

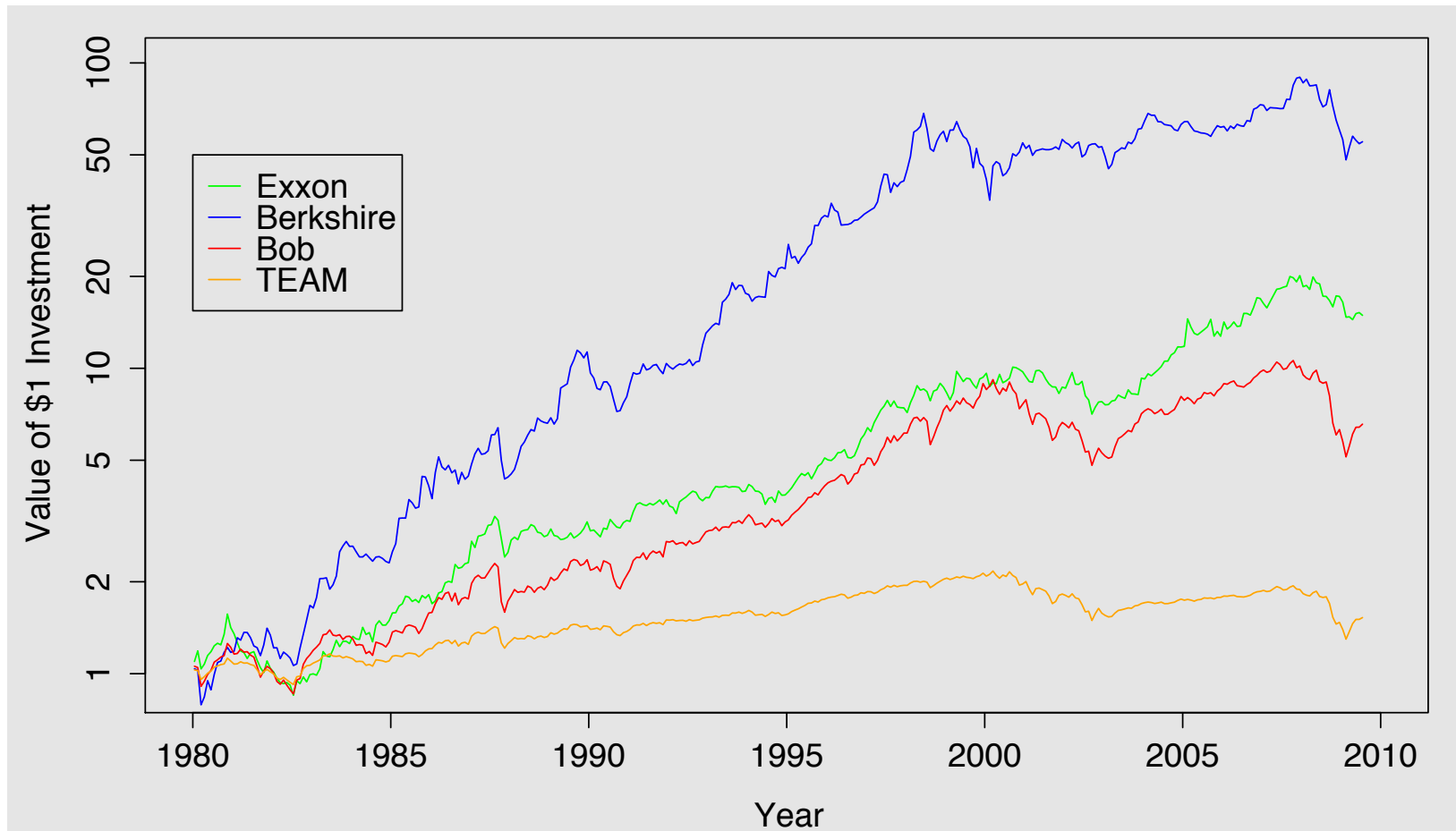
Valuing Investments A Statistical Perspective

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Challenge

- Value of \$1 initially invested in 1980 and reinvested



A Special Opportunity!

- While you are thinking about those dice, here's a special opportunity...

The Bob Fund

- Guarantees 2% excess annual returns above any benchmark you want. Guaranteed.
- Rest assured, it's not a Ponzi/Madoff scheme.
- Contact me after the talk...

Overview

Principles

- Focus on returns, not cumulative value or prices
 - One good year \neq continuing success
- Remove market performance (CAPM)
 - Leverage was good until lately
- Watch for unseen volatility (Peso problem)
 - Just because it has not happened does not make it impossible.
- Adjust for multiplicity
 - Happy to see resurgence of E-trade ads

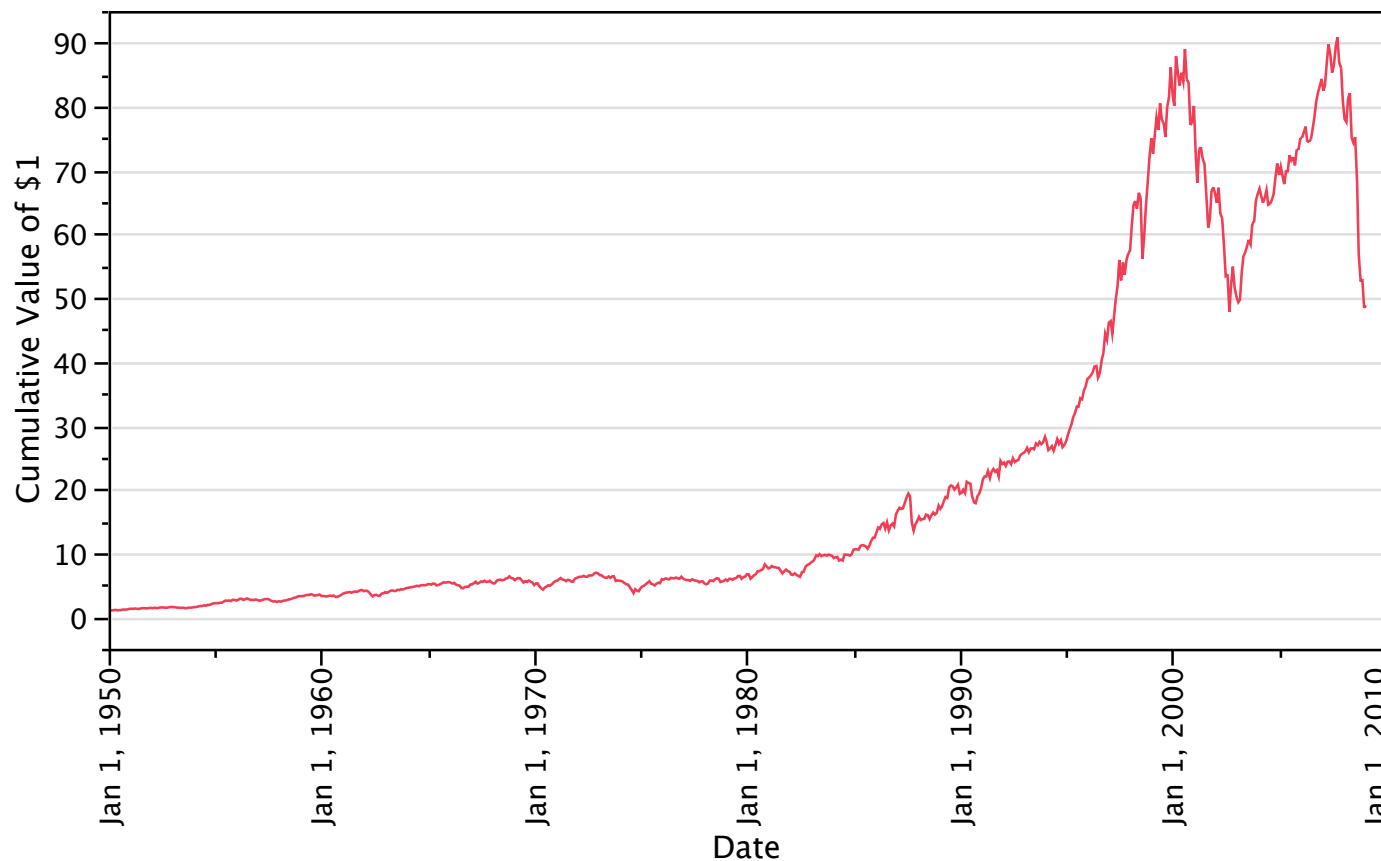
How to evaluate investments ...

