Model Risk in Finance

Bob Stine
Department of Statistics
Wharton, University of Pennsylvania

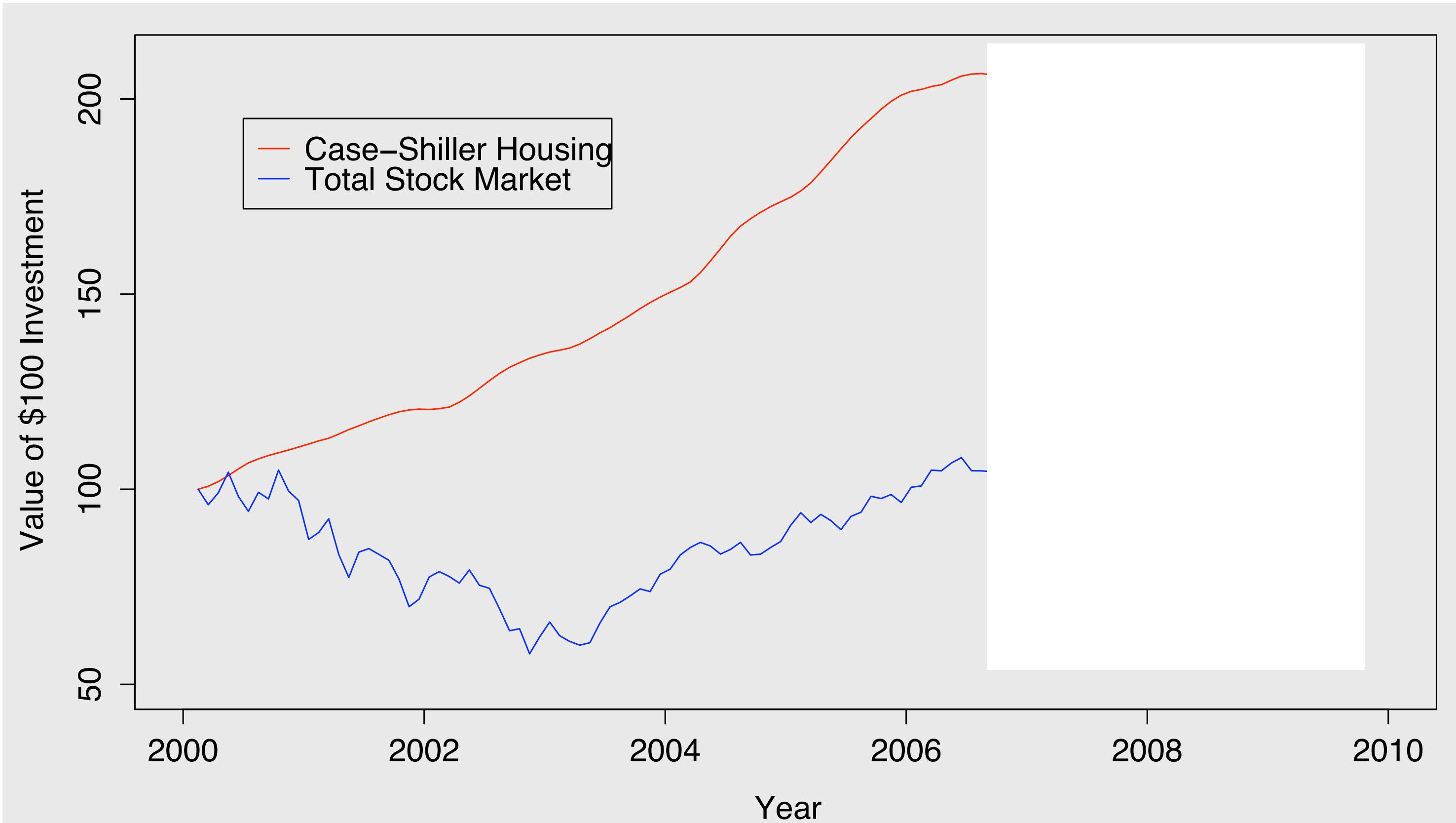
Collaboration with Dean Foster and Peyton Young

Overview

- Background
  - Various types of financial risk
  - Avoiding some of these risks, or at least deciding if the risk is one worth taking
- Statistical issues
  - Multiplicity
  - Transformations to obtain "independence"
  - Orthogonality
  - Robust tests CERT
- Examples

Clearly Risky Time to Invest

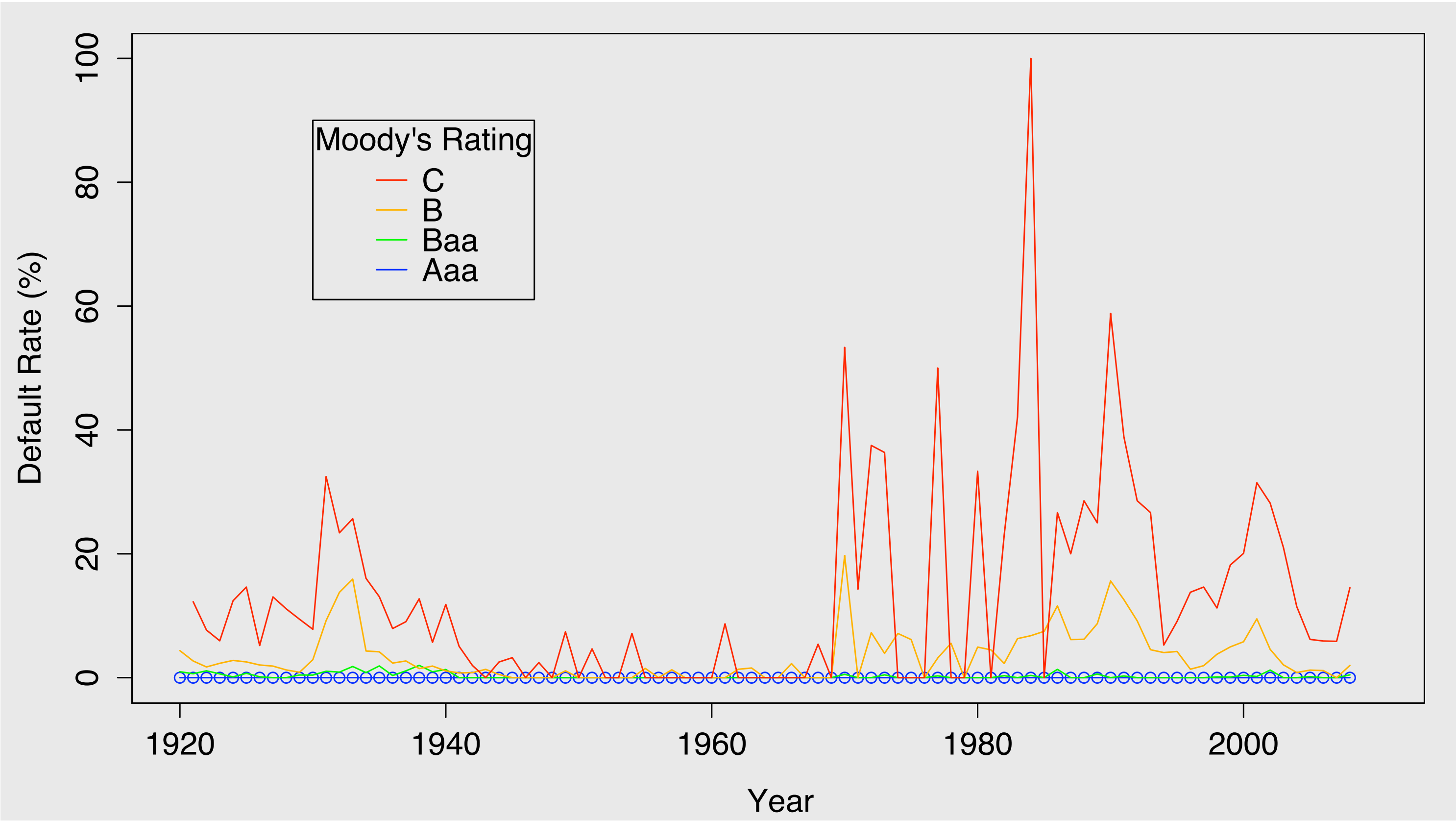
Financial risks are much more apparent now than several years ago.



