Solutions, Assignment #3 STAT 430

.5 $X = \#\text{ heads} - \#\text{ tails}$ in n tosses

X can take on values n (all heads), $n-2$ (one tail), …, $-n$ (all tails), which we can write as $n-2j$, $j=0,\dots,n$.

.6 $P(X=3) = P(\text{all heads}) = 1/8 = P(X=-3) = 1/8$

$P(X=1) = P(X=-1) = 3/8$ (3 ways to put the head/tail among 3 tosses)

1a Ignoring the "end effect" (ie, let k grow large) then $1/3$ are divisible by 3, $1/5$ are divisible by 5, etc.

b (opt) For $\mu(n)=0$, we need it to have a repeated prime. That is, n must be a multiple of 2^2, 3^2, 5^2, etc. Hence, thinking conditionally,

$$P(\mu(n)=0) = 1 - \underbrace{\left(\tfrac{3}{4}\right)}_{\text{not factor of 4, etc}}\left(\tfrac{8}{9}\right)\left(\tfrac{24}{25}\right)\cdots = 1 - \frac{6}{\pi^2}$$

3 $p_X(x) = \binom{4}{x}\left(\tfrac{1}{2}\right)^4$, $x=0,\dots,4$ $p_Y(y) = \binom{4}{y+2}\left(\tfrac{1}{2}\right)^4$, $y=-2,\dots,2$ $Y = X-2$

Draw the PMF; it just gets shifted to the left by 2.

0 X are winnings

Possible outcomes

	X	prob
W_1	$+1$	$18/38$
$L_1 L_2 L_3$	-3	$(20/38)^3$
$L_1 L_2 W_3$	-1	$(20/38)^2(18/38)$
$L_1 W_2 L_3$	-1	$(20/38)^2(18/38)$
$L_1 W_2 W_3$	$+1$	$(20/38)(18/38)^2$

a. $P(X>0) = P(X=1) = \frac{18}{38} + \frac{20}{38}\left(\frac{18}{38}\right)^2$

b. See part c

c. $EX = -.108 = 1\left[\frac{18}{38} + \frac{20}{38}\left(\frac{18}{38}\right)^2\right] - 1\left[2\left(\frac{20}{38}\right)^2\left(\frac{18}{38}\right)\right] - 3\left[\frac{20}{38}\right]^3$

21 a. Expect EX to be larger since more likely to pick passenger on a bus with more passengers.

b. $EX = 40\left[\frac{40}{n}\right] + 33\left[\frac{33}{n}\right] + 25\left[25/n\right] + 50\left[50/n\right]$ $n = 148$

$= 39$

21 (cntd) $EY = \frac{1}{4}(40+33+25+50) = \frac{148}{4} = 37$

(selection bias!)

27 $E[\text{profit}] = \underbrace{(C-A)}_{\text{charge for policy}} p + C(1-p) = C - pA$

Solve for $E[\text{profit}] = .1A \Rightarrow C - pA = .1A \Rightarrow C = (p + \frac{1}{10})A$

,0 a. Ha, ha. If you could answer this from expected value, Yes — but you cannot.

b. Sure.

,5 $X = \text{winnings} = \begin{cases} \$1.10 & \text{same color} \\ -\$1.00 & \text{different} \end{cases}$

$P(\text{match}) = 2 \frac{\binom{5}{2}\binom{5}{0}}{\binom{10}{2}} = \frac{4}{9}$ (pick first, then $\frac{4}{9}$ of remaining match)

$E[X] = 1.10\left(\frac{4}{9}\right) - 1.00\left(\frac{5}{9}\right) = -.07$

$Var[X] = EX^2 - (EX)^2 = (1.10)^2\left(\frac{4}{9}\right) + 1^2\left(\frac{5}{9}\right) - (.07)^2$

$= 1.09$

38. (a) $E(2+X)^2 = E(4+4X+X^2) = 4 + 4 + EX^2 = 14$

$Var\, X = EX^2 - (EX)^2 = 5 \Rightarrow EX^2 = 6$

(b) $Var(4+3X) = 3^2 Var\, X = 45$