- As random process
- Offerings of financial advisors
- Using data

Returns

Overall Market Performance

- Cumulative value of a \$1 investment in the S&P 500 on January 1, 1950.

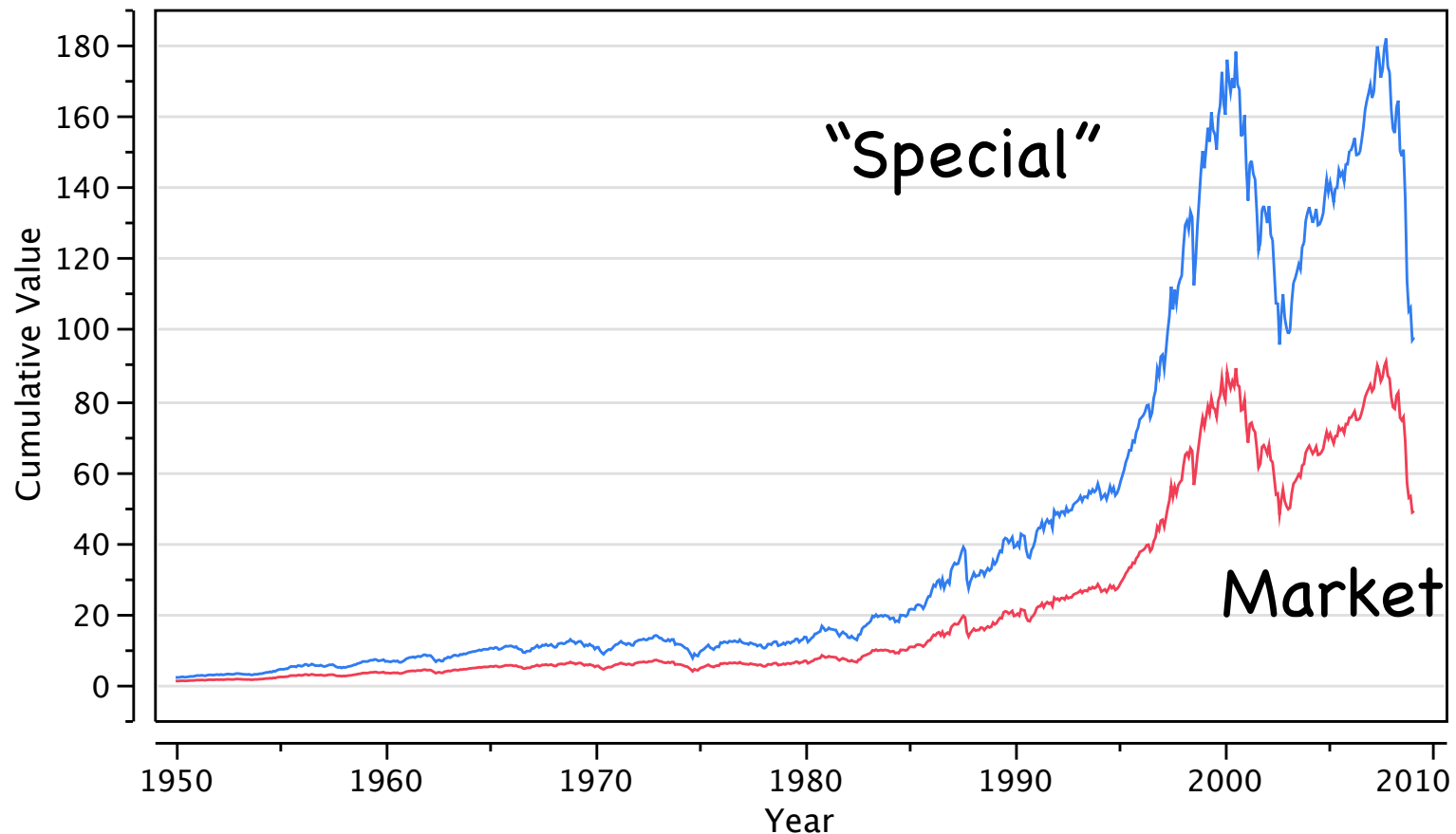


7% Annual growth

Log Scale

Cumulative Returns?

- Too easy to be deceived...

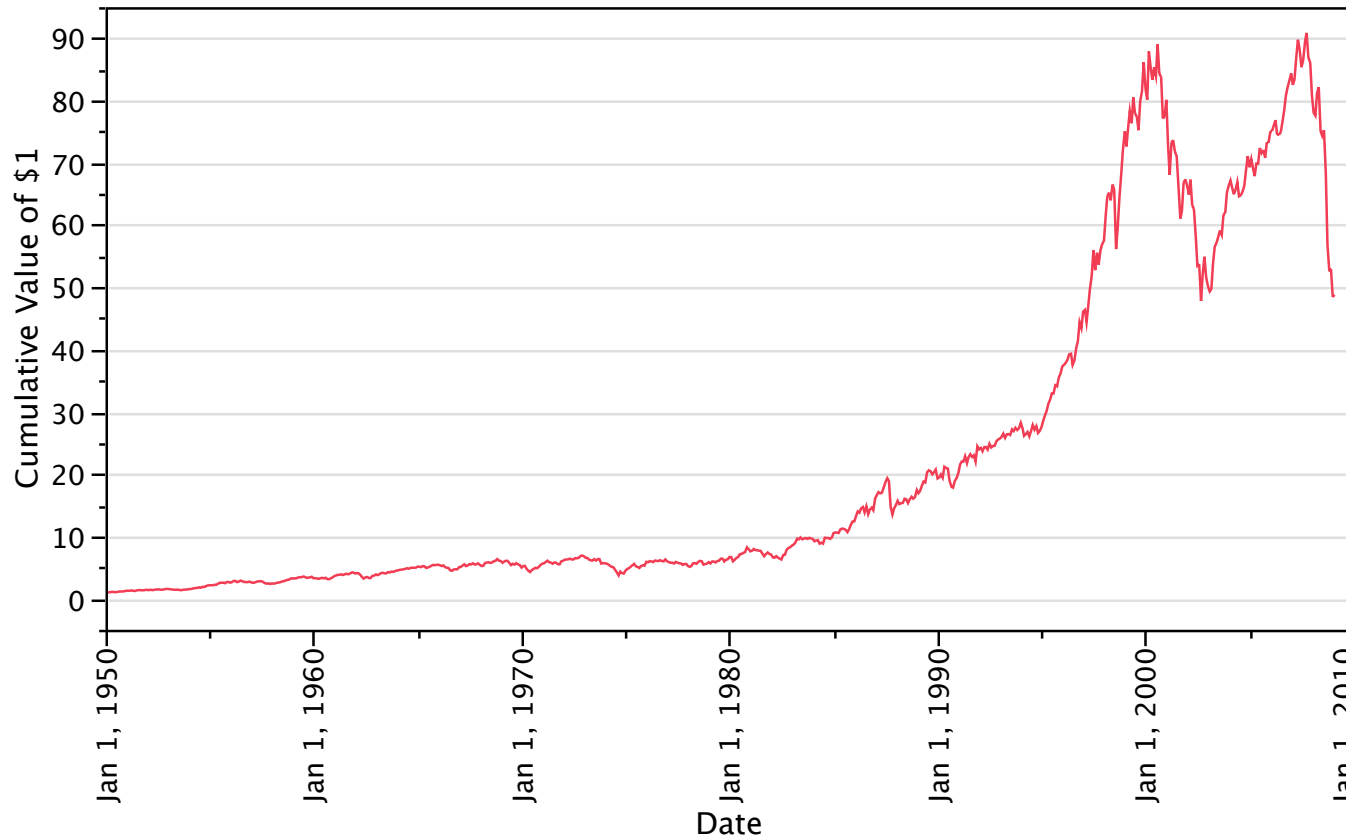


Moral: Stick to returns...

