Space-Time Models for Retail Credit

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The views expressed in this presentation do not necessarily reflect the views of the Federal Reserve Bank of Philadelphia or the Federal Reserve System.
Questions to Consider
Question 1

Is there spatial variation in credit and macro-economic conditions in the US?

Often hear numbers like “the” unemployment rate or level of disposable income...

How much variation around the overall numbers is present?

What is the spatial distribution of the variation?
Question 2

- Do local economic conditions affect the fit of models of retail credit risk?

- Models routinely incorporate “bank data” that includes past default rates, utilization, ... 
- Do economic variables such as local employment levels add value beyond information in the bank data?
  \[ P(\text{default}|\text{bank,macro}) = P(\text{default}|\text{bank})? \]
Question 3

Does spatial variation in economic conditions produce a form of stress-testing?

- Stress-test
  Does an overall model fit well when applied in times of economic distress?
- Back-testing prescribed in Basel II
- Tricky to prescribe conditions for stress test
- Does spatial variation provide a natural framework for exploring model accuracy in periods of high economic stress?
County-Level Data

- **Trend Data (TransUnion)**
  - Quarterly, 1992Q1 - 2004Q1 (49 quarters)
    - % 60 days past due among retail bank cards
    - % utilization among bank cards
    - average number of bank cards

- **Macro-economic data**
  - Monthly unemployment (Bureau of Labor Statistics, LAUS, Department of Labor)
  - Annual median income, % in poverty (SAIPE, US Census Bureau)
    - I’ll use estimates derived from 2000 Census
    - Census now estimates annually as part of the American Community Survey (replacing old “long form”)

National Trends

% 60 days past due
retail bank card

% Unemployed

Utilization

Avg number cards

BCNARB

National Trends

% 60 days past due
retail bank card

% Unemployed

Utilization

Avg number cards

BCNARB
Regional Variation

Log % 60 Days Past Due

Year
-2.0 -1.5 -1.0 -0.5

Log Bank Card 60 Days
-2.0 -1.5 -1.0 -0.5

Variation in Unemployment

Quite large, with diminishing variation.
Spatial View of Data
Regions

- County
  - Continental US has 3,000 counties
  - Diverse range of shapes and sizes
Spatial Clustering

- Concentrations of high population
- Evident urban clusters
- Confounding: urban and population
Spatial Patterns

Concentrations of high and low default

Log Past Due, 1992

[Map showing spatial patterns of high and low default concentrations across the United States]
Spatial Patterns

Concentrations of high and low default
Spatial Patterns

Concentrations of high and low default

Log Past Due, 1998
Spatial Patterns

Concentrations of high and low default

Log Past Due, 2001
Spatial Patterns

Concentrations of high and low default
Unemployment

Substantial regional concentrations in Mississippi valley, Appalachia, West
Spatial Patterns

Poverty also concentrated in southeastern US

% in Poverty, 2000
Return to Questions

1. Is there adequate spatial variation to support modeling credit risk?
   Yes. Maybe too much!

2. Do local macro-economic variables add value beyond usual bank information?

3. Do models suffer under local economic stress?
Models
Models

- Predict percentage late payments
  - Log scale, one point in time

- Baseline model ignores covariates
  \[ \log(\text{Late}_{t}) = b_{0,t} + b_{1,t} \log(\text{Late}_{t-1}) + e_{t} \]

- More complex models include bank variables plus macroeconomic variables
  - Add lagged covariates of several types
    \[ \log(\text{Late}_{t}) = b_{0,t} + b_{1,t} \log(\text{Late}_{t-1}) + \]
    \[ \text{“bank”} \quad b_{2,t} \log(\text{Util}_{t-1, t-2}) + b_{3,t} \log(\text{Cards}_{t-1, t-2}) + \]
    \[ \text{“macro”} \quad b_{4,t} \log(\text{Un}_{t-1, t-2, t-3, t-4}) + b_{5,t} \log(\text{Pov}) + e_{t} \]
Residual Plots

- Models are well-calibrated, with fitted values linearly related to response.
- Larger residuals randomly scattered.
Uh-Oh!