Wharton
Department of Statistics

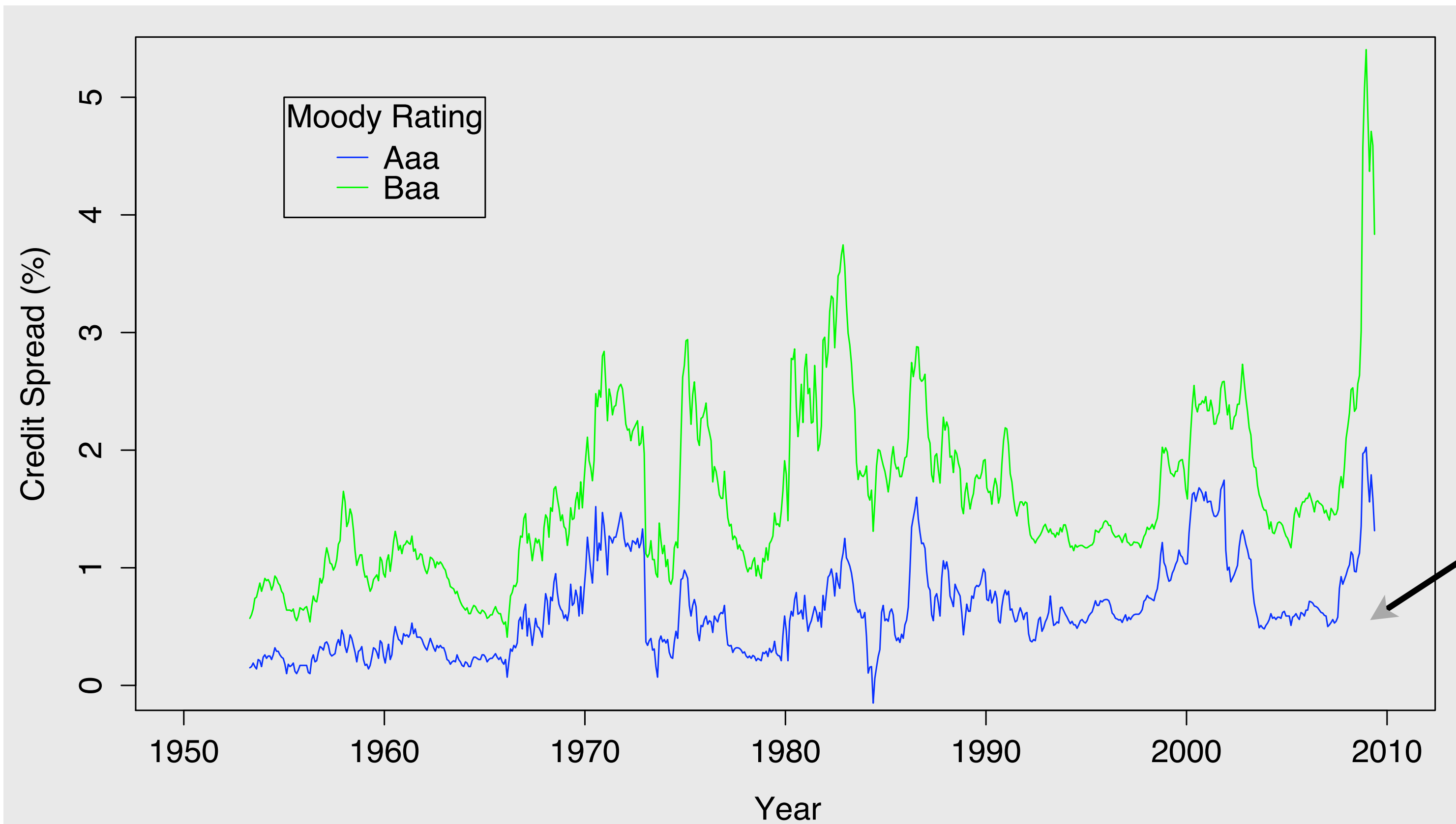
Corporate Bonds Risky Too

- Default rates
  - Junk bonds frequently default.
  - Never seen an Aaa immediately fail.



Credit Spreads

- Lower ratings mean higher credit spreads
- Evident recent increase anticipates increase in default rates?



Why not zero?

Wharton
Department of Statistics

Many Types of Financial Risk

- Default risk
  - Credit risk
  - Settlement risk
  - Liquidity risk
- Variety of other types of risk beyond default
  - Correlation risk
  - Regulatory risk
  - Reputation risk
  - Operational risk
  - Systemic risk, market risk
  - Specific risk, idiosyncratic risk
  - Model risk
- Focus on a particular problem…

Should I invest in an asset?

