

## POSITIVE DEFINITE MATRIX PROBLEM

ABSTRACT. The self-imposed rule of the *Cauchy-Schwarz Master Class* was to keep matrix algebra to a bare minimum. This decision was made to impose a discipline of simplicity, but many babies were thrown out with the bath water. Here is one that baby that is simple enough to have been included, even as a warm-up problem. It's also useful — as a tool and as a metaphor.

**Problem:** Give a necessary and sufficient condition on  $\alpha$  and  $\beta$  in order that

$$T^2 + \alpha T + \beta I$$

be positive definite for each self-adjoint matrix  $T$ .

**Comment:** One of the things I like about this problem is that it gives us a nice class of positive definite matrices from which we can construct other examples. It also speaks to the metaphor of a positive definite matrix as an analog of a positive real number.

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