Auctions

Administrative Things

➢ Discussion of final exam on Thursday.

• Please look over the exam before then so that we can resolve any questions and discuss what you need to do for the questions.

• Work alone on the exams, so all questions must come to me.

Today’s Topics

➢ Trees and the extensive form of games

• Role of trees in exploring alternatives

• Strategic games in extensive form

➢ Auctions

• Different types

• Subjective confidence intervals and Bonferroni

Review from Last Time

➢ Game theory

• Prisoner’s dilemma, tit-for-tat, and cooperation

• Special type of payoff matrix which is not zero-sum allows cooperation

• Simultaneous concealed moves

• Repeated encounters

• Simple strategies and predictability
Trees and Organizing Information

➢ Questionnaire on causes of car not starting

  • Complete the questionnaire on causes of your car not starting.
  
  • What is the role of having more alternatives available to you?
    
    Generally find that with more alternatives, the probability assigned to any one alternative gets smaller.
  
  • Brainstorming
    
    Free exchange of ideas is crucial when determining subjective confidence intervals.
    
    If you start judging the alternatives before the ideas are on the table, then you will explore a smaller part of the space of possibilities.
    
    Get as many scenarios out before start criticizing any of them.
  
➢ Shed load case

  • Trees are useful devices for organizing information.
    
    A better alternative to the columns of pros and cons on a sheet of paper.
  
  • Track the implications of a sequence of decisions

  • What is your interpretation of the data in this case?

➢ Trees in game theory

  • Most “games” in game theory have simultaneous moves.
  
  • Are most decisions of this type, or sequential.
    
    Even though some choices are sequential, some choices can remain hidden so that in effect the decisions are simultaneous. (e.g., allocations for research and development. You might see their budget for this year next year.)
  
  • You get a different sense of the way a game when you present it in a tree.

    Table – Normal form

    Tree – Extensive form
MAD game

<table>
<thead>
<tr>
<th></th>
<th>Column a</th>
<th>Column b</th>
</tr>
</thead>
<tbody>
<tr>
<td>Row A</td>
<td>1,2</td>
<td>1,2</td>
</tr>
<tr>
<td>Row B</td>
<td>2,1</td>
<td>0,0</td>
</tr>
</tbody>
</table>

- Arrange this table of payoffs as a tree, with row getting the first chance to move.

- Interpretation #1 First strike policy in Europe.
  Row is USSR, with “A” denoting status quo vis-à-vis US, and “B” denoting a ground invasion of Europe. Column is US, with “a” denoting no action and “b” denoting nuclear reply.

- Interpretation #2 Patent-infringement lawsuit
  Row is a small competitor of 3M, with action “A” denoting business as usual and action “B” denoting a copying of one of 3M’s patented products. Column is 3M, with “a” denoting no action and “b” denoting legal fight to the end.

- Predictable or not?
  For these games, the column player wants to be predictable. If its strategy is vague, it can be put in the position of having to take choice “b” and accept a turnabout of the payoffs.

Auctions

- Pull together several ideas we have discussed

  - Strategy
    You are bidding against others and must consider what they are likely to do.

  - Subjective confidence
    In order to bid effectively, you need to know your own range and how to use it to make effective bids. You also hope to know a bit about the intervals of others as well.
• Multiplicity

Bonferroni ideas appear when you think about who wins.

➢ Coin auction

• Nothing like having an auction to appreciate what happens.

• I am going to auction a jar of coins. All are circulating US coins.

• Rules for the auction (known as a first-price, sealed-bid auction)
  – You can inspect the jar.
  – Sealed private bids, please.
  – Jar is sold to the highest bidder.

• Type of auction typically used in public works bidding.

• Submit three pieces of information on your bid form
  (a) Your name
  (b) Your bid
  (c) Your subjective interval for the value of the coins in the jar

➢ Features of this auction

• “Common value” auction. The coins are worth the same to all of us.

  An example of an auction which does not have a common value would be an oil firm bidding for tracts that are expected to hold a particular type of crude that is of particular benefit to its refining operations (e.g., low sulfur content, sweet crude). Another would be an antique auction where certain items were of special value to a subset of bidders.

  The cost of the item being auctioned is sufficiently small so as to represent a small percentage of your total wealth – thus utility issues are not relevant here.

• The common value is unknown to all bidders.

• Bids are made independently of one another.

➢ Results of coin auction
• Winning bid and associated interval

• Do you think that the winning bid is likely to be too low or too high?

Auction Results

➢ The jar had $3.85… 13 out of the 30 intervals submitted with bids actually covered the true value of the jar.

➢ Notice that most, but not all, of the intervals include the bid value.

• Most bids were located at the middle or lower edge of the subjective confidence interval.

• Some were lower than the lower bound…
**Winner’s Curse**

- Often by winning an auction of this type, you have overbid and will pay too much.

- You’re not alone
  - Bidding for the services of professional athletes
  - Bidding for a construction or service contract
  - Buying a house
  - FCC auctions for airwaves and the subsequent bankruptcy of several winning bidders.

