

Statistics 434 Homework No. 4  
ARMA, ACF, PACF, and a Betting Simulation

**Reading**

- Review the material in Zivot and Wang on `arima.sim`, `arima.mle`, and `arima.forecast`. Also review the information on these functions using the S-Plus `help()`.

**Simulation and Estimation of an AR(2) or ARIMA(2,0,0) process**

- Let  $\phi_1 = 0.2$ ,  $\phi_2 = 0.15$ , and  $\sigma = .25$ . Simulate a series of length 1000 from the AR(2) model with these parameters by making appropriate use of the `innov` and `start.innov` function parameters. Use `set.seed(31415)`. Call the series  $X$ . Calculate the sample mean and variance of  $X$ . Comment on the relationship of this sample variance to the conditional variance of the model ( $\sigma = .25$ ). How does your sample variance compare with the unconditional variance of the AR(1) model with  $\phi_1 = 0.35$ . Don't press for "right" answer here. There is one, but this is mainly a chance to exercise your intuition.
- Forget how you got the mysterious series  $X$ , and imagine that it came to you as honest empirical data. Plot the sample ACF and the sample PACF for  $X$ . Do these plots give you a good hint for an appropriate model? Be bold, but be honest. Does our first-cut model selection technology work well, modestly, or poorly on this simulated data?
- Use a shotgun approach with `arima.mle()` and fit all of the AR(p) models  $p = 1, 2, 3, 4$  and 5. Do you have a preference? Comment on consistencies and inconsistencies.
- Now let's think about making bets using your AR(2) simulation model.
  1. Use simulation to decide what percentage of your bets would win if you used the rule bet that  $y_{t+1}$  is positive if  $y_t$  is positive.
  2. Use simulation to decide what percentage of your bets would win if you used the rule bet that  $y_{t+1}$  is positive if  $y_t$  is positive and  $y_{t-1}$  is positive.
- **Present your results with a one-page executive summary. Attached supporting code and graphics as you feel appropriate.**

**A Modest Model and the Possibility of Good Bets**

This problem suggests a caricature of a trading system, and, if you're favorably predisposed to systems, it may suggest ideas for making profitable bets. Whenever we decide that a series is well modeled by an AR(p), the signs of the coefficients suggests how recent values may influence the signs of future values. Under normal circumstances, just getting the signs right can be quite lucrative, and, as Dashiell Hammet might have said, good guesses are "the stuff that dreams are made of."