

## Timing Production Runs

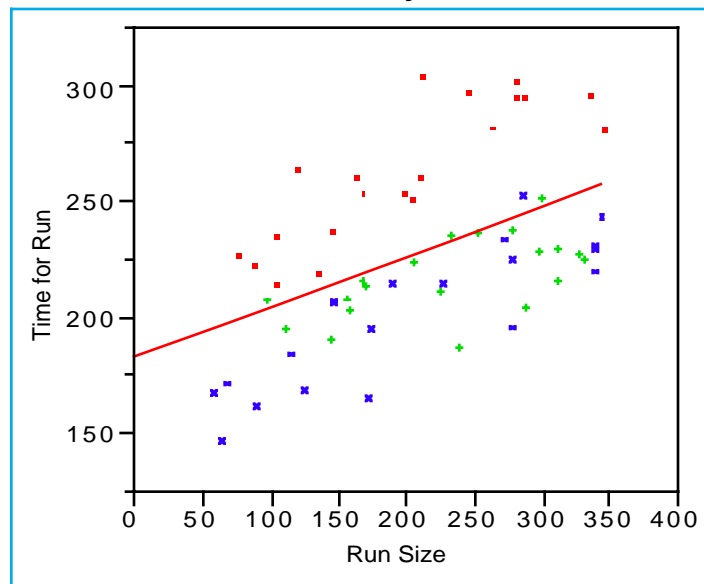
ProdTime.jmp

An analysis has shown that the time required in minutes to complete a production run increases with the number of items produced. Data were collected for the time required to process 20 randomly selected orders as supervised by three managers.

How do the managers compare?

The initial plot below is vague until coded using the *Manager* indicator variable as shown below. (Use the *Col/Marker by Col...* command from the *Rows* menu.) The overall fit is indeed positive in slope, with an average time per unit (the slope) of 0.22 per unit.

**Time for Run by Run Size**

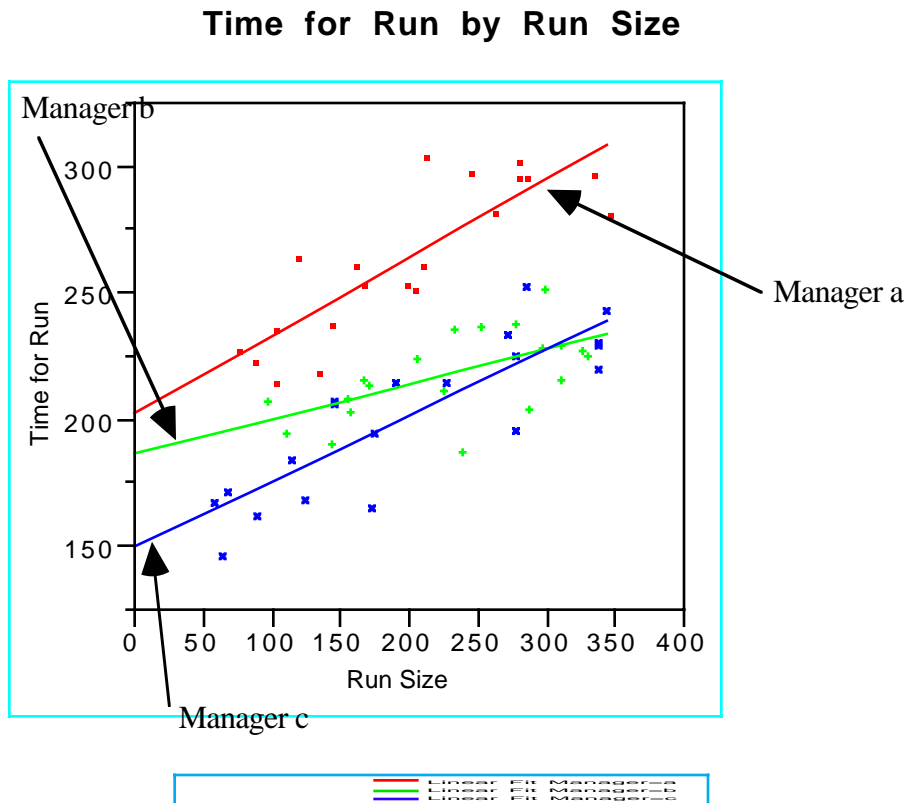


### Linear Fit

RSquare 0.256  
Root Mean Square Error 32.108

Term	Estimate	Std Error	t Ratio	Prob> t
Intercept	182.31	10.96	16.63	<.0001
Run Size	0.22	0.05	4.47	<.0001

In contrast to one overall regression, might the fit differ if restricted to the 20 runs available for each manager? When the regression is fit separately for each manager as shown next, it is clear that the fits are different.



Are the differences among these three fitted lines “large” and significant, or might they be the result of random variation? After all, we haven’t much data for each manager. Summaries of the three fitted regressions appear on the next page.

