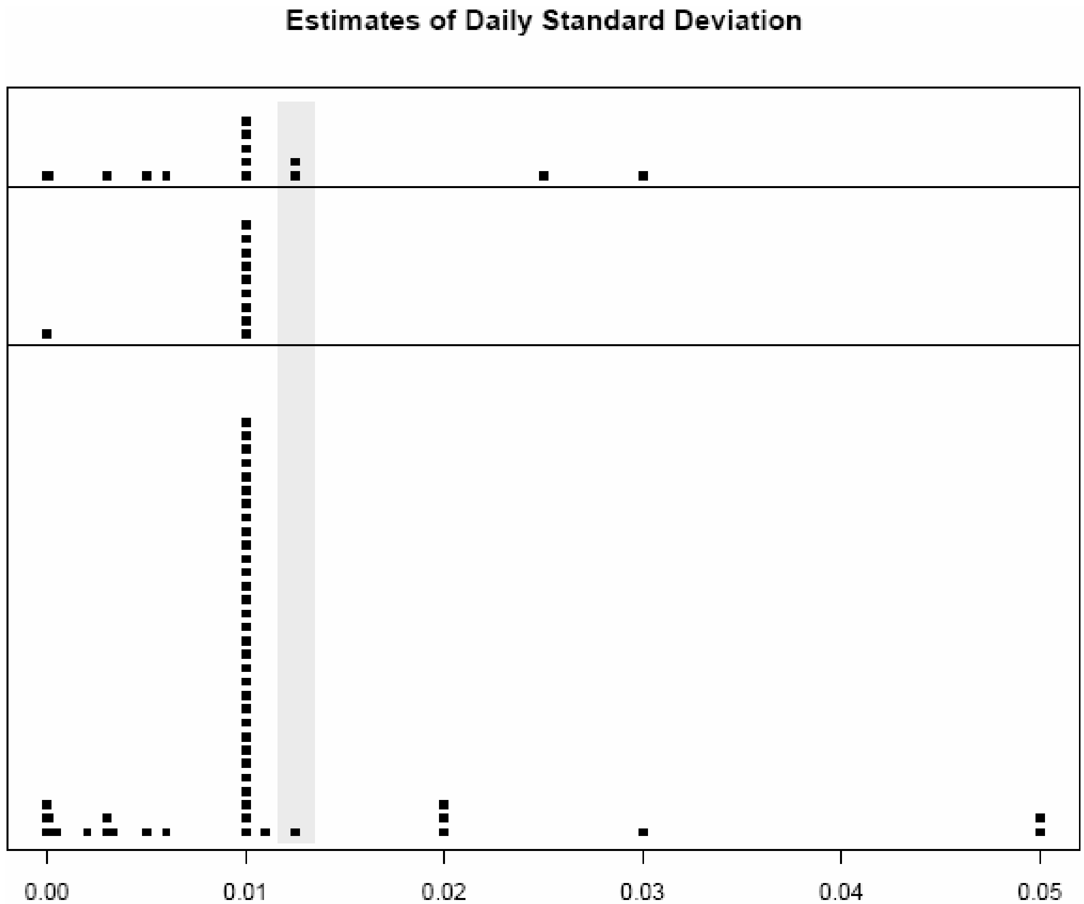


Figure 1: Estimates of standard deviation for a stock with a mean absolute deviation of .01. The top plot comprises the responses of investment professionals, the middle plot those of graduate students in financial engineering (excluding one outlier at .1), and the bottom plot those of professional portfolio managers and analysts. The correct answer under the stated Gaussian assumptions (.0125) is shaded in gray.

## RESULTS

Figure 1 shows frequency histograms of responses to the daily sigma question, converted to decimal notation. Only 3 of the 87 respondents arrived at the correct answer of .0125. The modal answer of .01 made up more than half of the responses received. Nine people handed in responses of blanks or question marks. Far from symmetrical, the ratio of underestimations of volatility to overestimations was 65 to 10.

Performance for yearly sigmas was even worse, with no one submitting a correct response. Here, the modal answer was .16, which appears to be the correct annualization of the incorrect daily volatility. Eleven people answered with blanks or question marks. The ratio of underestimations to overestimations was 76 to 9.

## CONCLUSION

The error is more consequential than it seems. The dominant response of 1% shown in Figure 1 suggests that even financially- and mathematically-savvy decision makers treat mean absolute deviation and standard deviation as the same thing. Though a Gaussian random variable that has a daily percentage move in absolute terms of 1% has a standard deviation of about 1.25%, it can reach up to 1.9% in empirical distributions (emerging market currencies and bonds). Mean absolute deviation is by Jensen's inequality lower (or equal) than standard deviation[1]. In a world of fat tails, the bias increases dramatically. Consider the following vector of dimension $10^6$, composed of 999,999 elements of 0 and a single one of $10^6$. V= $\{0,0,0,0,...., 10^6\}$. There the standard deviation would be 1,000 times the average move. For a Student's t with 3 degrees of freedom (often used to model returns in financial markets in Value-at-Risk simulations, see Bouchaud and Potters, 2003, and Glasserman, Heidelberger and Shahabuddin, 2002), standard deviation is 1.57 times mean deviation; $\frac{\sum |\boldsymbol{x}|}{\sqrt{\sum \boldsymbol{x}^2}} = \frac{2}{\pi}$.

Debriefings with respondents revealed that they rarely had an immediate understanding of the error when it was pointed out to them. However when asked to present the equation for "standard deviation" they expressed it flawlessly as the root mean square of deviations from

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1 More technically, the norm $L^p = \left(\frac{1}{\boldsymbol{n}}\sum|\boldsymbol{x}|^{\boldsymbol{p}}\right)^{1/\boldsymbol{p}}$ increases with p.

the mean. Whatever reason there was for their error, it did not result from ignorance of the concept. Indeed, most participants would have failed a basic statistics course had they not been aware of the mathematical definition. And yet, when given data that is clearly not a standard deviation, they treat it as one. Kahneman and Frederick (2002) discuss a similar problem of statisticians making basic statistical mistakes outside the classroom, "the mathematical psychologists who participated in the survey not only should have known better–they did know better...most of them would have computed the correct answers on the back of an envelope".

Why this is relevant? This sloppiness in translation between mathematics and applications can have severe effects, especially when considering that practitioners speak with co-workers, customers and the media about "volatility" on a regular basis. There are reported instances in the financial media to that effect, in which the journalist, while explaining the volatility index ***VIX*** to the general public, makes the same mistake.

Either we have the wrong intuition about the right volatility, or the right intuition and the measure of volatility is the wrong one. Two roads lead out of this unfortunate situation. Either we can continue defining volatility as we do, and conduct further research to see if the error can be made to disappear with training. Or we can do as the Enlightenment probabilists did; when the intuitions of ***hommes éclairés*** did not align with the valuations of the expected value formula, it was the mathematicians who dreamed up something that more intuitive, namely, expected utility (Daston, 1988). Perhaps, one day, Finance will adopt a more natural metric than standard deviation. Until then, operators should rely on definitions, not intuitions, where volatility is concerned.

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