Risk and Dependence

Lecture 3, SMMD 2005
Bob Stine
Review

- Key points from prior class
  - Hypothesis tests
  - Actions, decisions, and costs
  - Standard error and t-ratio
  - Role of assumptions, models
  - One-sample, two-sample comparisons
- Questions
Risk and Investing

Would either of these stocks be a good investment, going forward?
- EBAY (on-line auctions)
- OSIP (cancer medication)

What information can help you make this decision?

Speculative markets
- You might not want to invest in stocks, but
- You’ll often have to decide whether to invest in projects whose outcome is uncertain.
Historical Performance

Here are the results for these two companies for the first 150 days of 2004.

Which looks better from this point of view, remembering you are speculating on the future performance?

<table>
<thead>
<tr>
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<th>OSIP</th>
</tr>
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<tbody>
<tr>
<td>January 2</td>
<td>63.00</td>
<td>32.62</td>
</tr>
<tr>
<td>August 6</td>
<td>73.76</td>
<td>52.31</td>
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What do you think after seeing data? Are these series simple, or do you see trends?
Too Little Information?

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<tr>
<td>Return</td>
<td>17%</td>
<td>60%</td>
</tr>
</tbody>
</table>

Graph:
- EBAY Price (red line)
- OSIP Price (green line)
What have you done for me lately?

Returns show daily performance

“Current interest rate”

After all, money could have been invested elsewhere each day.

Conversion to returns gives a better view of the performance of investments

Percentage change = 100 times return

\[ R_t = \frac{P_t - P_{t-1}}{P_{t-1}} \]
Early 2004, No Outlier

Hardly seems like the same data...
Outlier concealed the simple variation
Because the returns have simple variation, we can summarize both with histograms.

What about the outlier? Keep it?
Histogram Summaries

Or remove it?

Without outlier, we can see bell-shaped distributions, but should we do this?

<table>
<thead>
<tr>
<th>EBAY Return</th>
<th>OSIP Return</th>
</tr>
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<tbody>
<tr>
<td><strong>Moments</strong></td>
<td><strong>Moments</strong></td>
</tr>
<tr>
<td>Mean</td>
<td>-0.0021</td>
</tr>
<tr>
<td>Std Dev</td>
<td>0.0011</td>
</tr>
<tr>
<td>Std Err Mean</td>
<td>0.0203</td>
</tr>
<tr>
<td>upper 95% Mean</td>
<td>0.0017</td>
</tr>
<tr>
<td>lower 95% Mean</td>
<td>-0.0021</td>
</tr>
<tr>
<td>N</td>
<td>149.0000</td>
</tr>
<tr>
<td>Mean</td>
<td>0.0017</td>
</tr>
<tr>
<td>Std Dev</td>
<td>0.0319</td>
</tr>
<tr>
<td>Std Err Mean</td>
<td>0.0044</td>
</tr>
<tr>
<td>upper 95% Mean</td>
<td>0.0026</td>
</tr>
<tr>
<td>lower 95% Mean</td>
<td>-0.0073</td>
</tr>
<tr>
<td>N</td>
<td>149.0000</td>
</tr>
</tbody>
</table>
**Risk**

- Finance defines risk of an investment as the variance of returns on the investment.
- Comparison of EBAY and OSIP, including the outlier shows trade-off.
- OSIP has higher average return.
- OSIP also has higher Standard Deviation.

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<tr>
<td>Mean</td>
<td>0.0011</td>
<td>0.0072</td>
</tr>
<tr>
<td>Var</td>
<td>0.02²</td>
<td>0.12²</td>
</tr>
</tbody>
</table>
**Variance**

- Variance of a random quantity is the average squared deviation of the values from their mean.
- **Population**
  \[ \text{Var}(X) = \text{E} (X - \mu)^2 = \text{SD}(X)^2 \]
- **Sample**
  \[ s^2 = \frac{\sum_{i=1}^{n} (Y_i - \overline{Y})^2}{n - 1} \]
Trading Risk for Return

- Where to strike a balance?
- Various approaches
  - Capital assets pricing model (CAPM)
  - Risk preferences
  - Volatility adjusted return (long-run return)

Volatility adjusted return adjusts for the manner in which variation “eats away” at the value of an asset.

Example: What happens if you get a 10% increase one year, followed by a 10% cut the next year?
Volatility Adjusted Return

Vol Adj Return = Avg(Return)-(1/2)Var(Return)

Weird units, but this is the right formula!

volatility drag = (1/2)Var(Return)

Example

OSIP basically all volatility

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<tr>
<td>Var=SD²</td>
<td>0.02</td>
<td>0.12</td>
</tr>
<tr>
<td>Vol Adj Return</td>
<td>0.0009</td>
<td>0</td>
</tr>
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</table>
What happens next?

- Prices over the full year
- OSIP remained volatile
- EBAY more steady, consistent performer
Returns

- Two big outliers...
- OSIP would be a very different investment without these two. Cannot omit them!
Returns

Summarize both as histograms, relying upon the simplicity of returns.

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<tr>
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<th>EBAY Return</th>
<th>OSIP Return</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.0025</td>
<td>0.0061</td>
</tr>
<tr>
<td>Std Dev</td>
<td>0.0192</td>
<td>0.0965</td>
</tr>
<tr>
<td>Std Err Mean</td>
<td>0.0012</td>
<td>0.0061</td>
</tr>
<tr>
<td>upper 95% Mean</td>
<td>0.0049</td>
<td>0.0181</td>
</tr>
<tr>
<td>lower 95% Mean</td>
<td>0.0001</td>
<td>-0.0058</td>
</tr>
<tr>
<td>N</td>
<td>252.0000</td>
<td>252.0000</td>
</tr>
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</table>
Performance

- Last months of 2004 were great in the US stock markets
- From August 6 through the end of 2004
- EBAY outperformed OSIP

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<td>73.76</td>
<td>52.31</td>
</tr>
<tr>
<td>December 31</td>
<td>116.34</td>
<td>74.85</td>
</tr>
<tr>
<td>Return</td>
<td>58%</td>
<td>43%</td>
</tr>
</tbody>
</table>
Another Approach to Risk

Commonly used by financial advisors selling investment packages to customers...