What is the interpretation of the slope and intercept in each equation?

***Linear Fit Manager=a (Red)***

RSquare	0.710
RSquare Adj	0.694
Root Mean Square Error	16.613
Mean of Response	263.050
Observations (or Sum Wgts)	20

Term	Estimate	Std Error	t Ratio	Prob> t
Intercept	202.53	9.83	20.59	<.0001
Run Size	0.31	0.05	6.65	<.0001

***Linear Fit Manager=b (Green)***

RSquare	0.360
RSquare Adj	0.325
Root Mean Square Error	13.979
Mean of Response	217.850
Observations (or Sum Wgts)	20

Term	Estimate	Std Error	t Ratio	Prob> t
Intercept	186.49	10.33	18.05	<.0001
Run Size	0.14	0.04	3.18	0.0051

***Linear Fit Manager=c (Blue)***

RSquare	0.730
RSquare Adj	0.715
Root Mean Square Error	16.252
Mean of Response	202.050
Observations (or Sum Wgts)	20

Term	Estimate	Std Error	t Ratio	Prob> t
Intercept	149.75	8.33	17.98	<.0001
Run Size	0.26	0.04	6.98	<.0001

To see if these evident differences are significant, we embed these three models into one multiple regression.

The multiple regression tool *Fit Model* lets us add the categorical variable *Manager* to the regression. Here are the results of the model with *Manager* and *Run Size*. Since we have more than two levels of this categorical factor, the “Effect Test” component of the output becomes relevant. It holds the partial *F*-tests. We ignored these previously since these are equivalent to the usual *t*-statistic if the covariate is continuous or is a categorical predictor with two levels.

### Response: Time for Run

RSquare	0.813
RSquare Adj	0.803
Root Mean Square Error	16.376
Mean of Response	227.650
Observations (or Sum Wgts)	60

Term	Estimate	Std Error	t Ratio	Prob> t
Intercept	176.71	5.66	31.23	<.0001
Run Size	0.24	0.03	9.71	<.0001
Manager[a-c]	38.41	3.01	12.78	<.0001
Manager[b-c]	-14.65	3.03	-4.83	<.0001

### Effect Test

Source	Nparm	DF	Sum of Squares	F Ratio	Prob>F
Run Size	1	1	25260.250	94.2	<.0001
Manager	2	2	44773.996	83.5	<.0001

The two terms  $\text{Manager}[a-c]$  and  $\text{Manager}[b-c]$  require some review. These two coefficients represent the differences in the intercepts for managers  $a$  and  $b$ , respectively, from the average intercept of *three parallel* regression lines. The effect for manager  $c$  is the negative of the sum of these two (so that the sum of all three is zero). That is,

$$\text{Effect for Manager } c = - (38.41 - 14.65) = - 23.76$$

The effect test for *Manager* indicates that the differences among these three intercepts are significant. It does this by measuring the improvement in fit obtained by adding the *Manager* terms to the regression simultaneously. One way to see how it's calculated is to compare the  $R^2$  without using *Manager* to that with it. If we remove *Manager* from the fit, the  $R^2$  drops to 0.256. Adding the two terms representing the two additional intercepts improves this to 0.813. The effect test determines whether this increase is significant by looking at the improvement in  $R^2$  per added term. Here's the expression: (another example of the partial  $F$ -test appears in Class 5).

$$\begin{aligned} \text{Partial } F &= \frac{\text{Change in } R^2 \text{ per added variable}}{\text{Remaining variation per residual}} \\ &= \frac{(R^2_{\text{complete}} - R^2_{\text{reduced}}) / (\# \text{ variables added})}{(1 - R^2_{\text{complete}}) / (\text{Error degrees of freedom})} \\ &= \frac{(0.813 - 0.256)/2}{(1 - 0.813)/56} \\ &\approx \frac{0.28}{.0033} \\ &= 83.4 \end{aligned}$$

What about the clear differences in slopes?

We need to add interactions to capture the differences in the slopes associated with the different managers.

### Response: Time for Run

RSquare	0.835
RSquare Adj	0.820
Root Mean Square Error	15.658
Mean of Response	227.65
Observations (or Sum Wgts)	60

