Appendix

```
(* * Calculation of  the  projective  scores *
   * and their informations for the circular *
   *           normal problem of Chapter 6   * *)

SetAttributes[rho,Constant]

(* Define a recursive function f[.], such that f[0] returns
   U_0, and f[t] returns the V_t. The replacement operator /.
   uses the identities given on p. 146.    *)

f[0] = U0[lambda]
f[1] = V[lambda]
f[2] = - rho (U0[lambda] + rho)   + f[1]^2
f[x_] := f[x] = Simplify[
                        D[f[x-2],{lambda,2}] +
                        2 D[f[x-2],lambda] f[1] +
                        f[x-2] D[f[1],lambda] + f[x-2] f[1]^2 /.
                        {V'[lambda] -> -rho (U0[lambda] + rho),
                         U0'[lambda] -> 1/rho V[lambda],
                         V''[lambda] -> - V[lambda],
                         U0''[lambda] -> - (U0[lambda] + rho)}
                       ]

(* The list ``simp'', is a list of replacement rules
   that evaluate the expectations of the products
   of U0 and V_1. It contains the formulae
   for the k^{th} moment of a central normal distribution. *)

simp = Join[
            Table[V[lambda]^(2 i) -> (2 i)!/(i! 2^i) rho^(2 i),
                  {i,14,1,-1}],
            Flatten[
                    Table[{U0[lambda]^i -> i!/((i/2)! 2^(i/2)),
                    U0[lambda]^(i-1) -> 0},{i,14,2,-2}]]]

(* The function m[i,j] is the (i,j) element of the
   extended information matrix. Note that it is
   only calculated for EVEN values of V_t. The
   {5,5} will take it up to the 10^{th} Bhattacharyya
   basis function. *)
```

```
Array[m[i_,j_] := Expand[f[2 (i-1)] f[2 (j-1)]] /. simp ,{5,5}]

(* i22 calculates returns the information in the
   U2, U4, U6 and U8 projective scores. *)

i22 = Table[
            m[1,1] -
            Table[m[1,i],{i,2,k}] .
            Inverse[Table[m[i,j],{i,2,k},{j,2,k}]] .
            Table[m[1,i],{i,2,k}],
            {k,2,5}]

(* The next statement outputs the information to
   a file, data.data, and is what was used to form the
   basis of Figure 6.2. *)

(*
Table[i22 /. rho -> i,{i,0.05,3.05,.05}] >>> data.data
*)

(* Uproj[k] calculates the k^{th} projective score,
   defined for k greater than or equal to  2,
   Uproj[k] returns the 2(k-1)^{th} projective score. *)

Uproj[k_] := Simplify[(U0[lambda] -
                       Table[m[1,j],{j,2,k}] .
                       Inverse[Table[m[i,j],{i,2,k},{j,2,k}]] .
                       Table[f[2 i],{i,1,k-1}]) ]

(* Uhat[k], calculates the k^{th} projective score, evaluated
   at the mle for the nuisance parameter lambda,
   defined for k greater than or equal to  2,
   Uhat[k] returns the 2(k-1)^{th} projective score
   evaluated at \hat\lambda_\rho. *)

Uhat[k_] := Simplify[(U0[lambda] -
                      Table[m[1,j],{j,2,k}] .
                      Inverse[Table[m[i,j],{i,2,k},{j,2,k}]] .
                      Table[f[2 i],{i,1,k-1}]) /.
                      {U0[lambda]->(r - rho),V[lambda]->0}]
```

VITA

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