Low risk, less return

High risk, high return
Key Role of Variance

Whether using volatility adjusted return or a heuristic to elicit level of “risk aversion”

Need to know variance of returns!
Portfolios

Portfolio is a blend of several investments

- Diversify: Rather than invest wealth in one stock, spread wealth over several
- “Don’t put all your eggs in one basket”

Risk of portfolio

- Quantify how the mixing of assets reduces the risk
- Manipulate components of portfolio to achieve desired balance of risk vs return
- Again computed as a variance
Two More Stocks

- Look at monthly returns for Dell and Microsoft over 1990-2003
- Monthly returns seem “simple enough” to put into histograms

<table>
<thead>
<tr>
<th>Moments</th>
<th>Dell</th>
<th>Microsoft</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.0518</td>
<td>0.0289</td>
</tr>
<tr>
<td>Std Dev</td>
<td>0.1646</td>
<td>0.1103</td>
</tr>
<tr>
<td>Std Err Mean</td>
<td>0.0127</td>
<td>0.0085</td>
</tr>
<tr>
<td>upper 95% Mean</td>
<td>0.0769</td>
<td>0.0457</td>
</tr>
<tr>
<td>lower 95% Mean</td>
<td>0.0267</td>
<td>0.0121</td>
</tr>
<tr>
<td>N</td>
<td>168.0000</td>
<td>168.0000</td>
</tr>
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**Performance**

- Summary statistics for monthly returns
- Dell has higher mean return and higher volatility.
- Common: larger return with larger risk
- Compensation for risk.
- Dell has larger volatility adjusted return

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<td>0.0518</td>
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<tr>
<td>Var=SD²</td>
<td>0.1646²=0.0271</td>
<td>0.1103²=0.0122</td>
</tr>
<tr>
<td>Vol Adj Return</td>
<td>0.0383</td>
<td>0.0228</td>
</tr>
</tbody>
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Example of Portfolio

Split of wealth into two parts

For every $1 invested, put $0.50 in Dell and $0.50 in Microsoft

Return on portfolio is
Return(portfolio) = (1/2) Return(Dell) + (1/2) Return(MS)

Is this a good mix? Need to know variance.

Average return is easy

Variance requires more work
Manipulating Variance

Notation

X, Y are “random variables”

Variance (risk of asset)

\( \text{Var}(X) = \text{Avg squared deviation from mean} \)

Properties of variance

Scale

\( \text{Var}(a \cdot X) = a^2 \text{Var}(X) \)

Sums (if independent)

\( \text{Var}(X+Y) = \text{Var}(X) + \text{Var}(Y) \)

Together

\( \text{Var}(aX+bY) = a^2 \text{Var}(X) + b^2 \text{Var}(Y) \)
Properties of Portfolio

Portfolio = (1/2) Dell + (1/2) Microsoft

Volatility adjusted return for portfolio is between those for the separate stocks

Vol Adj Ret = 0.0403 - 0.1291²/2 = 0.032
Finding better Mix

Is there a portfolio that does better than either of the stocks making up the portfolio?

In general, want to be able to find the properties of a portfolio without having to make a new column for every possible choice.

Average return is easy to get...

\[ \text{Avg(Portfolio)} = \frac{1}{2} \text{Avg(Dell)} + \frac{1}{2} \text{Avg(Microsoft)} \]

Does this work for variance?
Why doesn’t this work?

- Variances add for independent random quantities, but for Dell and Microsoft
  \[(0.1646^2 + 0.1103^2)/4 \neq 0.1219^2\]
  \[0.0098 \neq 0.0149\]

- These don’t match. Observed risk is larger

- Variance of portfolio is less than prior calculations indicate

- Are returns on stocks independent, or do things seem to happen together?

- Independent or dependence
Dependence

Scatterplot shows the dependence of the two sets of returns

What would this plot look like were there no dependence?
Covariance

- Measures dependence in just the right way
- Variance of a sum
  \[ \text{Var}(aX+bY) = a^2\text{Var}(X) + b^2\text{Var}(Y) + 2ab\text{Cov}(X,Y) \]
- Stock example
  \[ \text{Cov}(\text{Dell, MS}) = 0.01011 \]
- Plug into formula
  \[
  \text{Var}((\text{Dell+MS})/2) = (1/4)\text{Var}(\text{Dell}) + (1/4)\text{Var}(\text{MS}) \\
  + 2 (1/2) (1/2) \text{Cov}(\text{Dell, MS}) \\
  = (0.0271/4)+(0.0122/4)+(0.0101)/2 \\
  = 0.0149
  \]
  which matches variance of portfolio
Why go to the trouble?

- Why do all of this calculation if we could build a portfolio in the first place?
- Formula allows easy optimization
- What choice of weights for the two stocks gets the desired performance?
- Optimal portfolio has weight > 1 on Dell
- What does that mean?
Key Concepts

- Speculative investing and portfolios
- Risk and variance
- Volatility adjusted return
  - Volatility drag
- Dependence
- Covariance
Software Notes

- Scatterplot shows dependence
  Analyze > Fit Y by X

- Covariance buried in a general command
  Analyze > Multivariate > Covariance matrix

- Setting aside outliers
  Rows > Exclude