- Stocks in general?
  - Take perspective that investors are not so risk averse that the answer to this is yes.
- Buy specific stocks?
  - Berkshire-Hathaway
  - Exxon
- Rely on a special investment strategy?
  - TEAM
  - Bob Fund
  - Details of these in the accompanying paper

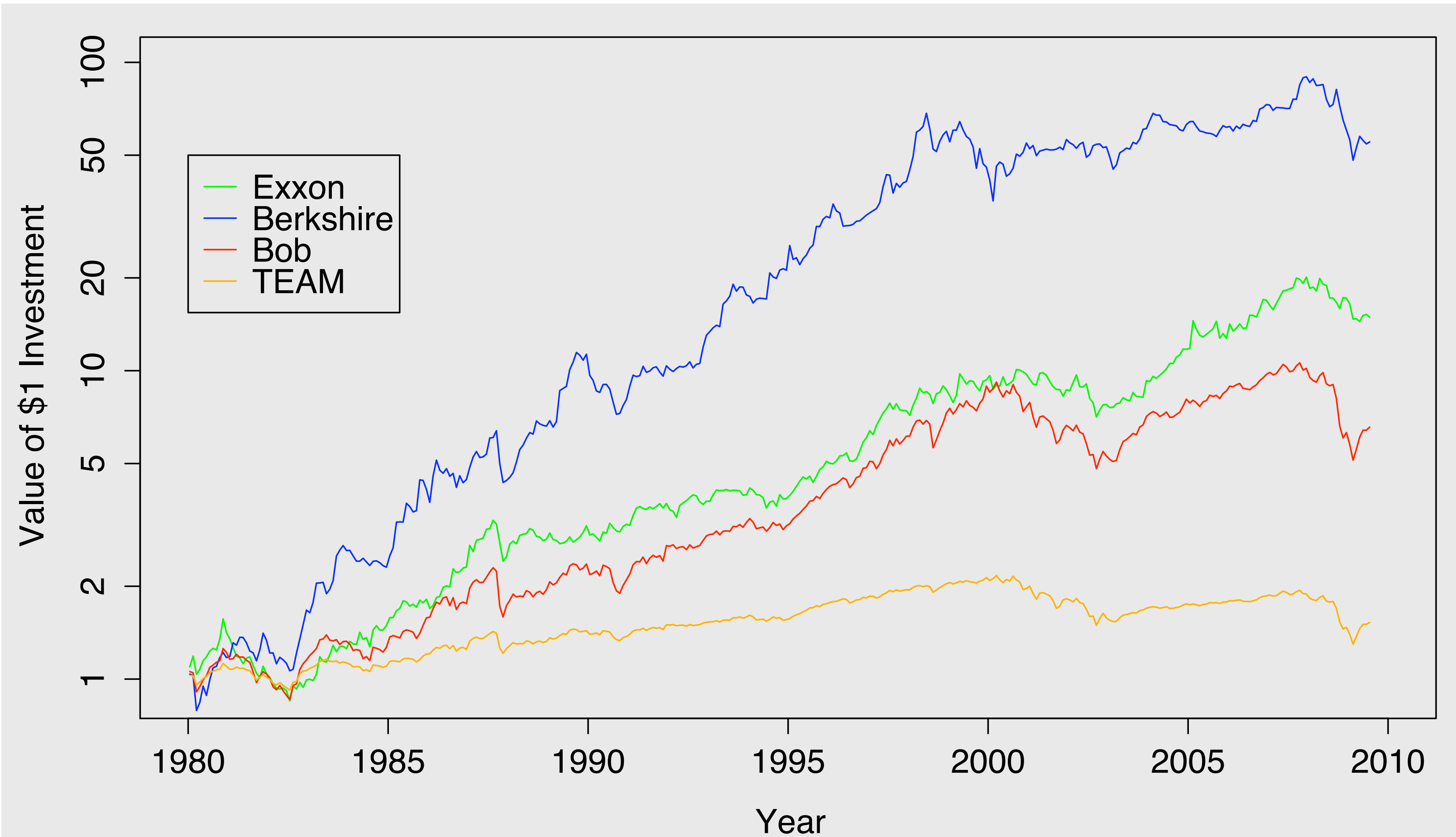
Wharton
Department of Statistics

Formulating an Answer

- Multiplicity
  - How many assets have you looked at in order to find one that "looks good"?
- Independence (or close to it)
  - Returns answer the important question
    "What have you done for me lately?"
- Orthogonality, regression, and CAPM
  - Risk-free rates
  - Market risk vs idiosyncratic risk
- Robust test using martingale (CERT)
  - Unrealized volatility
  - Distributional assumptions

Cumulative Performance

Value of $1 initially invested in 1980 and reinvested



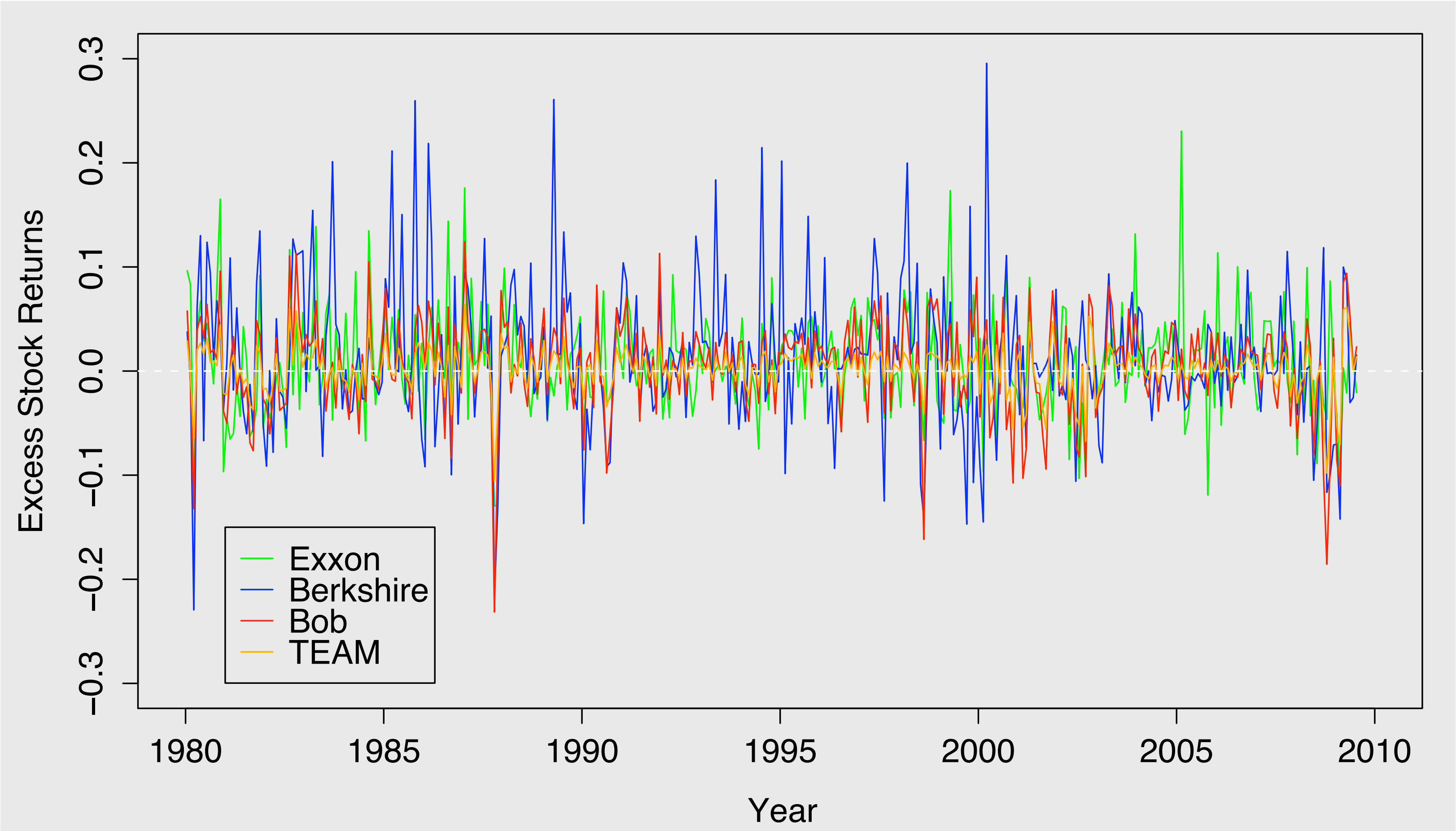
Wharton
Department of Statistics

Convincing going forward?

