POSITIVE DEFINITE MATRIX PROBLEM

Abstract. The self-imposed rule of the Cauchy-Schwarz Master Class was to keep matrix algebra to a bare minimum. This decision was made to impose a discipline of simplicity, but many babies were thrown out with the bath water. Here is one that baby that is simple enough to have been included, even as a warm-up problem. It’s also useful — as a tool and as a metaphor.

Problem: Give a necessary and sufficient condition on $\alpha$ and $\beta$ in order that

\[ T^2 + \alpha T + \beta I \]

be positive definite for each self-adjoint matrix $T$.

Comment: One of the things I like about this problem is that it gives us a nice class of positive definite matrices from which we can construct other examples. It also speaks to the metaphor of a positive definite matrix as an analog of a positive real number.