Monthly Returns

- Much simpler structure, almost iid...

$$\frac{P_t - P_{t-1}}{P_{t-1}}$$

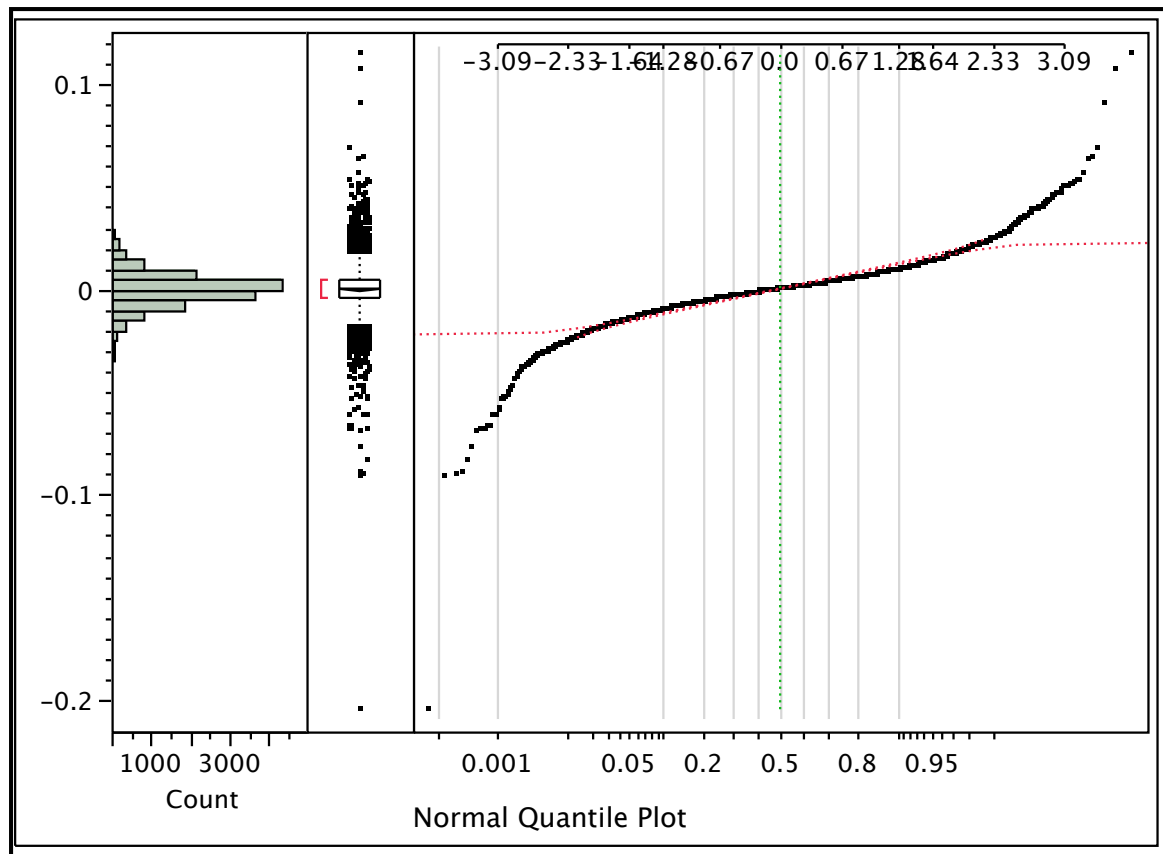


October 1987
"Black Monday"

August 1998
Long Term Capital
October 2008
Banking Crisis

Distribution of Returns

mean = 0.0064, $s = 0.0415$, $s^2 = 0.0017$

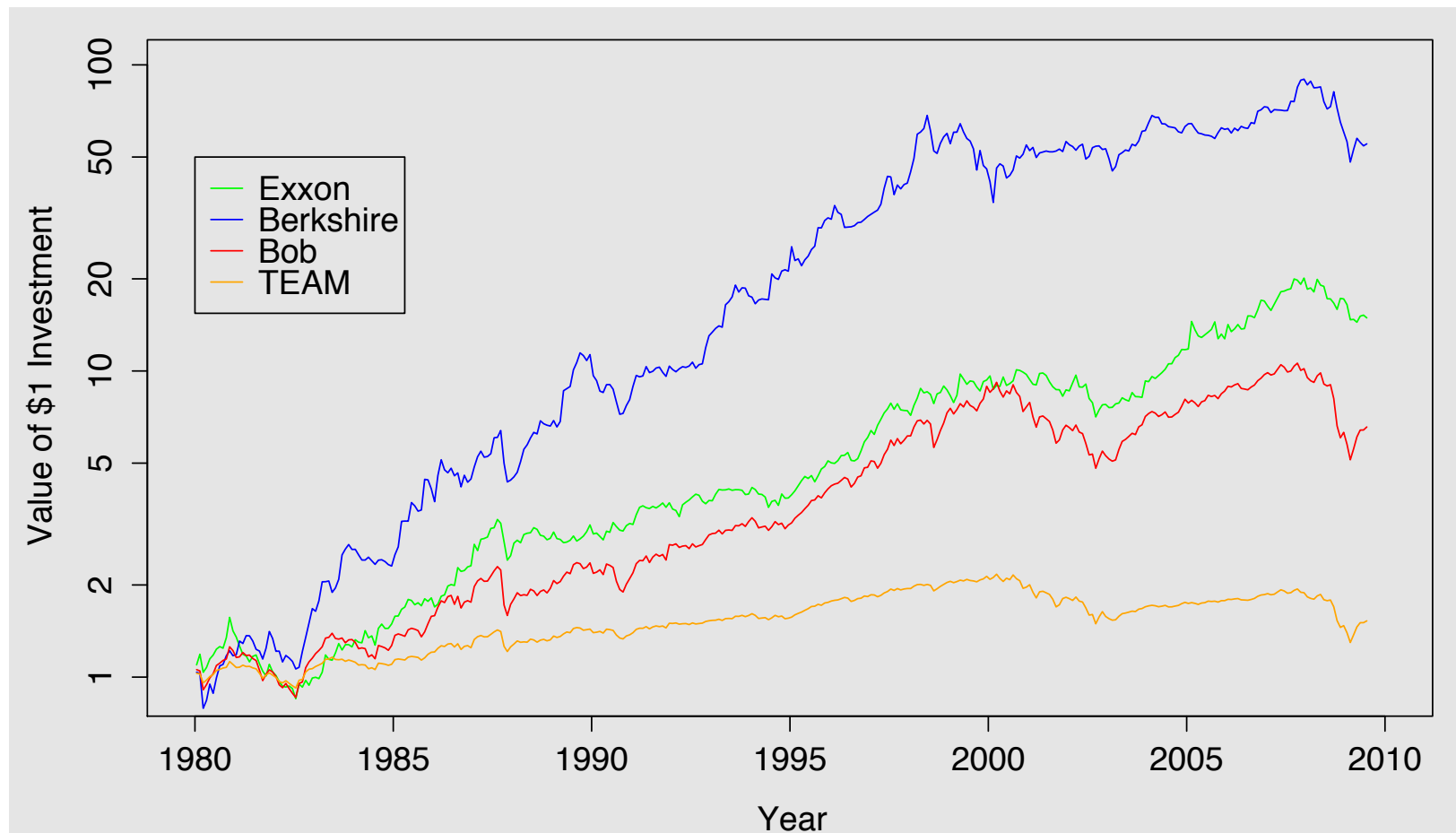


Fat tails more apparent in daily data.

mean = 0.0003, $s = 0.0095$, $s^2 = 0.0001$

Challenge Asset Returns

- Sequence of “bets” that appear nearly independent, but correlation remains between assets.



$$R_t = (P_t - P_{t-1}) / P_{t-1}$$

risk = variation in returns

Summary Statistics

Monthly returns, 1980-2009

	Mean Return	SD Return
Exxon	0.0089	0.0503
Berkshire	0.0137	0.0701
Bob	0.0064	0.0467
TEAM	0.0014	0.0217

Trade off return for risk?

Questions

- Two fundamental questions
- How much?
 - How much of my wealth should I invest to meet my financial goals?
- Which assets?
 - Start with the whole-market index
 - Which other investments in addition to an index based on the whole market?

How Much?

The Dice Game

What makes a good investment?

- Consider 3 investments...

Investment	Average Annual Return	SD Annual Return
Green	7.5%	20%
Red	71%	132%
White	0%	6%

- Questions

- Which of these do you like, if any?
- How do you decide: risk versus return?

