STAT 430

Jan, 2003

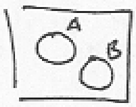
Assignment 1 Solutions (Chapter 2)

② $S = \{ (6), (-,6), (-,-,6), \dots \}$ where $-$ is any of 1, 2, 3, 4, 5. So, S has infinite outcomes. Only the outcome with $(-,-,\overset{n-1}{-},\dots,-,6)$ lies in E_n.
$\left(\bigcup_1^\infty E_n\right)^c$ is the event you never get a 6.

⑤ (a) $2\times2\times2\times2\times2 = 2^5 = 32$ by counting principle.

(b) $\{(1,1,-,-,-) \cup (-,-,1,1,-) \cup (1,-,1,-,1)\}$ (15 elements)

⑧ (a) $P(A \cup B) = .3 + .5 = .8$ since disjoint

(b) $P(A \cap B^c) = P(A) = .3$ since  $A \subset B^c$

(c) $P(A \cap B) = P(\emptyset) = 0$

⑨ $P(A \cup V) = P(A) + P(V) - P(A \cap V)$

$= .24 + .61 - .11 = .74$

C = cigarette

G = cigar

⑪ (a) $P(C^c \cap G^c) = 1 - P(C \cup G)$

$= 1 - [P(C) + P(G) - P(CG)]$

$= 1 - (.28 + .07 - .05) = .7$ (ie, 70%)

(b) $P(G \cap C^c) = P(G) - P(CG) = .07 - .05 = .02$

⑮ (a) $P(\text{flush}) = 4 \dfrac{\binom{13}{5}\binom{39}{0}}{\binom{52}{5}}$ also counting straight flush

$\Uparrow$

4 different suits

(c) $P(2 \text{ pairs}) = \binom{13}{2} \dfrac{\binom{4}{2}\binom{4}{2}\binom{44}{1}}{\binom{52}{5}}$

$\Uparrow$

ways to pick 2 denominations

(18) $P(\text{blackjack}) = \dfrac{\binom{4}{1}\binom{16}{1}}{\binom{52}{2}} = \dfrac{4 \cdot 16}{52 \cdot 51/2}$

(23) If arrange the sample space as an array of the form

$$\begin{array}{cccc} 1,1 & 1,2 & \cdots & 1,6 \\ 2,1 & 2,2 & \cdots & 2,6 \\ 3,1 & 3,2 & \cdots & 3,6 \\ 4,1 & 4,2 & \cdots & 4,6 \\ & \vdots & & \\ 6,1 & 6,2 & \cdots & 6,6 \end{array}$$

second die larger

equal on diagonal

Hence there are $\dfrac{36-6}{2} = 15$ outcomes that have a larger second die, and the prob $= {}^{15}/_{36} = {}^{5}/_{12}$

(27) Arrange all 10 draws as 10 "places."

$\underline{\underline{①}}\ \_\ \underline{\underline{③}}\ \_\ \underline{\underline{⑤}}\ \_\ \underline{\underline{⑦}}\ \_\ \underline{\underline{⑨}}\ \_$

We want prob that odd places catch first red ball. Cannot be ⑨ since there are 3 Reds. So

$$P(① \cup ③ \cup ⑤ \cup ⑦) \overset{\text{disjoint}}{=} P(①) + P(③) + P(⑤) + P(⑦)$$

$$= \frac{3 \cdot 9! + \frac{7!}{5!} \cdot 3 \cdot 7! + \frac{7!}{3!} \cdot 3 \cdot 5! + \frac{7!}{1!} \cdot 3 \cdot 3!}{10!}$$

Viewed using conditioning ideas gives same answer, just by a different means:

$$P(① \cup ③ \cup ⑤ \cup ⑦) = \begin{aligned} & P(①) = P(R) = {}^{3}/_{10} \\ & + P(③) = P(BBR) = P(R \mid BB) \cdot P(BB) \\ & \qquad = \frac{3}{8} \cdot \frac{7 \cdot 6}{10 \cdot 9} \\ & + P(⑤) = P(BBBBR) = P(R \mid BBBB)\, P(B\text{–}B) \\ & \qquad = \frac{3}{6} \cdot \frac{7 \cdot 6 \cdot 5 \cdot 4}{10 \cdot 9 \cdot 8 \cdot 7} \\ & + P(⑦) = P(B \cdots BR) = 3/\!\!. \end{aligned}$$

(3.7)

(a) $\dfrac{\binom{7}{5}\binom{3}{0}}{\binom{10}{5}} = \dfrac{7!/2!}{10!/5!} = \dfrac{\cancel{8}\cdot\cancel{4}\cdot\cancel{3}}{\cancel{10}\cdot\cancel{9}\cdot\cancel{8}} = \dfrac{1}{12}$

(b) $P(4 \text{ or } 5) = \dfrac{\binom{7}{4}\binom{3}{1}}{\binom{10}{5}} + \dfrac{1}{12} = \dfrac{1}{2}$

(45)

(a) Searches in random order:

$$P(k^{th} \text{ key works}) = P(\text{right key in } k^{th} \text{ position}) = 1/n$$

(b) Searches randomly (ie "samples with replacement)

$$P(k^{th} \text{ key works}) = P(\text{not})\, P(\text{not}) \cdots P(\text{not})\, P(\text{works})$$
$$= \left(\frac{n-1}{n}\right) \cdots \left(\frac{n-1}{n}\right) \frac{1}{n}$$
$$= \left(\frac{n-1}{n}\right)^{k-1} \frac{1}{n}$$

and observe that there are <u>independent</u> events.