9

Returns

- Sequence of "bets" that appear nearly independent, but correlation remains between assets.



$R_t = (P_t - P_{t-1})/P_{t-1}$ risk = variation in returns

Capital Asset Pricing Model

- Model describes <u>some</u> properties of returns
- Linear equation
  - Regress returns on asset on returns on market

$$R_t - r_f = \alpha + \beta\,(M_t - r_f) + \varepsilon_t$$

  - r_f = risk-free rate, M_t = market return
  - **α = 0**
- Orthogonal
  - Divide risk into systemic and specific components
  - Intrinsic returns uncorrelated with market

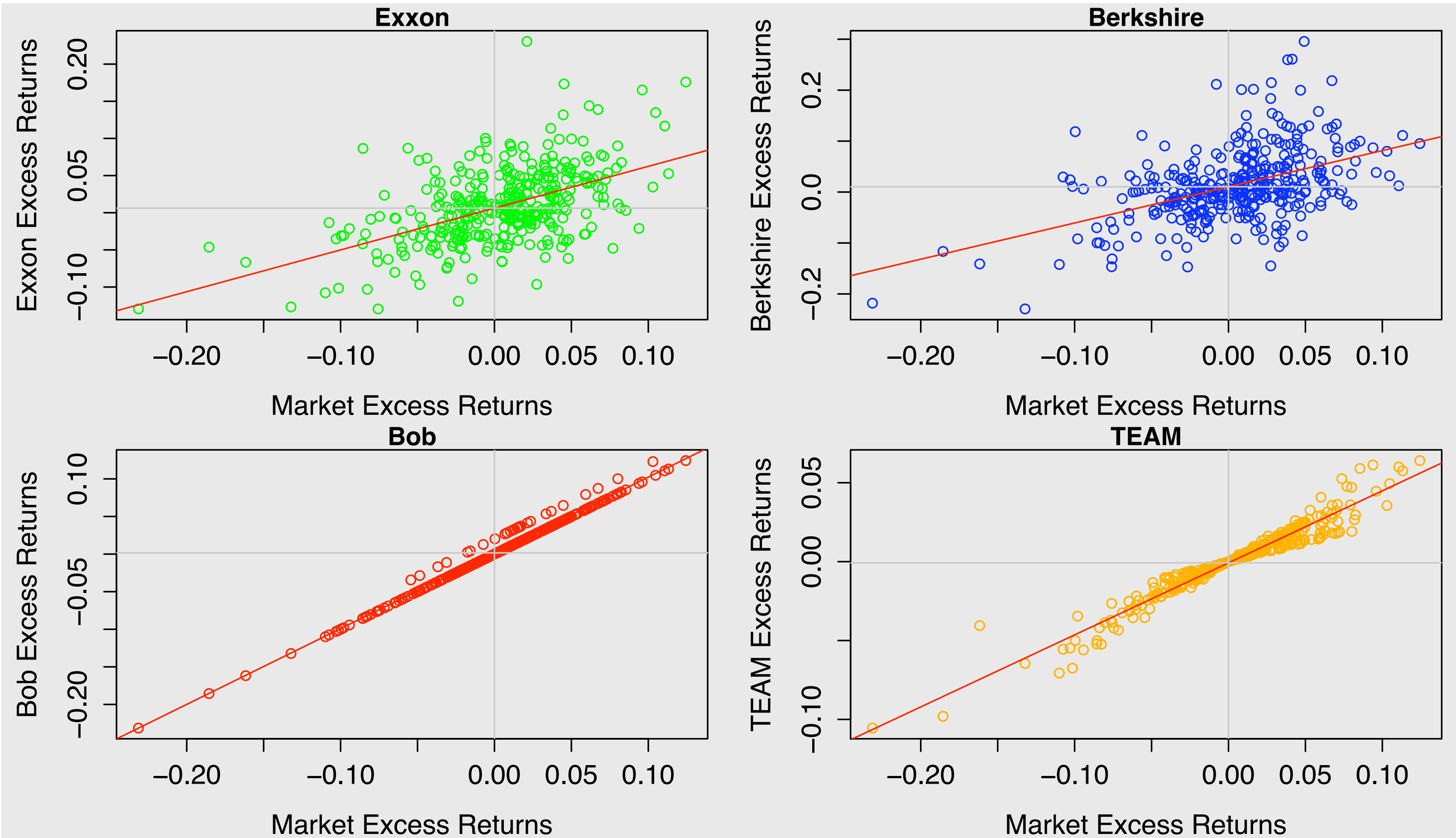
$$(R_t - r_f) - \beta\,(M_t - r_f) = \alpha + \varepsilon_t$$

- If $\alpha \neq 0$?
  - Intrinsic variation of asset has non-zero mean
  - Buy (or sell) some amount of it.