Hands-on Simulation

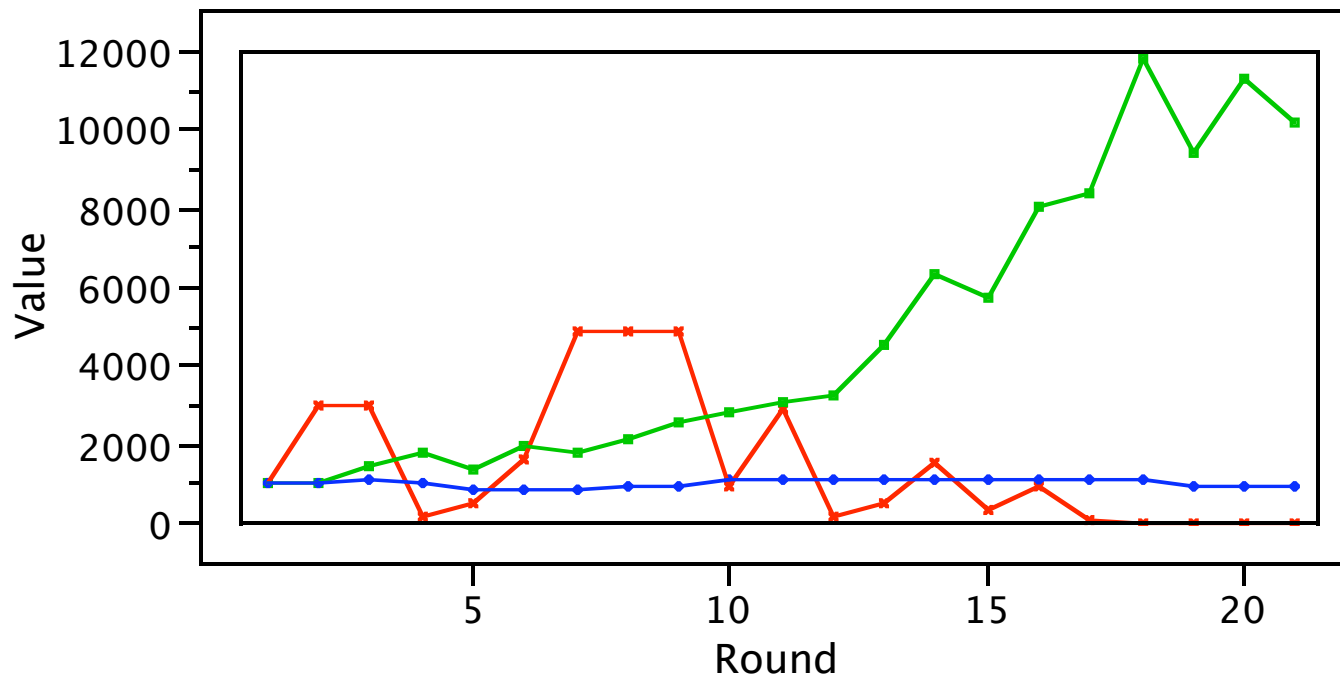
- 3 dice determine outcomes:

$$W_t = (\text{Table Result}) W_{t-1}$$

Outcome	Green	Red	White
1	0.8	0.06	0.9
2	0.9	0.2	1
3	1.05	1	1
4	1.1	3	1
5	1.2	3	1
6	1.4	3	1.1

Typical Results

- Red is “exciting” but generally loses value.
- Green offers steady growth.
- White goes nowhere.



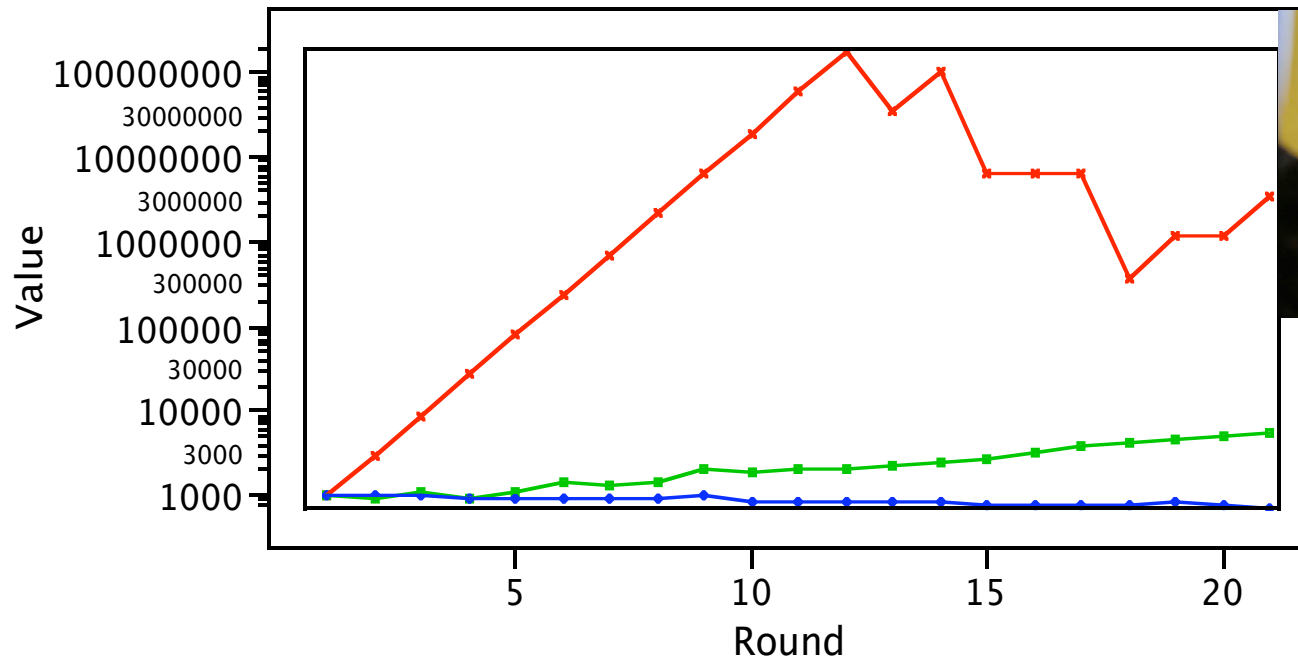
Green is calibrated to match annual excess returns on US stock market.

White is calibrated to match returns on Treasury Bills.

We made up Red!

Occasional Results

- Red soars...
 - In 20 rounds, the expected value of Red is $1.71^{20} = 45,700$ times initial value



Digesting the Results

- ◉ Something to ponder
 - ◉ Most simulations with the dice result in Red having lost most of its value.
 - ◉ A few simulations end with Red being fabulously wealthy, the “Warren Buffetts” of the class
- ◉ In the long run, Red will lose (w.p. 1)
 - ◉ How can I recognize that Red will lose without waiting for it to happen?
 - ◉ Even so, how can I take advantage of Red?

Investment Objective

- Long-run wealth

$$W_t = W_{t-1} (1+r_t)$$

$$= W_0 (1+r_1)(1+r_2) \dots (1+r_t)$$

- If the r_t are independent over time, then

$$W_t \approx W_0 (1 + E(r_t) - \text{Var}(r_t)/2)^t$$

Volatility Drag

	$E(r_t)$	$\text{Var}(r_t)$	$E(r_t) - \text{Var}(r_t)/2$
Green	0.075	$(0.20)^2 = 0.04$	$.075 - .04/2 = .055$
Red	0.71	$(1.32)^2 = 1.74$	-0.16
White	0	$(0.06)^2 = 0.003$	-0.002

Can buy this one

Diversifying is good.

- Mix investments rather than leaving everything in one.
- Pink is a 50/50 mixture of Red & White.

$$\begin{aligned} E(\text{Pink}) &= E(0.5 \text{ Red} + 0.5 \text{ White}) \\ &= E(\text{Red})/2 = 0.355 \end{aligned}$$

$$\begin{aligned} \text{Var}(\text{Pink}) &= \text{Var}(0.5 \text{ Red} + 0.5 \text{ White}) \\ &= \text{Var}(\text{Red})/4 = 0.435 \end{aligned}$$

Sacrifice **half** of the return to reduce the variance by 4.

- Long-run value of Pink is positive:
 $E(\text{Pink}) - \text{Var}(\text{Pink})/2 = 0.14$
even though neither Red nor White perform well taken separately.

Lessons from Dice Game

- Long-run return given by
 $E(\text{return}) - (1/2) \text{Var}(\text{return})$
- Over short horizons, a poor long-term investment might appear very attractive.
- Portfolios succeed by trading expected returns for reductions in variance

Cautions

- Real investments lack some properties of the investments in the dice simulation
- Independence
 - The dice fluctuate independently of one another. The returns of Red are not affected by what happens to Green.
- Stability
 - The properties of the dice stay the same throughout the simulation. The chance for a good return on Red does not change.
- Parameters known
 - We know the properties of the random processes in the dice game.

Challenge: Summary Stats

Monthly returns, 1980-2009

	Mean Return	SD Return	Long run Return
Exxon	0.0089	0.0503	0.0076
Berkshire	0.0137	0.0701	0.0113
Bob	0.0064	0.0467	0.0053
TEAM	0.0014	0.0217	0.0012

How much to invest?