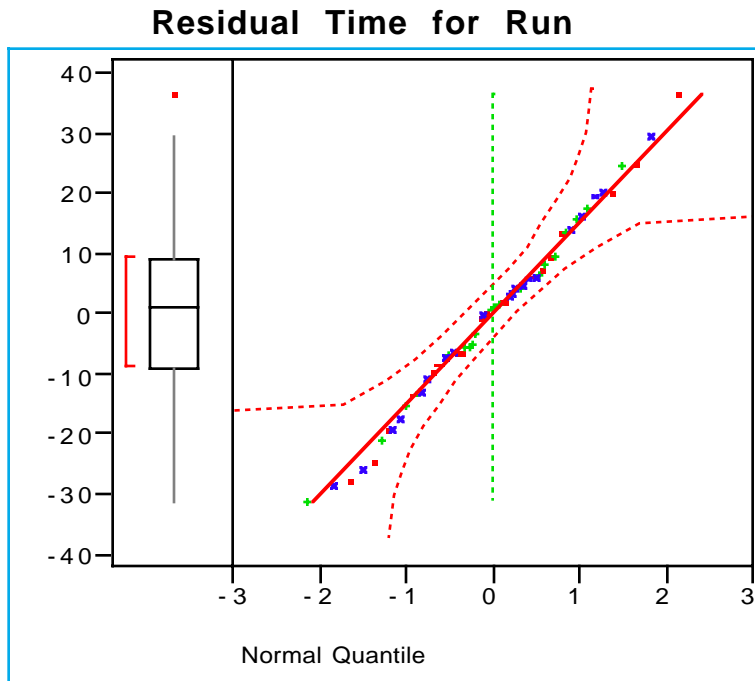
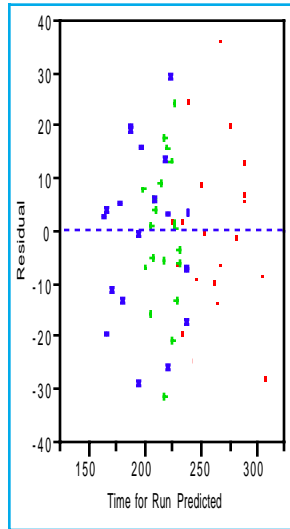
Term	Estimate	Std Error	t Ratio	Prob> t
Intercept	179.59	5.62	31.96	0.00
Run Size	0.23	0.02	9.49	0.00
Manager[a-c]	22.94	7.76	2.96	0.00
Manager[b-c]	6.90	8.73	0.79	0.43
Manager[a-c]*Run Size	0.07	0.04	2.07	0.04
Manager[b-c]*Run Size	-0.10	0.04	-2.63	0.01

### Effect Test

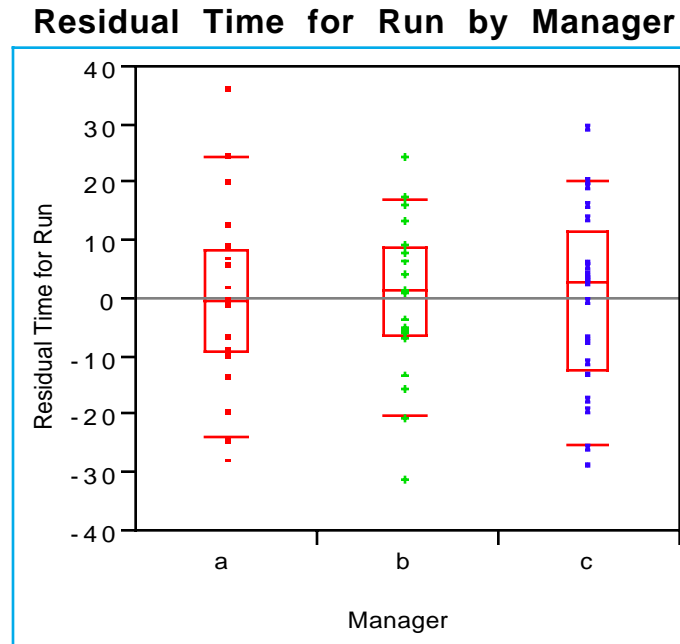
Source	Nparm	DF	Sum of Squares	F Ratio	Prob>F
Run Size	1	1	22070.614	90.0	<.0001
Manager	2	2	4832.335	9.9	0.0002
Manager*Run Size	2	2	1778.661	3.6	0.0333

In this output, the coefficients Manager[a-c]\*Run Size and Manager[b-c]\*Run Size indicate the differences in slopes from the average manager slope. As before, to get the term for manager *c* we need to find the negative of the sum of the shown effects, 0.03. The differences are significant, as seen from the effect test for the interaction.

The residual checks indicate that the model satisfies the usual assumptions.



Since the model has categorical factors, we also need to check that the residual variance is comparable across the groups. Here, the variances seem quite similar.



### Means and Std Deviations

Manager	#	Mean	Std Dev	Std Err	Mean
a	20	0.00	16.2	3.6	
b	20	0.00	13.6	3.0	
c	20	0.00	15.8	3.5	



An analysis has shown that the time required in minutes to complete a production run increases with the number of items produced. Data were collected for the time required to process 20 randomly selected orders as supervised by three managers.

How do the managers compare?

The runs supervised by Manager *a* appear abnormally time consuming. Manager *b* has high initial fixed setup costs, but the time per unit for this manager is the best of the three. Manager *c* has the lowest fixed costs with a typical per unit production time.