establish that $\hat{\lambda} = u(\hat{\theta}) \to S^2(\hat{\lambda}) \leq S^2(\lambda)$ for all $\lambda$ in $\Lambda$. In the “classical” linear model $E_\theta(X) = b(\theta) = B\theta$ and $\text{Cov}(X) = \Lambda^{-1}. However, the assertion above holds even without this particular specification of $b(\theta)$ and $\Lambda$.

Indeed, as one can see, this invariance property could be claimed for any method of estimation which defines the estimate, if it exists, as the one minimizing the value of a suitable non-negative “distance” function.

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**When Successes and Failures are Independent a Compound Process is Poisson**

**J. M. STEELE* **

Let $N$ be a random variable which takes on non-negative integer values, and let $X$ be a random variable which takes on values $E_1, E_2, \ldots, E_r$. Now let $Y_k$ denote the number of occurrences of event $E_k$ in $N$ independent trials of the random variable $X$. If $N$ is Poisson, it has been observed ([1], p. 217) that $Y_1, Y_2, \ldots, Y_r$ are independent. In the case that $X$ is Bernoulli, and $E_1$ denotes success and $E_2$ denotes failure this yields the interesting situation that the random variables $Y_1$ and $Y_2 = N - Y_1$ are independent. It should be noted that the random variables determined by $Y_1$ conditioned on the event $N = n$ need not be independent, but with this precaution one can express the independence of $Y_1$ and $Y_2$ by saying that in a Poisson number of trials the number of successes is independent of the number of failures. The theorem of this note forms a converse to the preceding observations.

**Theorem.** If there exist two of the $Y_i$ which are independent then $N$ must be Poisson.

**Proof.** Suppose $Y_1$ and $Y_2$ are independent, and let $f(s)$ be the generating function for the random variable $N$. Then letting $p_i = P[X = E_i]$ we have that the generating function for $Y_1$ is $f(1 - p_1 + p_2 s)$. Calculating the bivariate generating function for $Y_1$ and $Y_2$ gives

$$\sum_{k, l} P[Y_1 = k, Y_2 = l] s^k t^l = f(1 - p_1 - p_2 + p_1 s + p_2 t).$$

(1)

Now, by the independence of $Y_1$ and $Y_2$ we have

$$f(1 - p_1 - p_2 + p_1 s + p_2 t) = f(1 - p_1 + p_2 s)f(1 - p_2 + p_2 t).$$

(2)

Letting $a = 1 - p_1 + p_2 s$ and $b = 1 - p_2 + p_2 t$ yields the equation

$$f(a + b - 1) = f(a)f(b).$$

(3)

If $f(0) = 0$ then setting $a = b = \frac{1}{2}$ in equation (3) gives $0 = f(\frac{1}{2})$, which is impossible since $f(s)$ is the generating function of a non-negative random variable. Hence we have $f(0) \neq 0$. Now let $g(s) = f(s)/f(0)$. We have

$$g(a)g(b) = f(a)f(b)/[f(0)]^2 = f(a + b - 1)/[f(0)]^2.$$  

(4)

Now note that letting $b = 0$ in (3) gives $f(a - 1) = f(a)f(0)$, and then replacing $a$ by $a + b$ gives $f(a + b)f(0) = f(a + b - 1)$. This yields

$$g(a)g(b) = f(a + b)f(0)/[f(0)]^2 = g(a + b).$$

(5)

Since $g(s)$ is monotone (5) has the unique solution $g(s) = e^{u s}$ where $\mu \geq 0$ is a constant.

Now we have $g(1) = 1/f(0)$ so $f(0) = e^{-\mu}$, and $f(s) = e^{-\mu + u s}$ which is the generating function of the Poisson distribution.

**Corollary.** If two of the $Y_i$ are independent then all of the $Y_i$ are independent.

REFERENCE


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