- If we accept the objective to maximize long-run wealth, then the proportion of our wealth p to put in an investment is

$$p = \frac{\mu - r_f}{\sigma^2}$$

r_f is the risk-free rate of interest

- Example suggests we're more risk averse...
 - μ and σ for the entire history of the market gives

$$p = 0.075/0.040 = 1.75$$

times wealth.

- Nonetheless, we ought to invest **some** fraction of our wealth in any asset for which we know $\mu \neq 0$ (short it if $\mu < 0$).

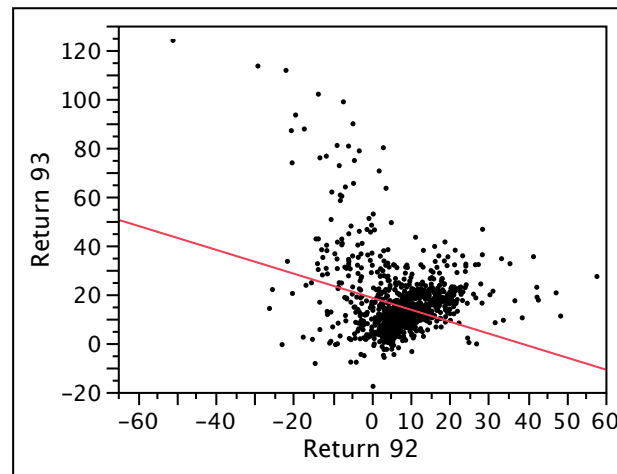
Which Investments?

Problem: So many choices?

- The simple analysis of how much to invest considers one asset, in isolation.
- Complications
 - Many many choices
 - Returns are correlated
- Role of dependence
 - Need to consider the correlation among the returns when investing in several
 - Messy problem of portfolio analysis is to anticipate correlations going forward.
- Theory from finance
 - Invest first in the market as a whole
 - Then consider other assets.

Dependence: Mutual Funds

- Regress growth in current year on prior growth
 - Annual results for 1500 mutual funds
- “Statistically significant”



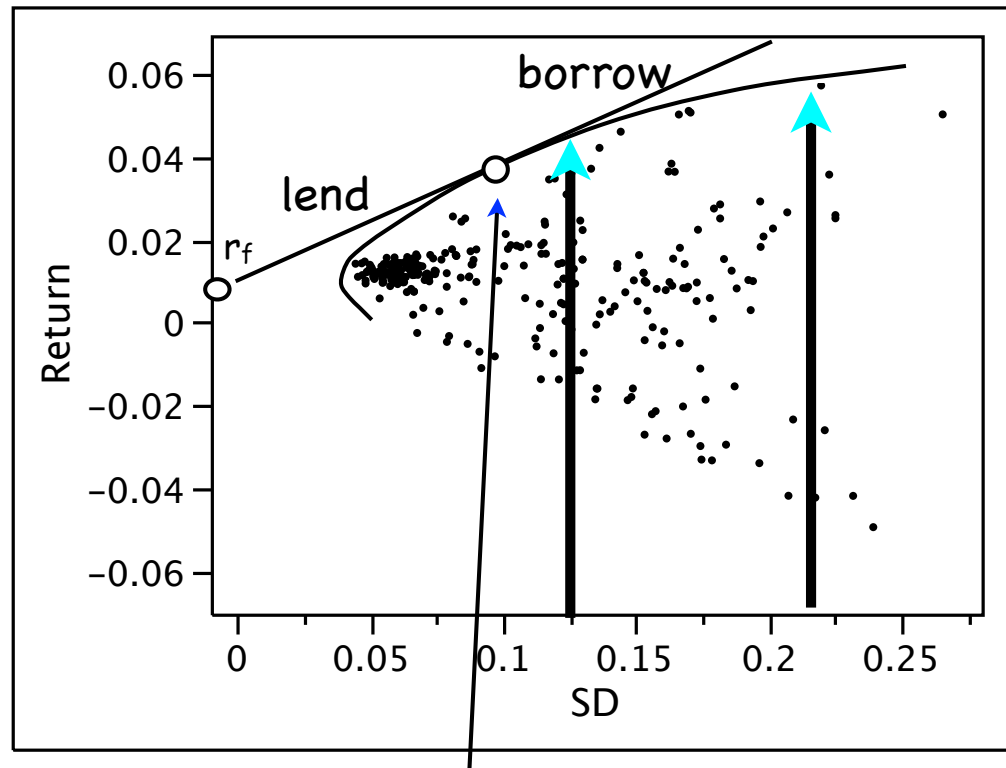
But the sign changes!

Term	Estimate	Std Error	t Ratio
Intercept	18.634313	0.467789	39.83
Return 92	-0.49079	0.04243	-11.57

Efficient Frontier

- Plot average return on SD of return for a collection of randomly formed portfolios

Leverage



Efficient Frontier

Mixing the "tangent" portfolio with cash obtains better performance

The tangent portfolio is the market portfolio.

Capital Asset Pricing Model

- Linear equation
 - Excess returns on an asset are related to those on whole market by a linear equation

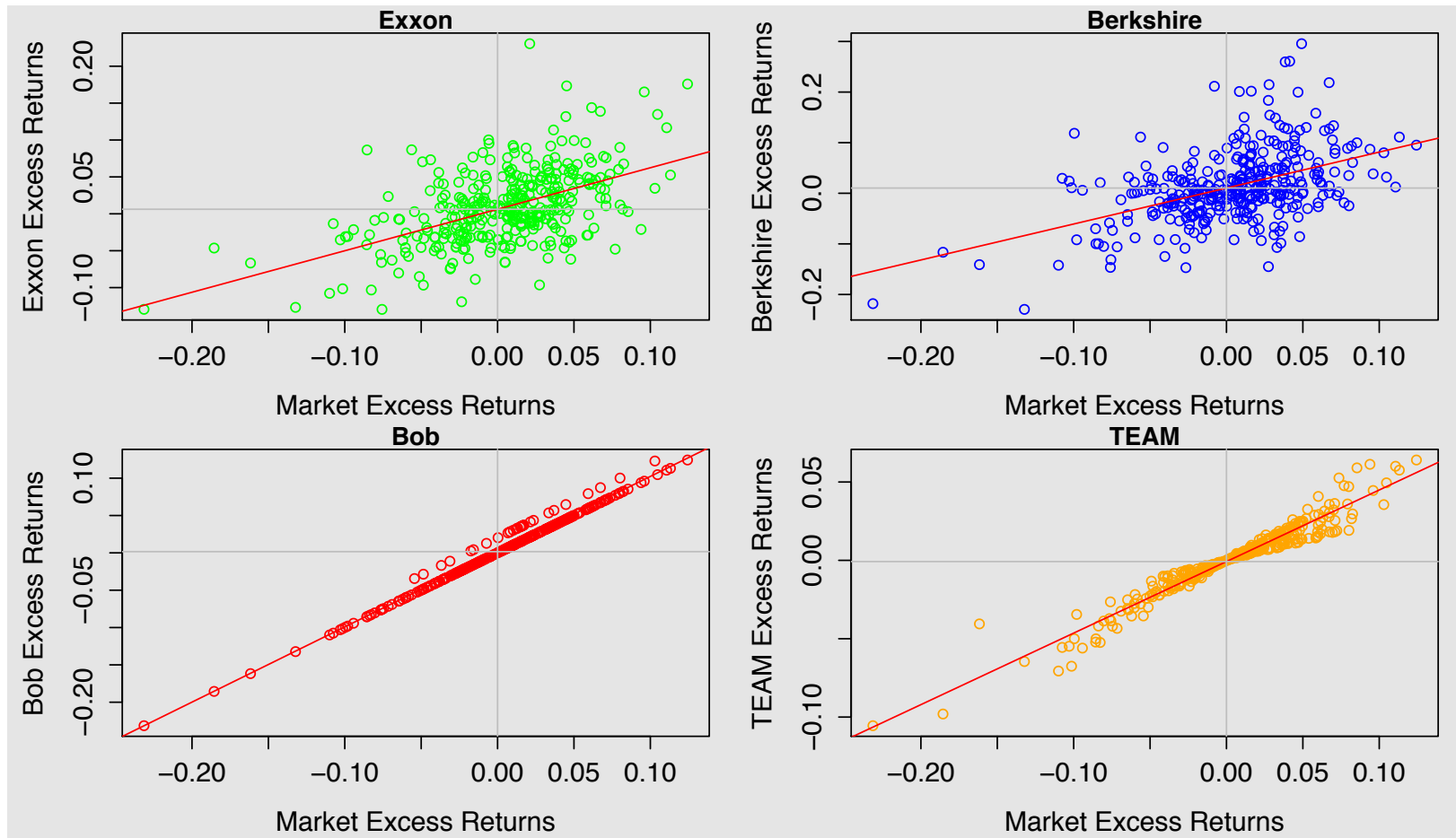
CAPM

$$R_t - r_f = \alpha + \beta (M_t - r_f) + \varepsilon_t$$

- r_f is the risk-free rate
 - $\beta = \text{Cov}(R_t - r_f, M_t - r_f) / \text{Var}(M_t - r_f)$
 - $\alpha = 0$
 - Orthogonal
 - Divide risk into market and specific
 - Specific returns uncorrelated with market
- $$(R_t - r_f) - \beta (M_t - r_f) = \alpha + \varepsilon_t$$
- If $\alpha \neq 0$?
 - Intrinsic variation in asset has non-zero mean
 - Buy (or sell) some amount of it.