Wharton
Department of Statistics

CAPM Regressions

Correlation with market (systematic risk) explains much of the dependence

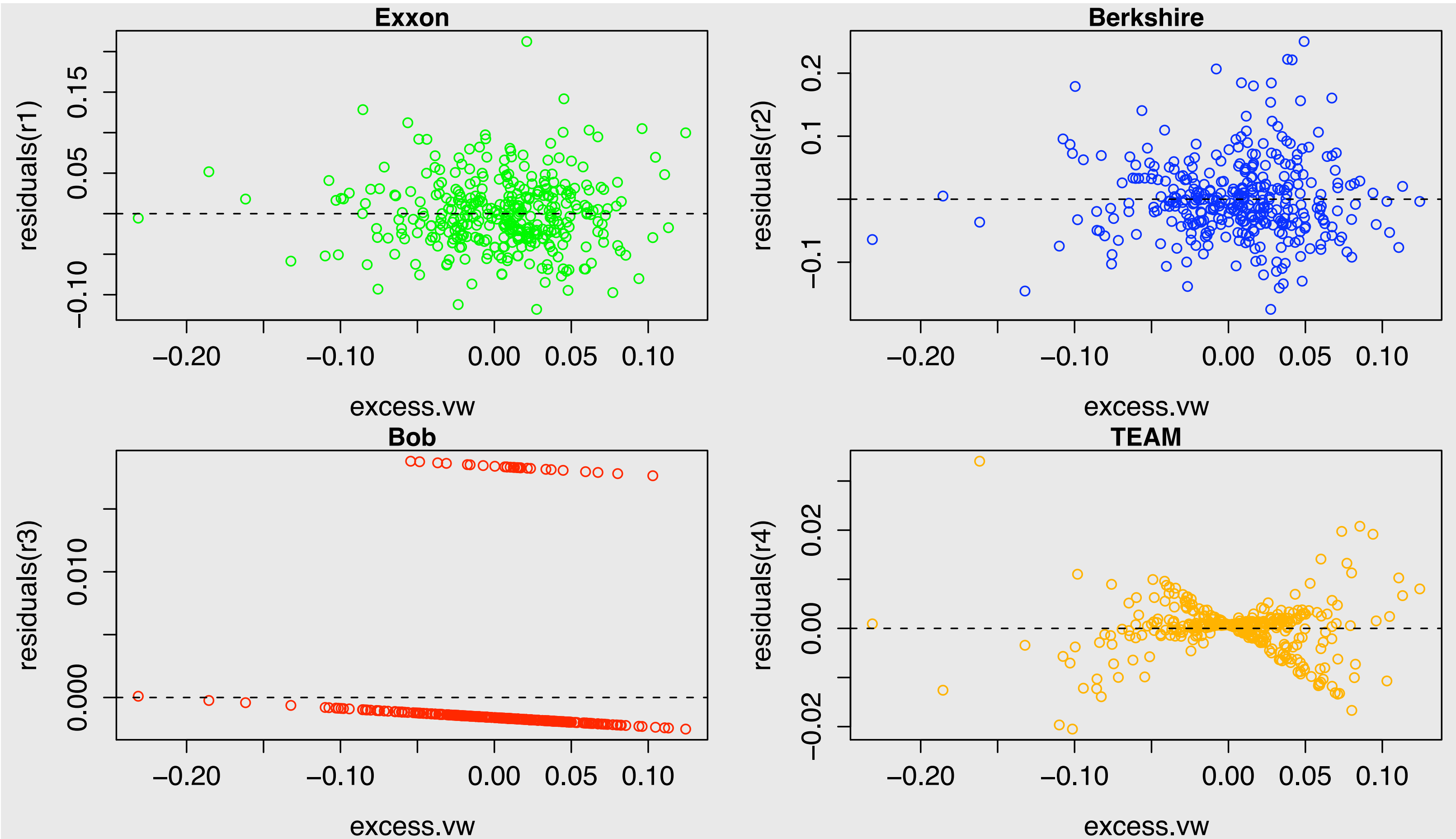


$M_t - r_f$ = market excess return

Wharton
Department of Statistics

CAPM Residuals

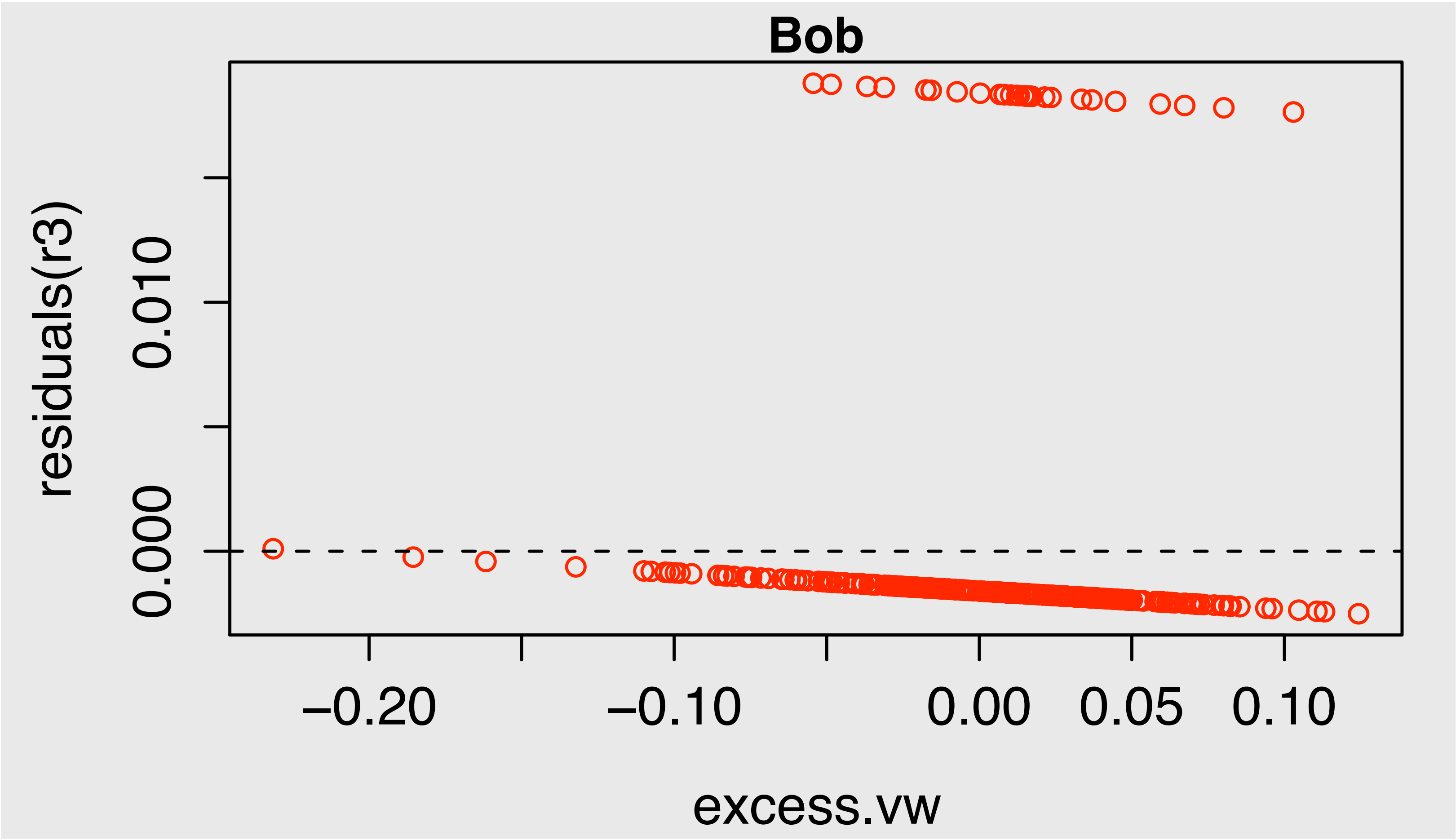
- Residuals need not look "well-suited" for the usual tests of regression parameters.



