(1) Two managers gave their intervals (in $1000US) as

Bob 1500 to 3500 Harry 2000 to 5000

(a) To pool the two subjective intervals, use two independent Cauchy sources with center (times 1000) 2.5 and scale 1 for Bob and center 3.5 and scale 1.5 for Harry. Using 10,000 simulated values and subsetting from the histogram of Bob-Harry, I got a 50% interval of **[2.3 to 3.6]** (thousand), roughly speaking, ranging from the upper part of Bob's interval to the lower part of Harry's – as we have seen in other examples.

(b) If Bob and Harry spent a great deal of time discussing the product, then these ranges are likely to represent dependent sources rather than independent sources. As such, one would not expect to gain as much by pooling them as we have seen here. Two correlated sources are not as informative as two independent sources, and the pooled interval would be wider than that suggested in this simulation.

(2) Suppose instead of the two intervals given above, two managers report these intervals

Bob 1500 to 3500 Dick 9000 to 11000

(a) With Dick represented as a Cauchy source with center at 10 and scale 1, we get very few matched observations. Again with 10,000, I found about 200 near zero using the brush tool. The approximate interval from the matched data for Bob or Dick is **[3.4 to 9.9]** (thousand). With this small sample, the results are quite variable from one sample to the next.

(b) When you compare your pooled interval to the two intervals offered by Bob and Dick, we see that Cauchy pooling indicates that the truth is somewhere between the upper side of Bob's interval and the lower side of Dick's, but we cannot be sure of much more than that.

(c) If we pool them as normal intervals, you will not find very much matched data using the simulation technique unless you have an unreasonably large sample size. Since both are normal, you can use the regression method that we used previously. If you regress the simulated values of (Bob+Dick)/2 on Bob-Dick, you get the interval as the intercept ± 2 RMSE. The answer should be an interval with center near 6.25 and a width of about 2 times .35 – a relatively narrow interval which is incompatible with either source. Another way to see this result is to imagine that you had the data from both Bob and Dick. If you merged the two samples, you would get a smaller variance with a mean in the middle of the two.

(3) Suppose that the intervals were a mixture of Cauchy and data,

Bob 1500 to 3500 Regression 95% interval 2000 to 5000

(a) When we pool these to construct a 50% source, matching and selecting the center bin of the histogram of Bob-Regr yields about 2000 observations (out of 10000) with an interval of **[2.7 to 3.6]** thousand, much like pooling two normal sources.

(b) The lower bound of the interval has moved up a bit, with the upper limit staying about the same. Thus, this interval is somewhat shorter and shifted to larger values, reflecting the fact that a 95% normal interval from 2 to 5 is more informative than a 50% Cauchy interval of the same size – but not that much different.

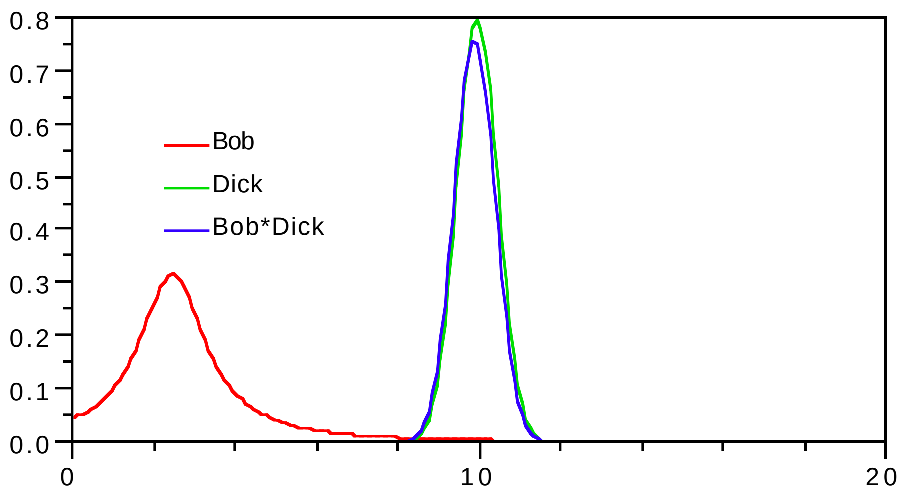
(4) As in #3, but suppose that the regression 95% interval is that from Dick in #2, [9000,11000].

(a) As in #2, there will be very little data, but in this case there will almost be too little to be useful unless you do a very large sample. Here's what I got with 20,000; about 250 were near to zero in the Bob-Regr

maximum	100.0%	12.100
	99.5%	11.957
	97.5%	11.167
	90.0%	10.906
quartile	75.0%	10.246
median	50.0%	9.840
quartile	25.0%	9.306
	10.0%	8.912
	2.5%	8.420
	0.5%	7.972
minimum	0.0%	7.905

maximum	100.0%	11.431
	99.5%	11.375
	97.5%	10.977
	90.0%	10.500
quartile	75.0%	10.244
median	50.0%	9.892
quartile	25.0%	9.574
	10.0%	9.197
	2.5%	8.946
	0.5%	8.672
minimum	0.0%	8.650

column. The results are a bit unstable since the samples are small. One 95% pooled interval is [8.4, 11.2] whereas the other is [8.9, 11]. Unlike the normal-normal problem, the simulation method works in this case since the Cauchy distribution has such larger outliers but they will be very rare. This method will not work in the case where both are normal. As you can see from these results, the interval is essentially the regression interval with little impact from the Cauchy. The density plot picture given below shows a more precise view of what happens. Basically, the normal dominates the Cauchy. This occurs because the normal is much more concentrated, whereas the Cauchy is quite diffuse (long tails). When the two compete in this fashion, the normal wins.



(b) Unlike the case in which both intervals were Cauchy, the normal dominates the Cauchy and essentially leads one to ignore the subjective Cauchy information. Not unreasonable in some sense if you believe that the normal-based regression interval and model are appropriate. The normal interval comes from real hard data whereas the Cauchy represents someone more-or-less accurate intuition. However, if you believe that the regression modeling process is itself subjective, it might not be such a good idea to ignore the subjective interval in this fashion and pool the results as Cauchy intervals.