Statistics 712		Assignment 5
<u>Spring, 1999</u>	Solutions	1

For this assignment, you will be forecasting the cellular telephone data series. The data cover the cellular market through the end of 1995. You have the task of forecasting the number of cellular subscribers for the next four periods through the end of 1997.

(1) The 4<sup>th</sup> order polynomial fit gives the long equation shown below which has an R<sup>2</sup> over 0.9996 based on the estimates fit to the available 23 points. The plot, believe it or not, shows the individual prediction intervals.

Subscribers = 489511 - 361816 Period + 110787 Period^2 - 8045.11 Period^3 + 289.362 Period^4



(a) Substituting into the this equation (or letting JMP do it for you using the \$ key offered by multiple regression) gives a prediction of 66,900,000 in period 27 (end of 1997).

(b) The individual prediction standard error is 1,040,000. Again, the multiple regression tool will do this calculation for you. Since the RMSE of the fit is only 213,000 the statistical extrapolation penalty is quite large.

(c) These estimates assume no change in the market will occur and that growth will continue as it has over this period. A peak at the residuals shows some pattern, but



nothing to indicate a dramatic change is coming. Nonetheless, remember the IBM forcast of growth. The arrival of a new technology would have huge impact not represented in this historical look.

(2)An alternative approach to this problem is to study the percentage growth of the series.



(a) You can do this one in several ways. For example, you can follow what was done in the casebook and get the fit summarized below, which predicts 14.9% growth in subscribers in period 27. Alternatively, from looking at this plot you might decide to remove the early data and fit a line through the last few as in this fit. The line is flat, so fit

Pct Growth = 7.50205 + 199.656 Recip(Period)

Summary of Fit	
RSquare	0.927104
RSquare Adj	0.923459
Root Mean Square Error	6.414511

Pct Growth = 23.3597 - 0.14489 Period

Summary of Fit		
RSquare	0.034381	
RSquare Adj	-0.06218	
Root Mean Square Error	2.90376	
Mean of Response	20.8241	
Observations (or Sum Wgts)	12	

a constant which gives 20.5% growth throughout. Now, to get the forecast number of subscribers, you have to compound these amounts. Using the latter approach with constant growth gives  $(1.205)^4 = 2.11$ , for an estimate of  $(33785661)(2.11) \approx 71,000,000$ .

(b) As to accuracy, this is hard and one has to speculate on the effects of error in estimating the compounding of growth. A model for the next period given the last observation is  $(33785661)(1.205\pm2(.03))$ . That is, the growth rate is roughly normal with mean 1.205 and SD .03. Take the product of four of these (not just one to the fourth power) you find a mean of 2.106 (a bit smaller than 2.11 and an SD of 0.1. So, the expected range for period 27 is 71,000,000  $\pm 2(3,600,000)$ . Again, this is probably too accurate since it ignores model error, but it seems a bit more reasonable than the accuracy claimed by the polynomial.

(3)All of these approaches suggest that the series will make 50 million. From the prior analysis, we can even estimate the probability of this. (Just estimate the fraction of simulated values larger than 50 million). Its all of them. Again, this is optimistic in that it ignores the chance for a technological innovation. Basically, your odds that this data will not reach 50 million are your odds of a big innovation.

() As to the use of the number of cell sites, these are correlated with the numbers of subscribers and improve a multiple regression using the  $4^{th}$  order polynomial unless you think of Bonferroni (since the p-value is 0.04). However, since the future number of cell sites is unknown, this probably would not improve the prediction from this model.