  Australian story illustrates what happens when the rules of the auction are vague. One group submitted as collection of bids, going up to quite large values guaranteed to win. As the bids were revealed, they would seek a partner. If none was found, the next bid was opened, etc. The process continued until someone else decided to join them (i.e., they sold the rights). *Rules in an auction are important!*

- So why does it happen?
  - Is it just a part of the nature of the bidding process?
  - Competitive nature of people
    
    We have seen in the case of prisoner’s dilemma when two will compete with each other rather than cooperate?
  - Auction a $20 bill.
    
    - Rules
      
      (1) Both the top bidder and the second largest bidder must pay.
      
      (2) Only the top bidder receives the $20 bill.
Understanding Winner’s Curse

- Notation for discussion
  - \( n \) bidders in a sealed bid, first price auction
  - \( V \)  True value of object, assumed unknown to the bidders
  - \( V_i \)  Valuation of the object by person \( i \). It’s the estimate of \( V \) for person \( i \).

- Common-value auction in this notation, from this perspective
  - We have a sample of \( n \) random variables denoted \( V_i \) representing the bids of the participants in the auction.
  - \( V_i \) all have the same mean value, namely the unknown true value \( V \).
  - \( V_i \) all have the same variance, say \( \sigma^2 \).
  - \( V_i \) are independent.
  - These assumptions thus not only presume independence, but in effect equal information about \( V \) among the bidders.

- Winning bid from this perspective is
  \[
  \max V_i
  \]

- Draw the histogram and you will have a clear idea of why winner’s curse happens in this setting.

Adjusting for Winner’s Curse – Bidding with No Regrets

- What are the properties of the high bid in the context just described?
  - Normality
  - If the \( V_i \sim N(V, \sigma^2) \), then the approximate size of the max was discussed in the context of Bonferroni, namely the mean plus a multiple of the SD
  \[
  \max V_i \approx V + \sigma \sqrt{2 \log n}
  \]
where \( n \) is the number of bidders in the auction.

- Draw the picture of the distribution of valuations and look at where the maximum is located.

**Implications for a bidding strategy**

- **Principle of bidding in an auction**
  
  Set your bid so that you are not sorry if you win.

- That is, treat your bid as though you have the maximum valuation and shift your bid down to adjust for the winner’s curse effect.

**How much to shift your bid?**

- If you knew \( \sigma \), then you could simply shift your bid down by the factor seen above, namely shift your bid to \( V_i - \sigma \sqrt{2 \log n} \).

- If you could interpret your subjective interval as a 95% statistical interval for \( V \), then the length of your interval would be \( L = 4\sigma \), so that your bid would be \( V_i - L/4 \sqrt{2 \log n} \).

- Note that this is typically quite a bit smaller for a bid than the one suggested by just shifting your bid to the left side of your interval, using \( V_i - L/2 \) as your bid.

**Cauchy uncertainty**

- If you model your uncertainty as Cauchy, the chance for being “clueless” and far from the truth is quite high, so it requires a much, much larger endpoint adjustment, namely to \( V_i - L/2 \frac{n}{\pi} \).

**General idea of these adjustments.**

- Given a model of the variation, estimate how the value of the maximum of a sample of \( n \) of them grows as a function of \( n \). This is the “easy” part.

- The hard part is to know the nature of the variation and the level of uncertainty in other bidders.
Further Discussion

➢ Key factors in the adjustments

• Scale (either $\sigma$ or the Cauchy length factor)
• Number of bidders

➢ Crucial question: What value to use for the scale and $n$?

• Are all of the bidders as uncertain as you are, or do some have less information or more information?
• That is, what is the $\sigma$ term for the other bidders?

➢ If you are among the informed bidders

• Say, you have access to an expert in van Gogh paintings
• You need to set $n$ to be the number of other equally well informed bidders.

➢ If you are among the uninformed bidders

• If they are all bidding properly, you have no real chance to win.
• If you do “win”, you have likely overpaid.

➢ Bidding against experts

• You never want to be in an auction against others who know more about the value (i.e., smaller $\sigma$ terms) than you.
• If you win against experts, you have likely paid too much and failed to appreciate your own uncertainty, not shifting left enough.
• That is, those that know more require a smaller shift to accommodate their uncertainty. There’s no way for you to win unless you have estimated the value incorrectly.

➢ Auctions from the seller’s point of view

• Winner’s curse phenomenon suggests that sellers may earn more than the value of the sold item.
• Seller may not be sure of the actual value of the item, and so attempt to learn this value via an auction.
- auction of an estate
- soliciting construction bids for home repairs

• Sellers like more bidders (larger n).

Recall the recent auction of the Kennedy jewelry.

• Sellers like bidders who fail to adjust for the winner’s curse problem since they increase the variation and on average make the maximum bid much larger.

• Sellers may want to

  - restrict the information to keep bidders in the dark if the bidders are believed to not know this effect
  - release more information to skilled bidders since then σ is smaller, and the adjustment becomes smaller as well, resulting in higher bids. Reduce the risk aversion of these skilled bidders.

Evaluations

➢ Please take a moment for written comments

➢ Suggestions for future versions of this course
  • Other courses that provide useful synergy?
  • Other topics to cover? Topics to remove?
  • What did you like about the course? What you did not like?
  • What would you change for the next time?

School’s out for summer…