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PROJECTIVE SCORE METHODS

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by

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Abstract

Nuisance parameter problems receive special attention because the classical optimality theory of maximum likelihood estimation can break down. Neyman and Scott (1948) showed that the maximum likelihood estimator for the parameter of interest need not be consistent, and even if consistent it may still be inefficient. This thesis provides a partial repair to the theory of maximum likelihood in nuisance parameter problems, in the context of what is termed a "square array" asymptotic embedding. The square array is a framework in which the number of nuisance parameters is equal to the number of observations within each stratum of the array.

The adjustment to maximum likelihood estimation is achieved by modifying the score equation for the parameter of interest, by removing from it, its projection onto a sequence of subspaces of mean zero estimating equations. These modified score equations are referred to as the *projective scores*. The motivation for using the particular subspaces comes from the theory of Locally Minimum Variance Unbiased Estimation.

It is shown that inefficiency of maximum likelihood estimates in Neyman-Scott square array problems can be attributed to a limiting bias. The projective scores remove this bias and hence retain full efficiency.

In a wider context, the projective scores can be used to approximate an optimal but possibly intractable conditional or efficient score. Some examples are presented.

The projection ideas have received recent attention from Small and McLeish (1988). The work in this thesis extends their results. One generalization is the consideration of a multivariate parameter of interest. The construction of the projective

scores is considered in detail, and the properties of a class of matrices, termed *permutation invariant*, are used to obtain the necessary simplifications that allow the theory to be implemented in realistic problems. A class of generalized linear models is investigated and the projective scores are used to obtain an approximate conditional inference.

Interpretation of the results is provided by certain model curvatures. The curvature aspect provides a link to work in the area of second order inference and some comparisons are made to Efron (1985).

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