Fat-tails: due to spatial heterogeneity?
Explanation of Fat Tails

- Residual variance related to pop size
- \( \text{Var}(e_t) \) does not fall off with population as rapidly as usual calculation would suggest

![QQ Plot](image)

- Theoretical Quantiles
- Sample Quantiles
- Log Residual^2
- Log Population
Stabilize Variance

After weighting by the estimated variance function, residuals are much nicer.
Much easier than spatial adjustments
Now that we have a reasonable model, take a look at its properties...
Goodness of Fit

- Fit improves over time
- Macro variables are statistically significant
- Gain worth the effort?
Coefficients over Time

- Estimated coefficient “drifts”
- Size of effect of lagged endogenous grows
- Less drift when use macro variables

![Graph showing coefficients over time with different lines representing different models: lag, lag+bank, and lag+bank+mac. The x-axis represents years from 1992 to 2004, and the y-axis represents the coefficient values ranging from 0.3 to 0.8.](image-url)
Borrowing Strength?

- I estimated the model $M_t$ with data for a specific quarter, ignoring models in prior quarters $M_{t-1}, M_{t-2}, \ldots$

- **Extension**
  - Smooth the models by capturing the dynamics of the drifting estimates and goodness of fit.

- **Caveat**
  - Must capture seasonal effects like that in unemployment rather than smooth over. Smoothing forces similarity.
Questions

1. Is there adequate spatial variation to support modeling credit risk?
   Yes. Maybe too much!

2. Do local macro-economic variables add value beyond usual bank information?
   Yes. The gain is small (albeit significant) and these variables stabilize the model structure

3. Do models suffer under local economic stress?
Spatial Effects
Plan

- Removed the heterogeneity due to population size, but still need to check for spatial dependence among residuals.

Methods

- Descriptive: variogram
- Conceptual: Markov random field
Spatial View of Residuals

Substantial clustering or natural variation?
Spatial Models

Broad class
- More models than those available in time series analysis because the dependence is not naturally “one-sided”
  - e.g., two types of first order autoregressions
  - 2-D plane only partially ordered

Neighborhood
- Which observations are “close”? 
- Easily defined in computer graphics because data live on well-defined grid
- Less clear for map-based geographical units
Neighborhoods

- Model dependence using adjacent counties
- Layers define neighborhoods
- CAR model assumes \( P(Y|\text{all}) = P(Y|\text{neighbors}) \)
- Conditional on neighbors, independent of others
Link to Neighbors

- Relationship of residuals to average residual over neighboring counties
- No evidence of dependence ($r \approx 0.15$)
Fit Local Models

- Spatially local fits require “small” samples
  - Want small enough area so that do not lose ability to localize spatial properties
  - Cannot spread over time since we know these models change over time

- Sample 300 spatially separated points around the US
  - None is adjacent to another
  - Use 5 “layers” to define each neighborhood
  - Each neighborhood has 50 to 100+ counties
Local Models

Fit locally defined model at each position.
Finding Spatial Deviations

- Estimate deviations from overall estimates
  - Replace $Y_t$ by $e_t$ as the response

- Only interested in meaningful deviations from the overall fit
  - 300 fits offer many chances for accidentally estimating large deviations from overall WLS fit
  - Akin to over-fitting in variable selection

- Solution
  Use an approach that avoids the problem in variable selection: shrinkage
  - Ridge (ie, Bayes estimator under normal prior)
  - Adaptive estimator (Polyshrink)
Deviations from Overall

Polyshrink estimates of the coefficient of the lagged endogenous variable, 2000 Q1
Deviations from Overall

Polyshrink estimates of the coefficient of the lagged endogenous variable, 2001 Q1
Deviations from Overall

Polyshrink estimates of the coefficient of the lagged endogenous variable, 2002 Q1
Deviations from Overall

Polyshrink estimates of the coefficient of the lagged endogenous variable, 2003 Q1
Deviations from Overall

Polyshrink estimates of the coefficient of the lagged endogenous variable, 2004 Q1
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3. Do models suffer under local economic stress?
   Models vary spatially as well as over time. Explanations?
What next?

- **Incremental**
  - Multivariate response (mortgage, installment)
  - More population/demographic information

- **Modeling**
  - “Global” model that describes the evolution of parameters over time and spatial clusters
  - Combine with micro-level data

- **Methods**
  - Stress testing
    - Comparison of accuracy in regions of changing parameters versus regions of stability
Thanks!

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