Wharton
Department of Statistics

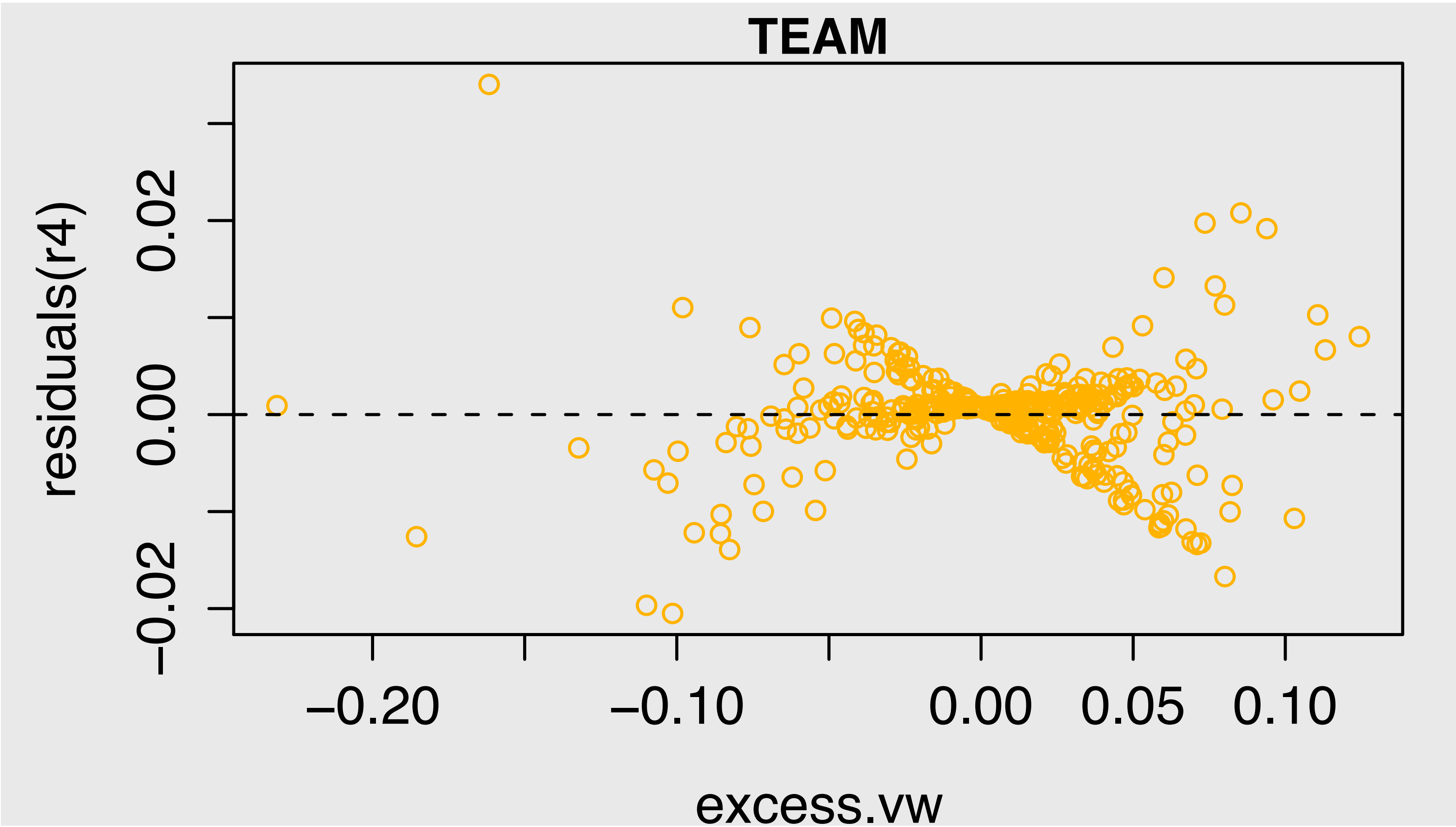
CAPM Residuals

- Residuals need not look “well-suited” for the usual tests of regression parameters.



CAPM Residuals

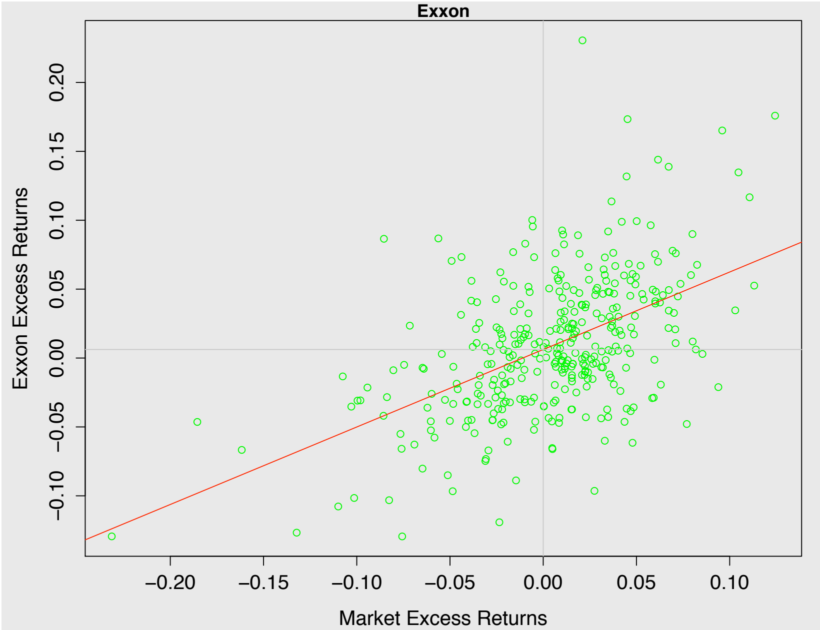
- Residuals need not look "well-suited" for the usual tests of regression parameters.



Wharton
Department of Statistics

Testing Alpha

- Example: Exxon
- Regress out market risk, obtaining estimates of α and β.
  - beta = 0.56
  - alpha = 0.0062
- Test H_0: $\alpha = 0$
  - Standard procedure relies on t-distribution to obtain p-value



	Estimate	SE	t	p
Alpha	0.0062	0.0023	2.68	0.0077
Beta	0.5623	0.0500	11.25	0

16

Summary of Tests

	estimate of alpha	t	p-value
Exxon	0.0062	2.7	0.008
Berkshire	0.0103	3.1	0.002
Bob	0.0016	5.5	0
Team	-0.0008	-2.6	0.009

Do you believe these results?