CAPM Regressions

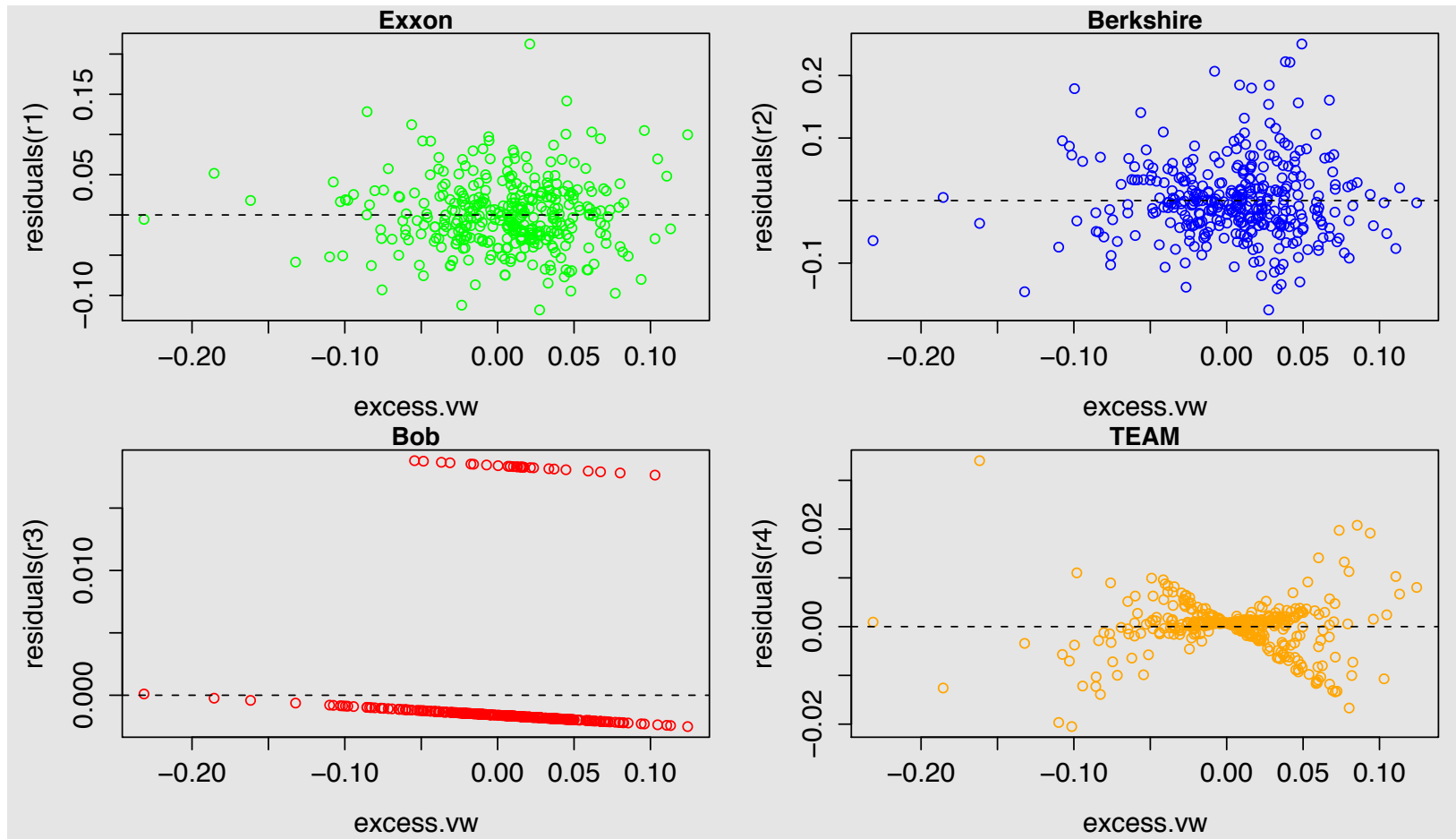
- All have substantial correlation with the returns on the whole market (market risk)



$$M_{t-r_f} = \text{market excess return}$$

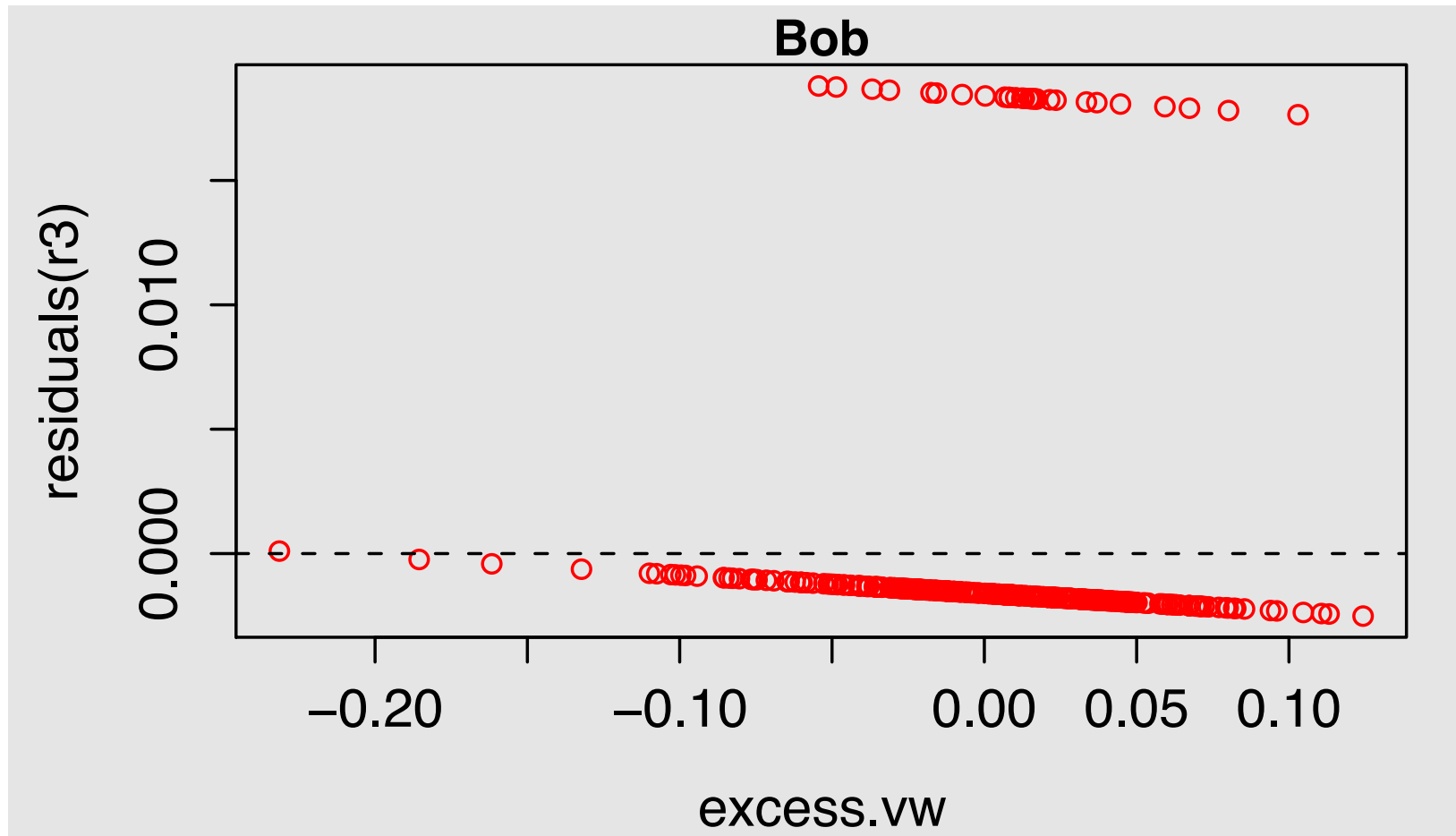
CAPM Residuals

- Residuals often deviate from the usual assumptions of simple regression.



