

Solutions by Mansur Boase, student, St. Paul's School, London, England and Edward T.H. Wang, Wilfrid Laurier University, Waterloo, Ontario.
 We give the solution by Boase.

$$\sum_{i=1}^n \frac{s - a_i}{a_i} = \sum_{i=1}^n \left(\frac{s}{a_i} - 1 \right) = \sum_{i=1}^n \frac{s}{a_i} - n$$

$$\sum_{i=1}^n \frac{a_i}{s} = 1 \quad \text{and} \quad \sum_{i=1}^n \frac{s}{a_i} \sum_{i=1}^n \frac{a_i}{s} \geq (\sum 1)^2 = n^2,$$

by the Cauchy–Schwarz inequality.

Thus

$$\sum_{i=1}^n \frac{s}{a_i} \geq n^2.$$

Hence $\sum_{i=1}^n \frac{s - a_i}{a_i} \geq n^2 - n = n(n - 1)$.

To prove the first inequality, first note that

$$\sum_{i=1}^n 1 \sum_{i=1}^n a_i^2 \geq \left(\sum_{i=1}^n a_i \right)^2 = s^2.$$

Hence $\sum_{i=1}^n a_i^2 \geq \frac{s^2}{n}$.

By the Cauchy–Schwarz inequality,

$$\sum_{i=1}^n a_i(s - a_i) \sum_{i=1}^n \frac{a_i}{s - a_i} \geq \left(\sum_{i=1}^n a_i \right)^2 = s^2.$$

Therefore

$$\begin{aligned} \sum_{i=1}^n \frac{a_i}{s - a_i} &\geq \frac{s^2}{\sum_{i=1}^n a_i(s - a_i)} = \frac{s^2}{s \sum_{i=1}^n a_i - \sum_{i=1}^n a_i^2} \\ &\geq \frac{s^2}{s^2 - \frac{s^2}{n}} = \frac{1}{1 - \frac{1}{n}} = \frac{n}{n - 1}. \end{aligned}$$

So, both inequalities are proved.