Testing Alpha

- Standard test procedure
  - Regress out the market
  - Test H_0: $\alpha = 0$ using regression estimates
- Model risk
  - Doubts about standard test.
  - What's the distribution of the t-statistic?
  - Some investments produce returns that are far from Gaussian, with large outliers (fat tails)
  - Evident lack of independence in CAPM residuals
  - ARCH processes
- Nonetheless want a p-value

Wharton
Department of Statistics

Martingale Test for Alpha

- Intrinsic returns after removing market

  $w_t = (R_t - r_f) - \beta\,(M_t - r_f) = \alpha + \varepsilon_t$

- Null hypothesis H_0: $\alpha=0$
  - Implies does not "beat the market"
  - Assume $E(w_t|w_{t-1},w_{t-2},\ldots) = 0$

- Compound returns are non-negative martingale

  $C_t = (1+w_1)(1+w_2)\ldots(1+w_t) \quad t = 1,2,\ldots,n$

- CERT p-value from Doob's inequality

  $P(\max C_1,\ldots,C_n \geq \gamma) \leq 1/\gamma$

- Easy to use

  To reject H_0 at 0.05 level, compound returns have to exceed 20 during observed period

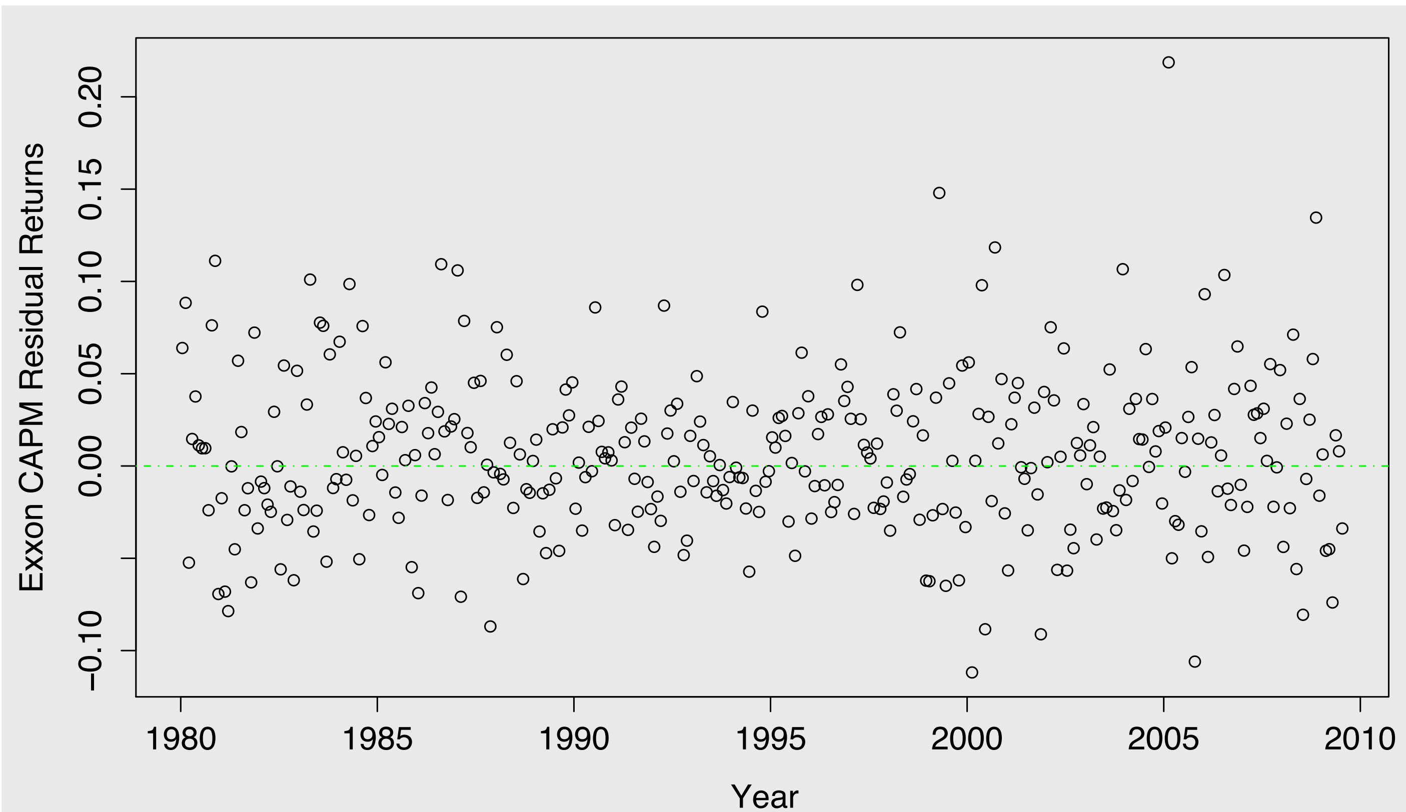
Wharton
Department of Statistics

Example

- "Residual" returns for Exxon,

$$(R_t - r_f) - b\,(M_t - r_f)$$

- Since the martingale test does not depend on n, we can use finely spaced data that essentially reveal β (if you believe it's fixed!)

