Introducing students to risk

- Hands-on simulation experiment
  - Avoid computer simulation by rolling dice instead

- Simulation successful in teaching…
  - Random variables as models for future returns
  - Variance
  - Equivalence between risk and variance of returns
  - Portfolios that trade risk for return

- Surprising side effect leads to a richer discussion of investing schemes
Classroom simulation

- Three dice simulate investments

<table>
<thead>
<tr>
<th>Investment</th>
<th>Expected Annual Return</th>
<th>SD of Annual Return</th>
</tr>
</thead>
<tbody>
<tr>
<td>Green</td>
<td>7.5%</td>
<td>20%</td>
</tr>
<tr>
<td>Red</td>
<td>71%</td>
<td>132%</td>
</tr>
<tr>
<td>White</td>
<td>0%</td>
<td>6%</td>
</tr>
</tbody>
</table>

- Which one do you like?
Dies outcomes

- Investments rise or fall according to these multipliers.

<table>
<thead>
<tr>
<th>Outcome</th>
<th>Green</th>
<th>Red</th>
<th>White</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.8</td>
<td>0.06</td>
<td>0.9</td>
</tr>
<tr>
<td>2</td>
<td>0.9</td>
<td>0.2</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>1.05</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>1.1</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>1.2</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>1.4</td>
<td>3</td>
<td>1.1</td>
</tr>
</tbody>
</table>
For most students, Red loses.
But for some, Red wins BIG
Red is highly volatile, with volatility drag ultimately pulling down long-run value.

<table>
<thead>
<tr>
<th>Color Die</th>
<th>Mean</th>
<th>Variance</th>
<th>Mean–Var/2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Green</td>
<td>0.075</td>
<td>$0.2^2 = 0.04$</td>
<td>0.055</td>
</tr>
<tr>
<td>Red</td>
<td>0.71</td>
<td>$1.3^2 = 1.69$</td>
<td>–0.135</td>
</tr>
<tr>
<td>White</td>
<td>0</td>
<td>$0.06^2 = 0.0025$</td>
<td>–0.002</td>
</tr>
</tbody>
</table>

Green calibrated to match US stock market, and generally ends simulation with highest value.
Second message

• What about the team that made $10,000,000 with Red while the other teams lost money?

• Call this team “Warren Buffetts”
  - Generally hear them while dice are rolling.
  - Used to try to avoid them by extending simulation.

• Pose questions
  What were your classmates doing to make money from Red that you were not doing?

  If you and they both apply the same “strategy”, then why were they successful but you were not?
Answering the question

- What’s an attractive investment?
  - If offered “dice-like” investments, then you would like to own some of any for which Mean(Return) > 0
  - Amount owned depends on utility, level of risk aversion
  - Want some Red, just don’t borrow against the house.

- What’s a dice-like investment?
  - Dice simulation creates independent investments
  - Allocating wealth among correlated investments is messy

- Use CAPM to build “dice-like” investments
CAPM Regression

- Simple regression

\[(r_s - r_f) = \alpha + \beta (r_M - r_f) + \varepsilon\]

- Use CAPM to isolate idiosyncratic variation in each stock, removing correlation with market

\[(r_s - r_f) - \beta (r_M - r_f) = \alpha + \varepsilon\]

- APT adds little in empirical analysis, with about 50% of the variation in stocks being idiosyncratic
Rule for investors…
Buy investments for which you “know” the mean orthogonal return $\alpha > 0$.

How do know if an investment has this property?

Two problems to overcome once you have taken care of the observed volatility are:
- Persistence problem
- Peso problem
Persistence problem

- Persistent search eventually discovers an investment that has a “statistically significantly” positive mean.
- Usual test has 1% or 5% chance for a false positive
- Multiplicity
- Protection
  - Bonferroni methods
  - Rather than test at nominal 5% level, adjust the level to reflect the amount of search used to discover the significant result.
Peso problem

- Unseen volatility

- Examples
  - Peso devaluation
  - Long-term Capital Management
  - Andy Lo’s “Capital Decimation” fund
  - The Bob Fund

- Protection
  - Assume the worst
  - Limited liability in the US implies 100%
Ponzironni test

- Orthogonalize returns on an investment.
- Adjust level of test for multiplicity
  - 1000 trading rules, then use 0.05/1000
- Test for non-zero mean using critical value from Bennett’s inequality
  - \( B_i \) are independent, bounded random variables, \( |B_i| < M \)
  - \( E B_i = 0, \sum \text{Var} B_i = 1 \)

\[
P \left( \sum_{i=1}^{n} B_i \geq \tau \right) \leq \exp \left( \frac{\tau}{M} - \left( \frac{\tau}{M} + \frac{1}{M^2} \right) \log(1 + M \tau) \right)
\]
Example: Avoiding Bob Fund

- Bob Fund
  - Guarantee to beat the market by 5% annually.
  - Method: Bet the whole amount in Vegas at 20 to 1 odds
  - 15 year history
Common sense vs Convention

- Once you know how the Bob Fund works, it's not so impressive.

\[
\text{Pr(15 consecutive hits)} = \left(\frac{20}{21}\right)^{15} = 0.48
\]

- The usual t-test is quite impressed, however.

\[
t = \frac{0.051}{0.00484/\sqrt{14}} = 40.76
\]

- More than large enough to overcome Bonferroni.
Bennett not so impressed

- Using the prior formula, Bennett bounds the p-value at 0.62.
- Rather close to the common-sense bound obtained when the underlying risk is revealed.
  - As an upper bound, it seems better than one might expect
Approximation to Bennett

• Maurer (2000) bound shows effect of bound more clearly

\[ P\left(\sum_{i=1}^{n} B_i \geq \tau\right) \leq \exp\left(-\frac{\tau^2}{2n(\sigma^2 + M^2)}\right) \]
Summary

- Dice simulation
  - Question of how to separate knowledge from luck

- Ponzironni approach
  - Orthogonalize returns to adjust for leverage
  - Correct for multiplicity
  - Protect from hidden volatility via Bennett

- More information in forthcoming introductory statistics book, and in the meantime...
  www-stat.wharton.upenn.edu/~stine