Statistics 930: Probability Theory

Professor J. Michael Steele

PREREQUISITES. This course is designed for graduate students in Statistics, OPIM, mathematics, applied mathematics, and related fields. (It is also open to *advanced* undergraduates who have had a course in real analysis that included Lebesgue integration.)

COURSE OBJECTIVES. The course is designed to provide a professional and rigorous understanding of the most fundamental theorems of probability theory. The central focus will be on sums of independent random variables, but the course will also deal to a lesser extent with martingales, stationary processes (Ergodic theory), and Markov processes. The level and extent of the course is well captured by the text: *Probability:Theory and Examples (Fourth Edition)* by Rick Durrett.

SPECIAL EMPHASIS ON PROBLEM SOLVING. The course will give much attention to problem solving. The intention is to engage probability theory as a living subject with discoveries just waiting to be made.

COURSE TOPICS:

Probability Spaces, Integration, and Independence: Sigma fields and why they matter. Tools of Expectation (Monotone Convergence Theorem, Dominated Convergence Theorem, Fatou's Lemma), Modes of convergence, especially convergence in probability and convergence with probability one, Inequalities of Markov, Chebyshev, and others, the Borel Cantelli Lemmas, Lovasz Local Lemma

Theory of the Law of Large Numbers: Easy SLLNs, Etemadi's Proof of SLLN, Kolmogorov's Three Series Theorem.

Characteristic Functions: Fundamental facts, including Fourier inversion. Applications in distribution theory. Essen's inequality.

Central Limit Theory: Proof of the CLT and its refinements using characteristic functions. Proof of the CLT by Lindeberg's method, Proof of the CLT by Stein's method.

Martingale Methods: Basic martingale theory including Doob's maximal inequality and several proofs of the martingale convergence theorem. The CLT for martingales.

Stationary Processes: The Ergodic theorem and several proofs. CLT for stationary processes.

TEXT, POLICIES, ETC.

Texts: *Probability:Theory and Examples (Fourth Edition)* by Rick Durrett

Homework: Regular homework will be assigned and graded. Homework solutions will **not** be provided; one is encouraged to ask about solutions, ideas, hints, etc. in class or office hours.

Grading: Grades are based on the homework (30%), midterm (20%) and a final exam (50%).

Auditors: Auditors are welcome. Homework by auditors will **not** be graded.

MORE INFORMATION?

Office Hours: Please see Steele's personal web page for current contact information.

Still More?: Google "Steele Probability Course" etc. for the Course Blog. All course information will be posed on the blog.