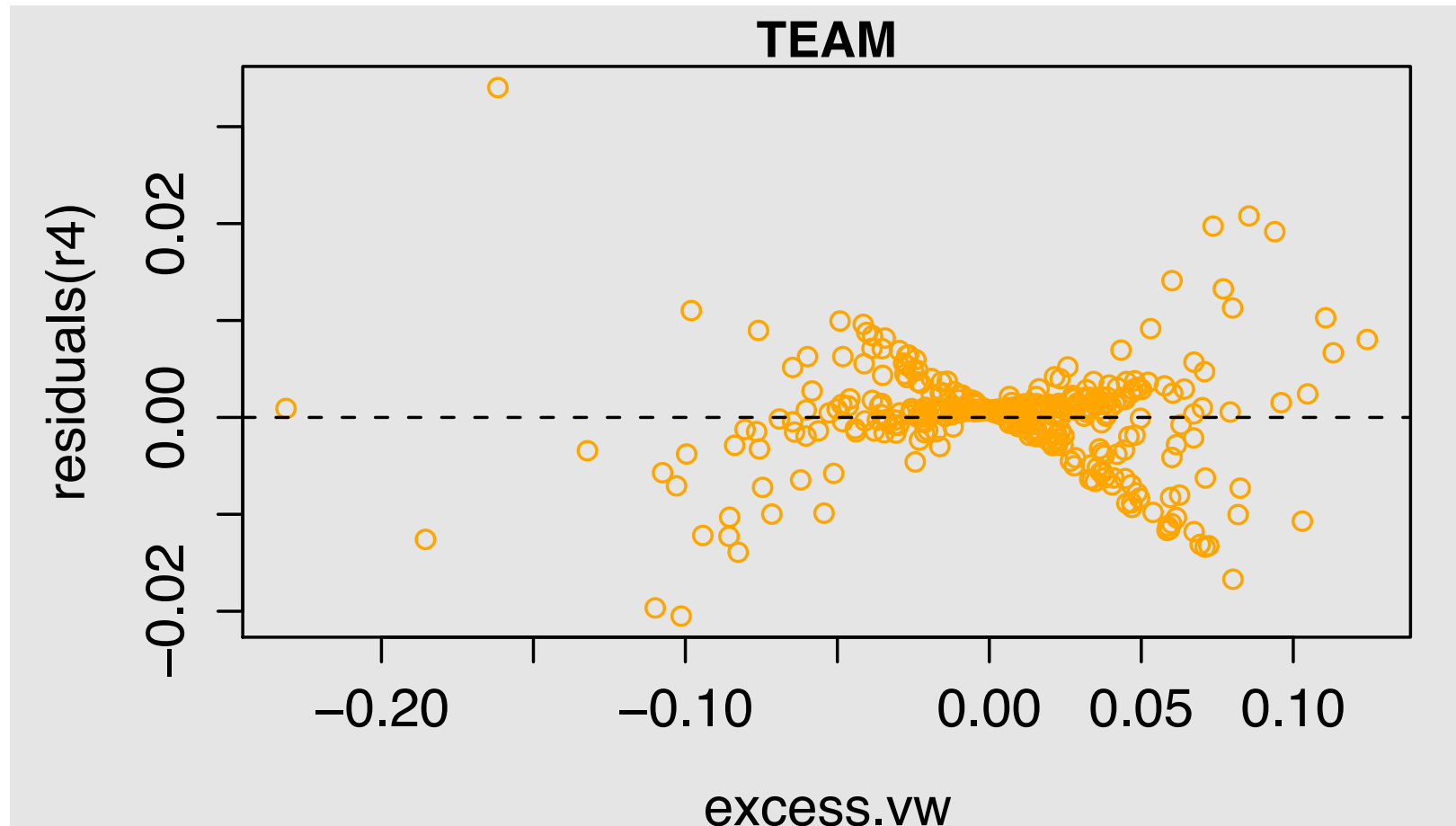
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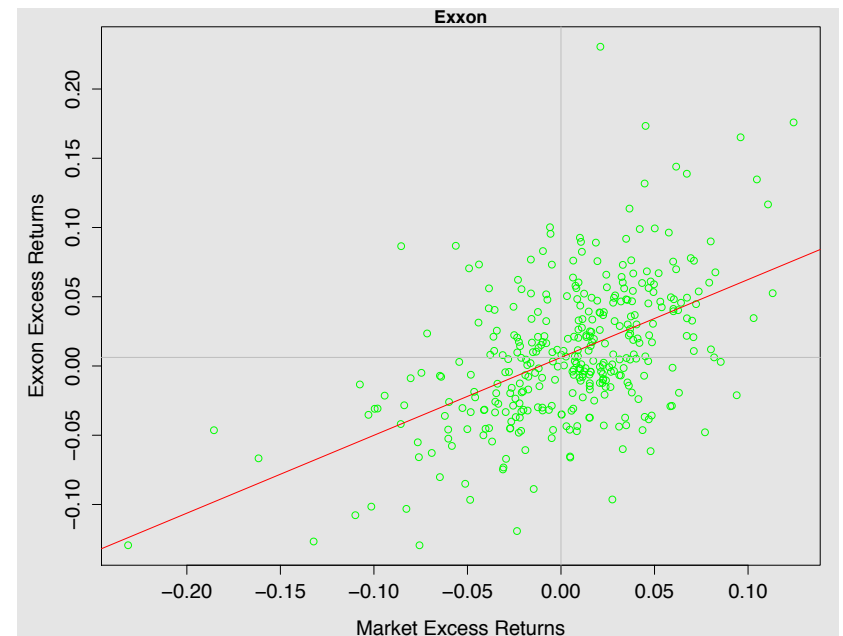
CAPM Residuals

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Usual Test of Alpha

- Example: Exxon
- Regress out market risk, obtaining estimates of α and β .
 - beta = 0.56
 - alpha = 0.0062
- Test $H_0: \alpha = 0$
 - Standard procedure relies on t-distribution to obtain p-value



	Estimate	SE	t	p
Alpha	0.0062	0.0023	2.68	0.0077
Beta	0.5623	0.0500	11.25	0

Summary of Tests

	estimate of alpha	t	p-value
Exxon	0.0062	2.7	0.008
Berkshire	0.0103	3.1	0.002
Bob	0.0016	5.5	0.000
Team	-0.0008	-2.6	0.009

Do you believe these results?

Robust Testing

Testing Alpha

- Standard test procedure
 - Regress out the market
 - Test $H_0: \alpha = 0$ using t-statistic
- Model risk
 - Doubts about standard test.
 - What's the distribution of the t-statistic?
Some investments produce returns that are far from Gaussian, with large outliers (fat tails)
 - Evident lack of independence in CAPM residuals
 - ARCH processes
- Nonetheless want a p-value

Martingale Test (CERT)

- Specific returns after removing market

$$w_t = (R_t - r_f) - \beta (M_t - r_f) = \alpha + \varepsilon_t$$

- Null hypothesis $H_0: \alpha=0$

- Implies does not “beat the market”

- Assume only that $E(w_t | w_{t-1}, w_{t-2}, \dots) = 0$ (not nec. iid)