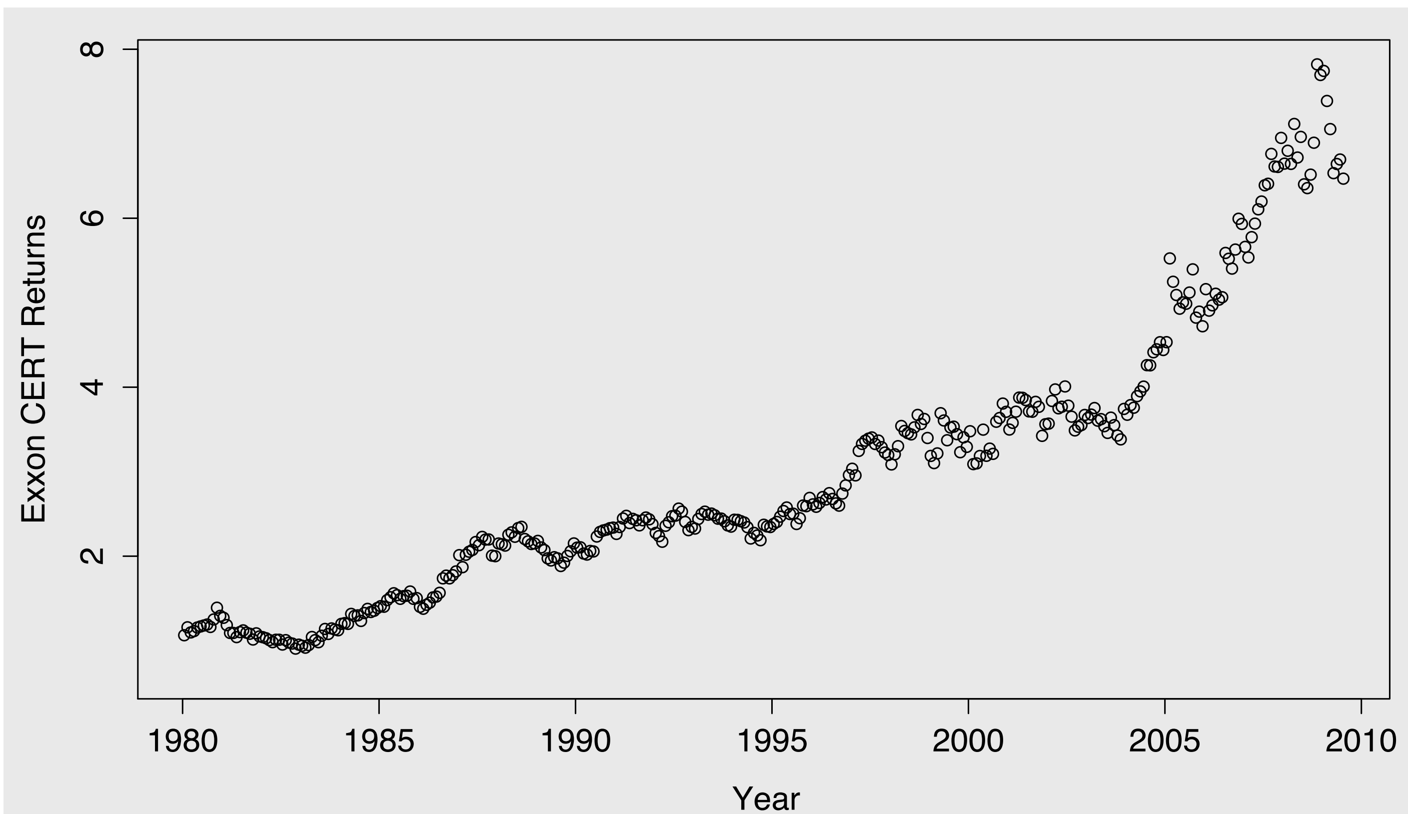


Example

- "Residual" returns for Exxon,

$$(R_t - r_f) - b\,(M_t - r_f)$$

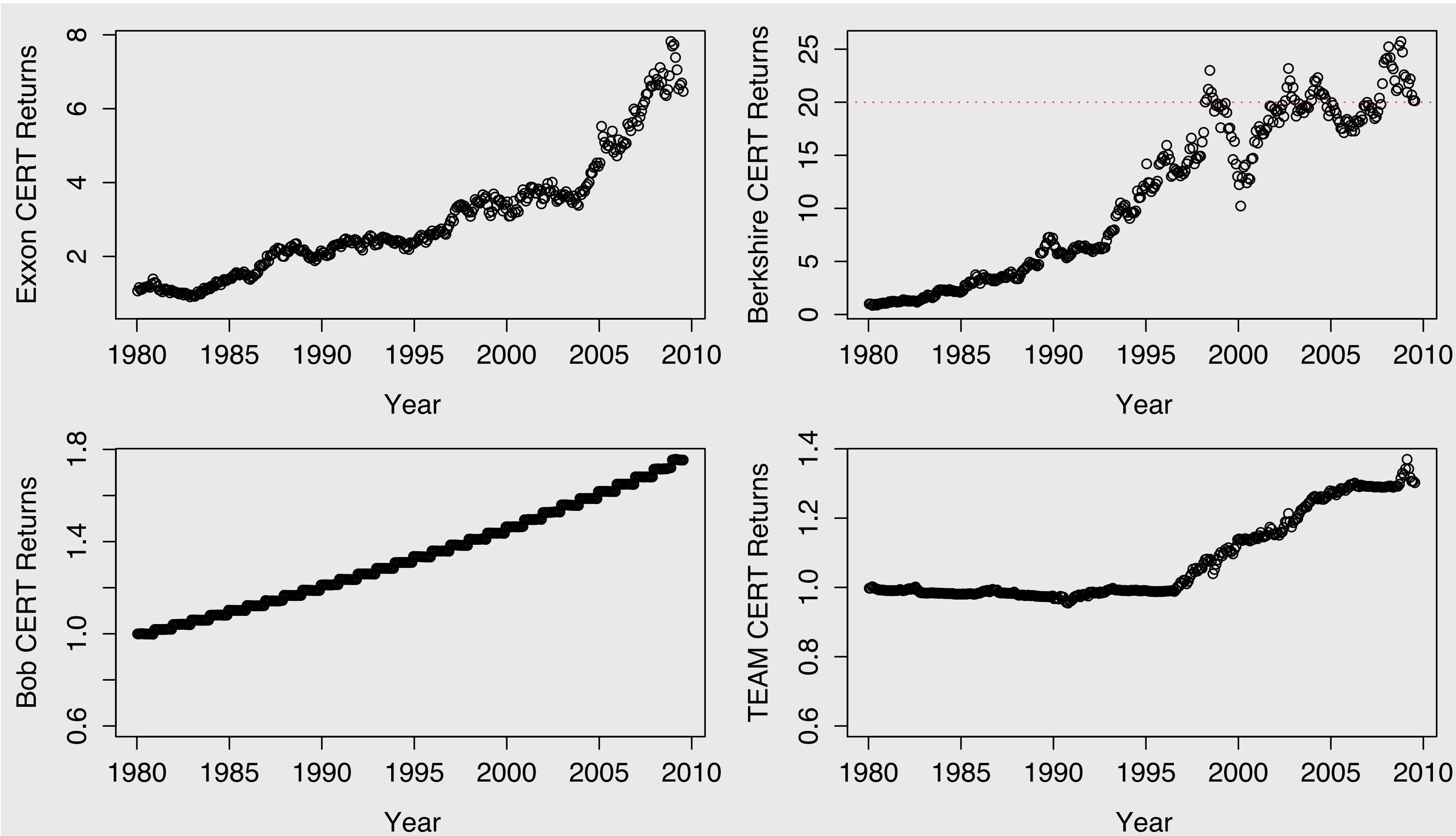
  - Since the martingale test does not depend on n, we can use finely spaced data that essentially reveal β (if you believe it's fixed!)



Wharton
Department of Statistics

CERT Results

- Only Berkshire Hathaway rejects the null, and then we have to consider multiplicity.



Wharton
Department of Statistics

22

Discussion

- Multiplicity
  A p-value of 1/20 does not overcome adjustments for multiplicity.
- Bonferroni p-value
  Multiply the p-value from martingale test by number of assets considered.
  - I bet that you have considered more than 4.
- Power
  The test is "tight" in the sense that there are processes you would not want to consider for which it gets the right answer.

Wharton
Department of Statistics

23

Bob Fund

- How do you guarantee 2% above benchmark returns?
- Unobserved volatility
  - $R_t = 1/k$ w.p. $k/(k+1)$
  - $R_t = -1$ w.p. $1/(k+1)$ busted
  - $E(R_t) = 0$
- Example
  - $k = 49$, so returns a bit more than 2% growth
  - Smaller k give more exciting performance
- Similar "unacceptable" funds obtain CERT bound

  $$P(\max C_t > 20) = 1/20$$
- Martingale test protects against the "until it happens" unobserved volatility

Summary

- Principles
  - Focus on returns, not cumulative value
  - Remove market performance
    - Regress out market from returns
  - Adjust for multiplicity
    - Bonferroni does fine, particularly since it's so hard to "count" the considered alternatives
- Use martingale test (CERT) to adjust for hidden volatility and avoid model risk.

Thanks!

www-stat.wharton.upenn.edu/~stine

Foster, Stine, Young (2008) "A martingale test for alpha"

25