Exercise for Homework #3

Consider fitting a sequence of autoregressions of increasing order p to a time series x_1, x_2, ..., x_n. Define the deviations for a given order as

$$w_{pt} = x_t - \phi_{p1} x_{t-1} - \cdots \phi_{pp} x_{t-p} \ .$$

The coefficients ϕ_{pj} are the solutions to the Yule-Walker equations,

$$\Gamma_p \ \boldsymbol{\phi}_p = \gamma_1^p, \quad \boldsymbol{\phi}_p = (\phi_{p1}, \phi_{p2}, \ldots, \phi_{pp})' \ ,$$

and $\gamma(j)$ are the covariances of the underlying process. (Since the process that generated x_1^n may not be AR(p) for any p, these are more like residuals, though we have not estimated ϕ_j.) Similarly, define the "reverse residuals" obtained by running the indices backwards:

$$\tilde{w}_{pt} = x_t - \phi_{p1} x_{t+1} - \cdots \phi_{pp} x_{t+p} \ .$$

(a) Show that the residuals at step $p+1$ are the following combination of those in the forward direction with those in the backward direction at step p:

$$w_{p+1,t} = w_{p,t} - \phi_{p+1,p+1} \tilde{w}_{p,t-p} \qquad (1)$$

(b) Interpret the result (1) in the context of a partial regression (*i.e.*, the characterization of multiple regression as the regression of residuals having removed the effects of other explanatory variables).

This approach is used to implement another method for estimating autoregressions known as Burg's method or the forward-backward method.