- Compound returns are non-negative martingale

$$C_t = (1+w_1)(1+w_2)\dots(1+w_t) \quad t = 1, 2, \dots, n$$

- CERT p-value from Doob's inequality

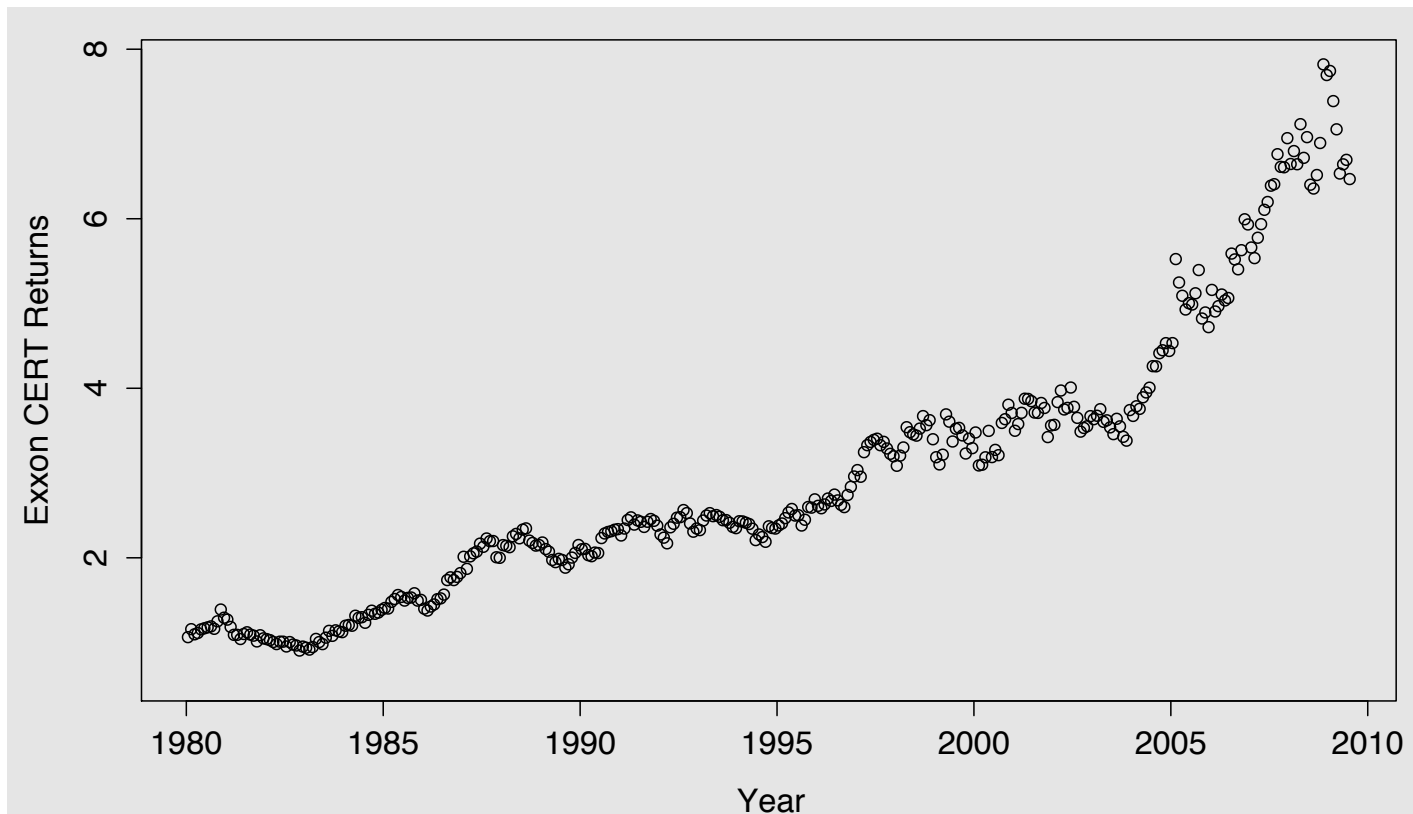
$$P(\max C_1, \dots, C_n \geq \gamma) \leq 1/\gamma$$

- Easy to use

To reject H_0 at 0.05 level, compound returns have to exceed 20 during observed period

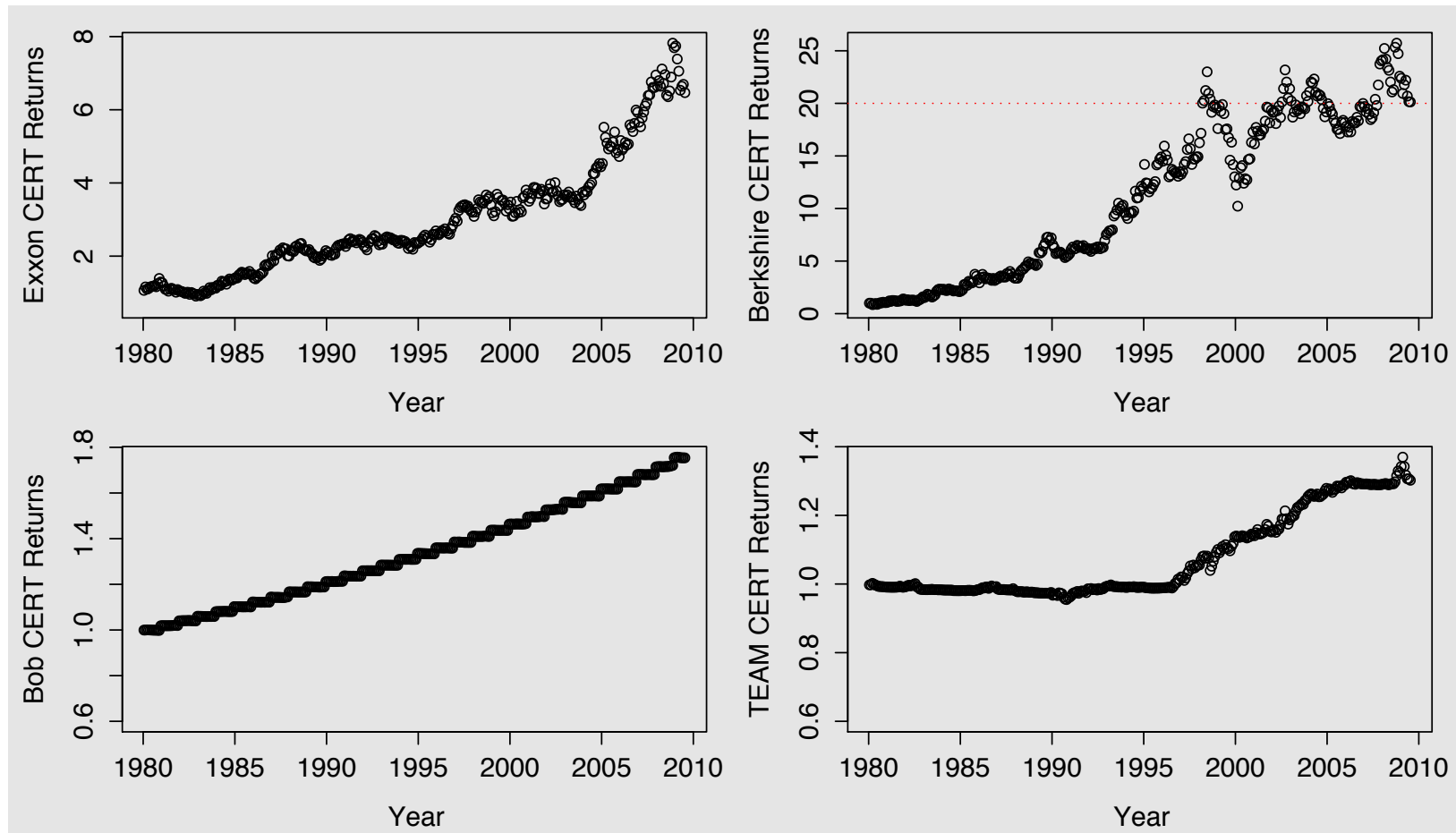
Example

- “Residual” returns for Exxon,
 $(R_t - r_f) - b (M_t - r_f)$
- Since the martingale test does not depend on n , we can use finely spaced data that essentially reveal β (if you believe it's fixed!)



CERT Results

- Only Berkshire Hathaway rejects the null, and then we have to consider multiplicity.



Wrapping Up

Discussion

- Multiplicity

A p-value of $1/20$ does not overcome adjustments for multiplicity.

- Bonferroni p-value

Multiply the p-value from martingale test by number of assets considered.

- I bet that you have considered more than 4.

- Power

The test is “tight” in the sense that there are processes you would not want to consider for which it gets the right answer, such as...

Bob Fund

- How do you guarantee those 2% above benchmark returns?
- Unobserved volatility
 - $r_t = 1/k$ w.p. $k/(k+1)$
 - $r_t = -1$ w.p. $1/(k+1)$
 - $E(r_t) = 0$
- Example
 - $k = 19$, so returns a bit more than 2% growth
 - Smaller k give more exciting performance
- For any choice of k
$$P(C_t \text{ of Bob Fund} > 20) = 1/20$$
- Martingale test protects against the “until it happens” unobserved volatility

busted!

Summary

- Focus on returns, not cumulative value
- Remove market performance
 - Regress out market from returns (CAPM)
- Watch for unseen volatility using robust test
 - Martingale test (CERT)
- Adjust for multiplicity
 - Bonferroni does fine, particularly since it's hard to "count" the considered alternatives

Thanks!

www-stat.wharton.upenn.edu/~stine

Foster, Stine, Young (2008) "A martingale test for alpha"